The complexity of star-forming filaments

OABO, Bologna, 29/05/2014

Simone Recchi IfA, Vienna

Alvaro Hacar, Arsen Palestini

Image credit: ESO/APEX (MPIfR/ESO/OSO)/A. Hacar et al./Digitized Sky Survey 2. Acknowledgment: Davide De Martin.

Outline

- **Setting the scene**
- **Non-isothermal filaments**
- **Rotating filaments**
- **Non-isolated filaments**
- **Conclusions and outlook**

- **Setting the scene**
- **Non-isothermal filaments**
- **Rotating filaments**
- **Non-isolated filaments**
- **Conclusions and outlook**

THE EQUILIBRIUM OF POLYTROPIC AND ISOTHERMAL CYLINDERS

J. Ostriker*
Yerkes Observatory
Received April 21, 1964

ABSTRACT

Infinite self-gravitating cylinders with pressure and density related by the equation $P =$ constant $\rho^{1+1/n}$ are considered; the purpose is to find the equilibrium distributions of pressure, density, and gravitational potential. Solutions in closed form are obtained for $n = 0$ (liquid), $n = 1$, and $n = \infty$ (isothermal perfect gas). It is proved that for all $0 \leq n < \infty$ the mass per unit length and the radius are finite; for $n = \infty$ the mass per unit length is finite, but the radius is not. Graphical and tabular material is included showing the run of density with radius, and the variation of radius, mass per unit length, and half-radius (radius within which half the mass is contained) as a function of polytropic index n.

I. INTRODUCTION

Gaseous rings occur in a variety of astronomical contexts. Previous workers (cf., in particular, Randers 1942) have pointed out that infinite cylinders provide the first term of a natural series expansion in which one may develop the theory of the equilibrium of such rings. However, in their detailed considerations, Randers and his predecessors restricted themselves to incompressible cylinders and rings. For a theory of gaseous rings, under the less restrictive assumption of polytropic or isothermal equilibrium, the theory of self-gravitating cylinders, with an underlying equation of state

$$P \propto \rho^{\mu}$$

provides a natural starting point. In this paper the equilibrium theory of such cylinders is developed; the extension to rings is considered in a following paper. Knowledge of polytropic and isothermal cylinders may also prove useful in other applications, e.g., in the study of gaseous filaments or of spiral arms, where it would provide less idealized models than some which have been considered in the past.

On the formal side, the theory follows the classical expositions on polytropic and isothermal gas spheres by Emden (1907) and Chandrasekhar (1938); on this account the theory need not be set out in too great detail.

II. POLYTROPIC CYLINDERS

a) *The Equations of Equilibrium*

The equation of hydrostatic equilibrium is

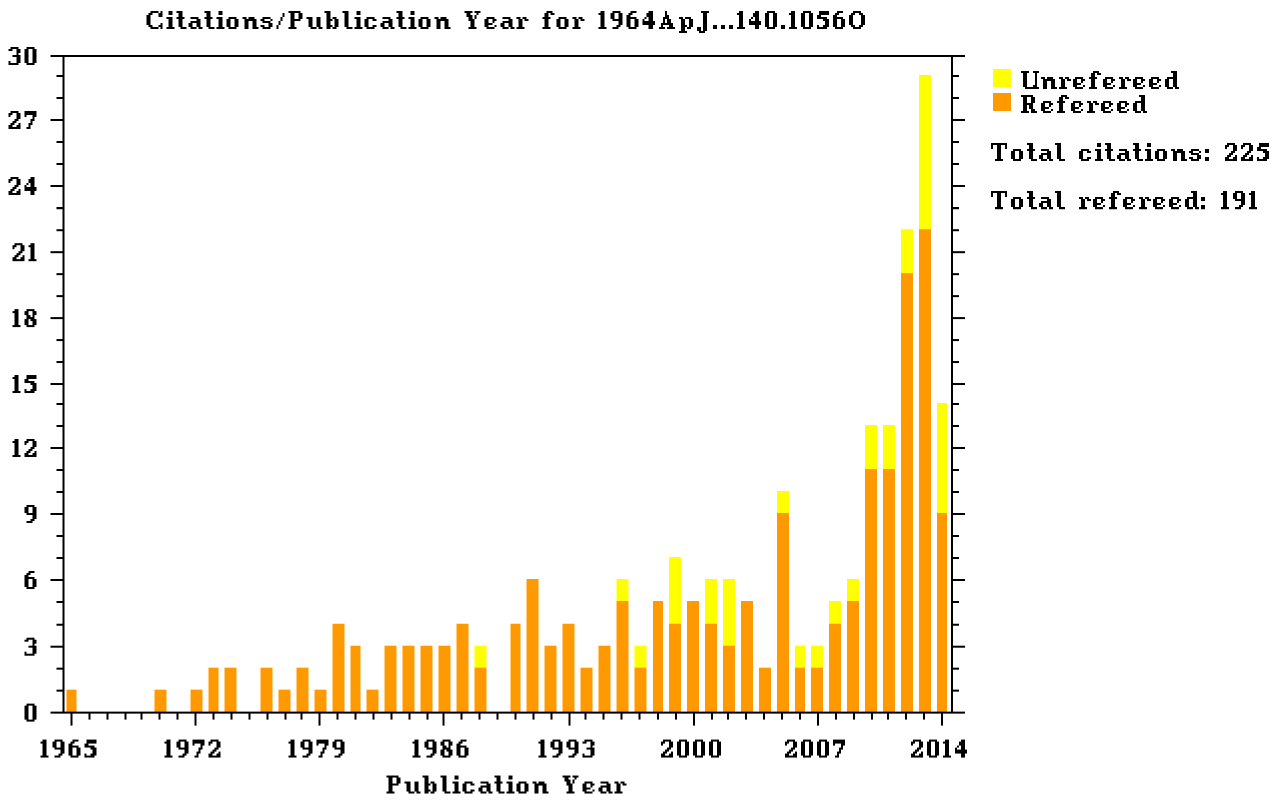
$$\nabla \mathfrak{V}^{i} = \frac{1}{\rho} \nabla P, \tag{1}$$

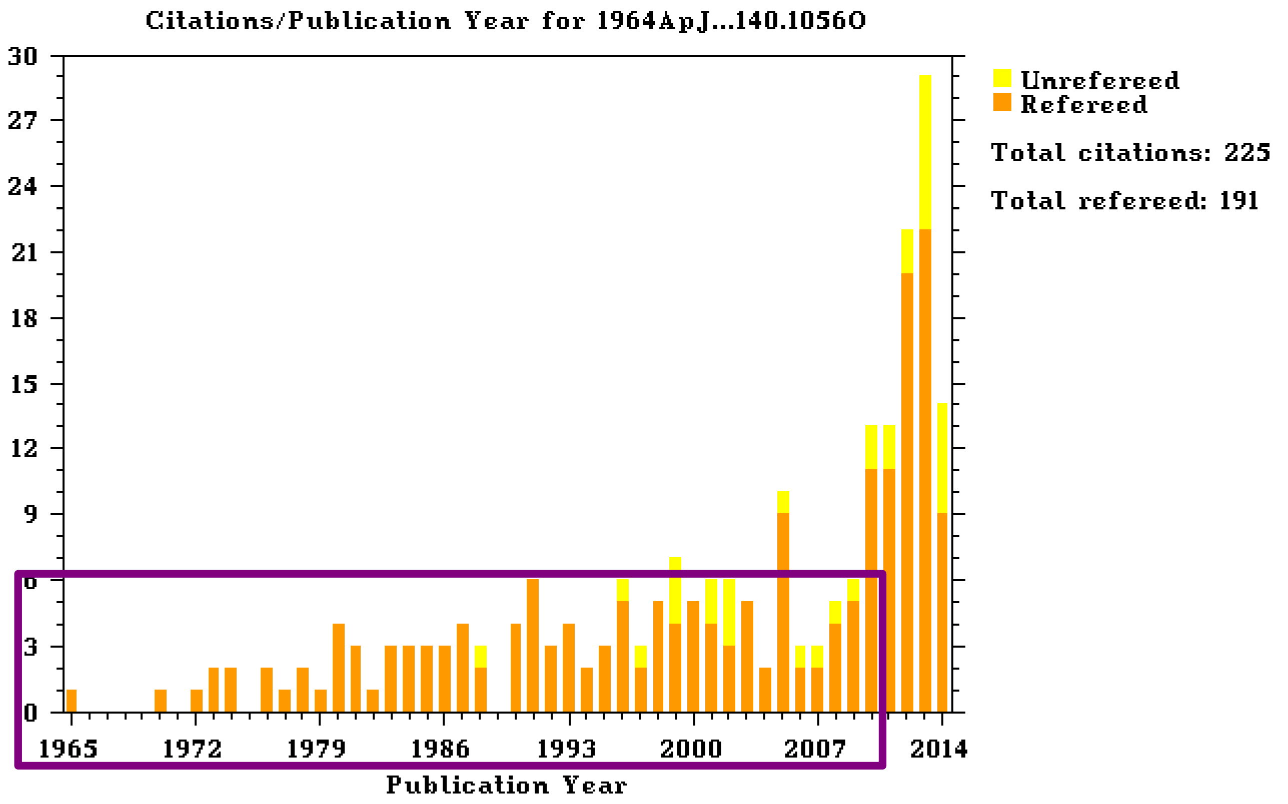
where $\mathfrak{V}^{i}$ is the gravitational potential within the cylinder and P and ρ are the pressure and the density. As stated in the Introduction, we shall suppose that

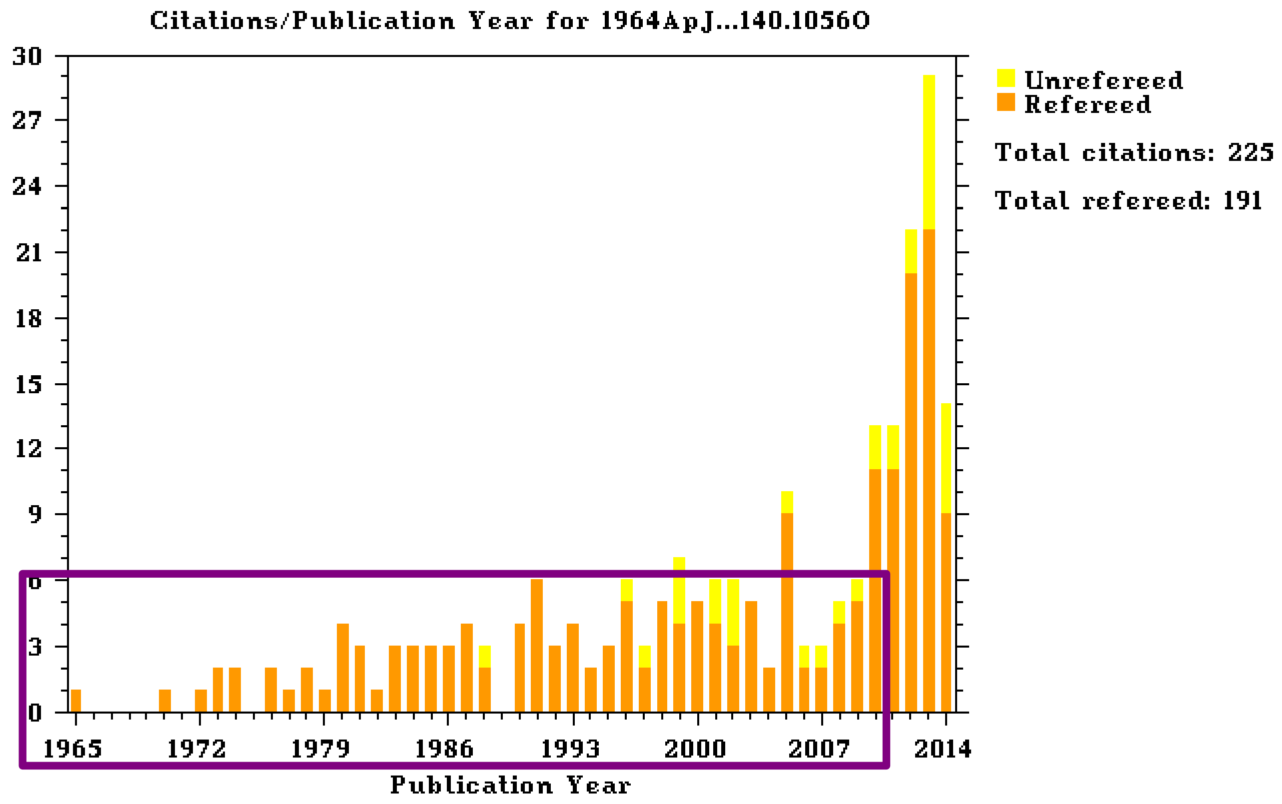
$$P = K_n \rho^{1+1/n} \tag{2}$$

were K_n and n are given constants. Such a relation can represent a broad spectrum of different possible conditions: $n = 0$ represents a homogenous liquid, $n = \frac{3}{2}$ a monatomic

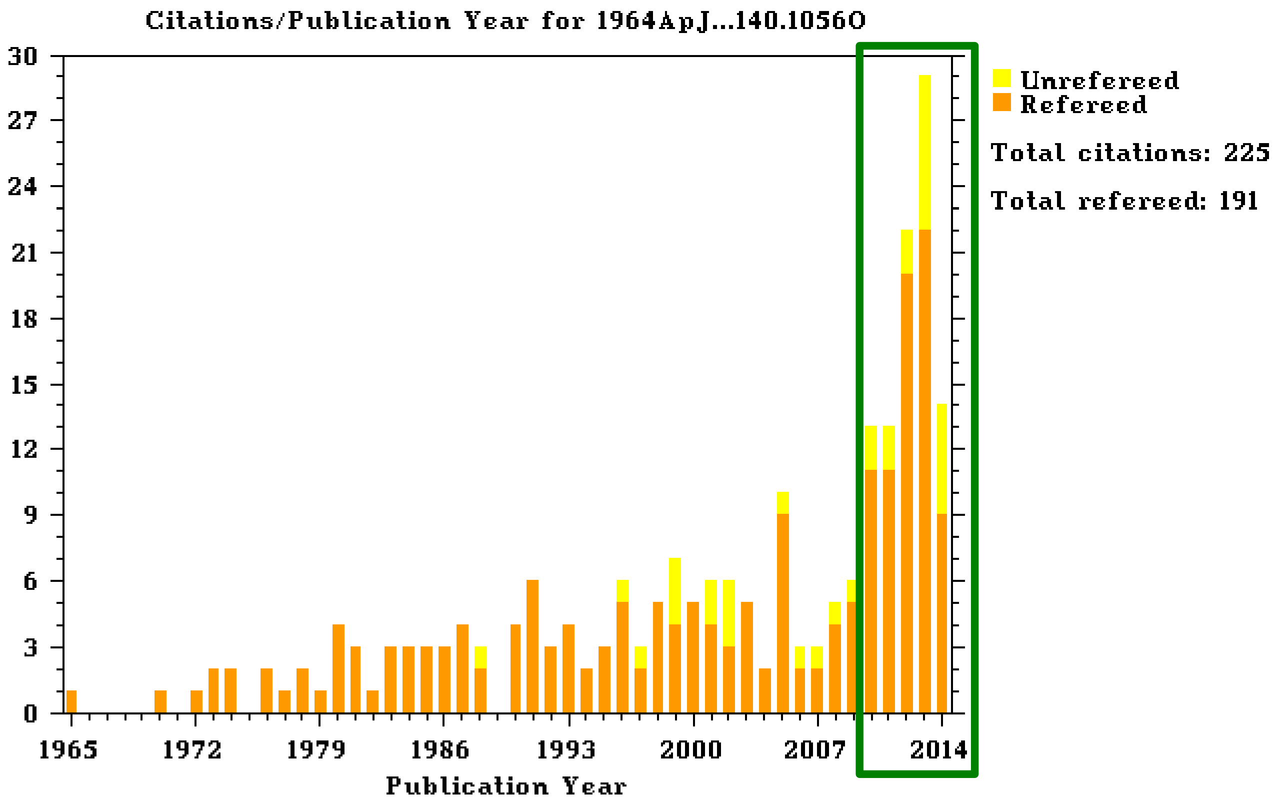
* National Science Foundation graduate fellow 1963, 1964.


Citations/Publication Year for 1964ApJ...140.1056O
30
27
24
21
18
15
12
9
6
3
0
1965
1972
1979
1986
1993
2000
2007
2014
Publication Year
Unrefereed
Refereed
Total citations: 225
Total refereed: 191


Citations/Publication Year for 1964ApJ...140.1056O
Unrefereed
Refereed
Total citations: 225
Total refereed: 191
30
27
24
21
18
15
12
9
6
3
0
1965
1972
1979
1986
1993
2000
2007
2014
Publication Year



Gaseous rings occur in a variety of astronomical contexts. Previous workers (cf., in particular, Randers 1942) have pointed out that infinite cylinders provide the first term of a natural series expansion in which one may develop the theory of the equilibrium of such rings.



Citations/Publication Year for 1964ApJ...140.1056O
30
27
24
21
18
15
12
9
6
3
0
1965
1972
1979
1986
1993
2000
2007
2014
Publication Year
Unrefereed
Refereed
Total citations: 225
Total refereed: 191

A&A 518, L102 (2010)
DOI: 10.1051/0004-6361/201014666

Astronomy & Astrophysics
Special feature

Herschel: the first science highlights

LETTER TO THE EDITOR

From filamentary clouds to prestellar cores to the stellar IMF: Initial highlights from the *Herschel*★ Gould Belt Survey★★

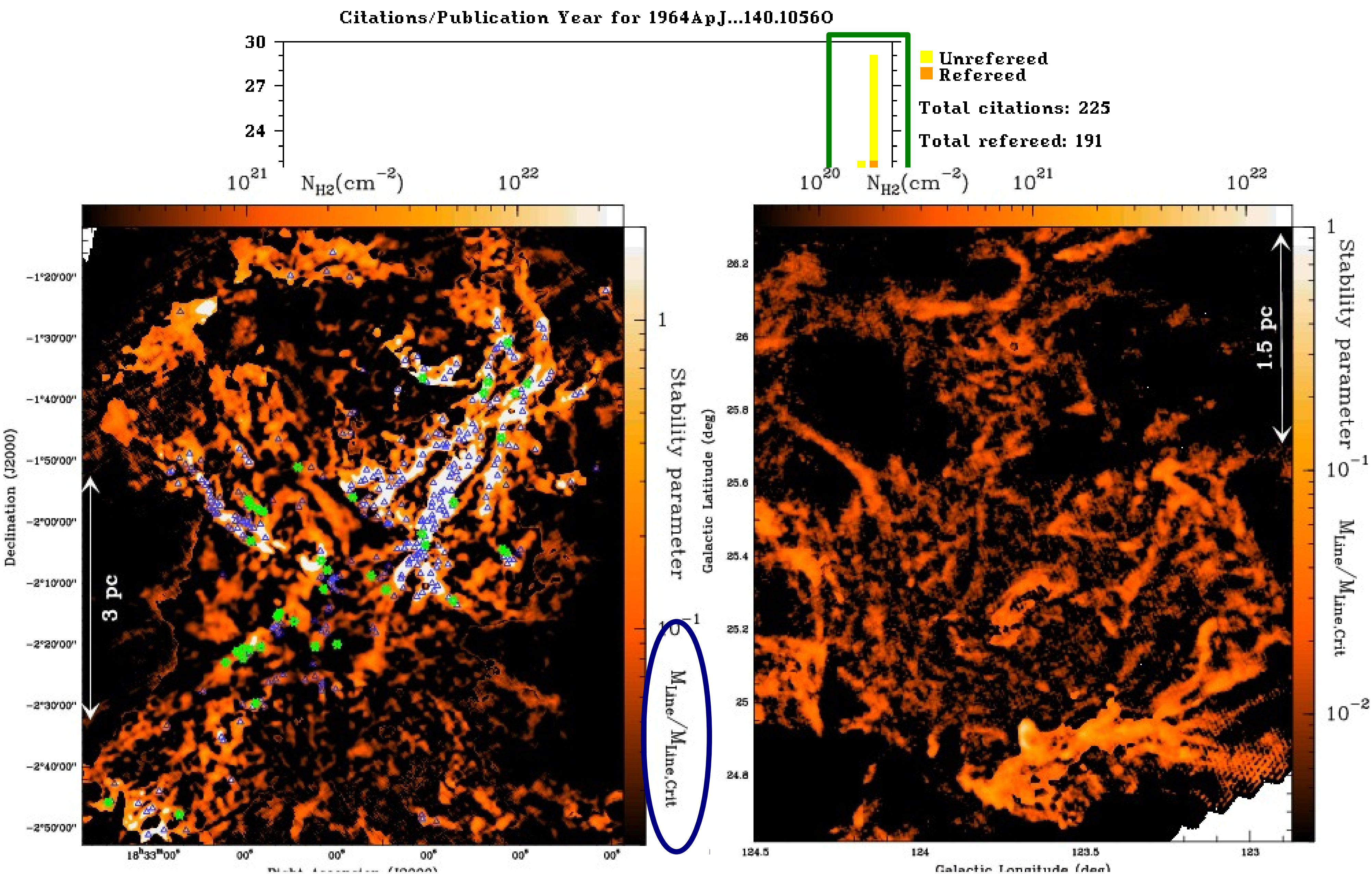
Ph. André[1], A. Men'shchikov[1], S. Bontemps[1], V. Könyves[1], F. Motte[1], N. Schneider[1], P. Didelon[1], V. Minier[1], P. Saraceno[5], D. Ward-Thompson[3], J. Di Francesco[10], G. White[18,22], S. Molinari[5], L. Testi[17], A. Abergel[2], M. Griffin[3], Th. Henning[11], P. Royer[7], B. Merín[13], R. Vavrek[13], M. Attard[1], D. Arzoumanian[1], C. D. Wilson[19], P. Ade[3], H. Aussel[1], J.-P. Baluteau[4], M. Benedettini[5], J.-Ph. Bernard[6], J. A. D. L. Blommaert[7], L. Cambrésy[8], P. Cox[9], A. Di Giorgio[5], P. Hargrave[3], M. Hennemann[1], M. Huang[12], J. Kirk[3], O. Krause[11], R. Launhardt[11], S. Leeks[18], J. Le Pennec[1], J. Z. Li[12], P. G. Martin[14], A. Maury[1], G. Olofsson[15], A. Omont[16], N. Peretto[1], S. Pezzuto[5], T. Prusti[21], H. Roussel[16], D. Russeil[4], M. Sauvage[1], B. Sibthorpe[20], A. Sicilia-Aguilar[11], L. Spinoglio[5], C. Waelkens[7], A. Woodcraft[20], and A. Zavagno[4]

(Affiliations are available in the online edition)

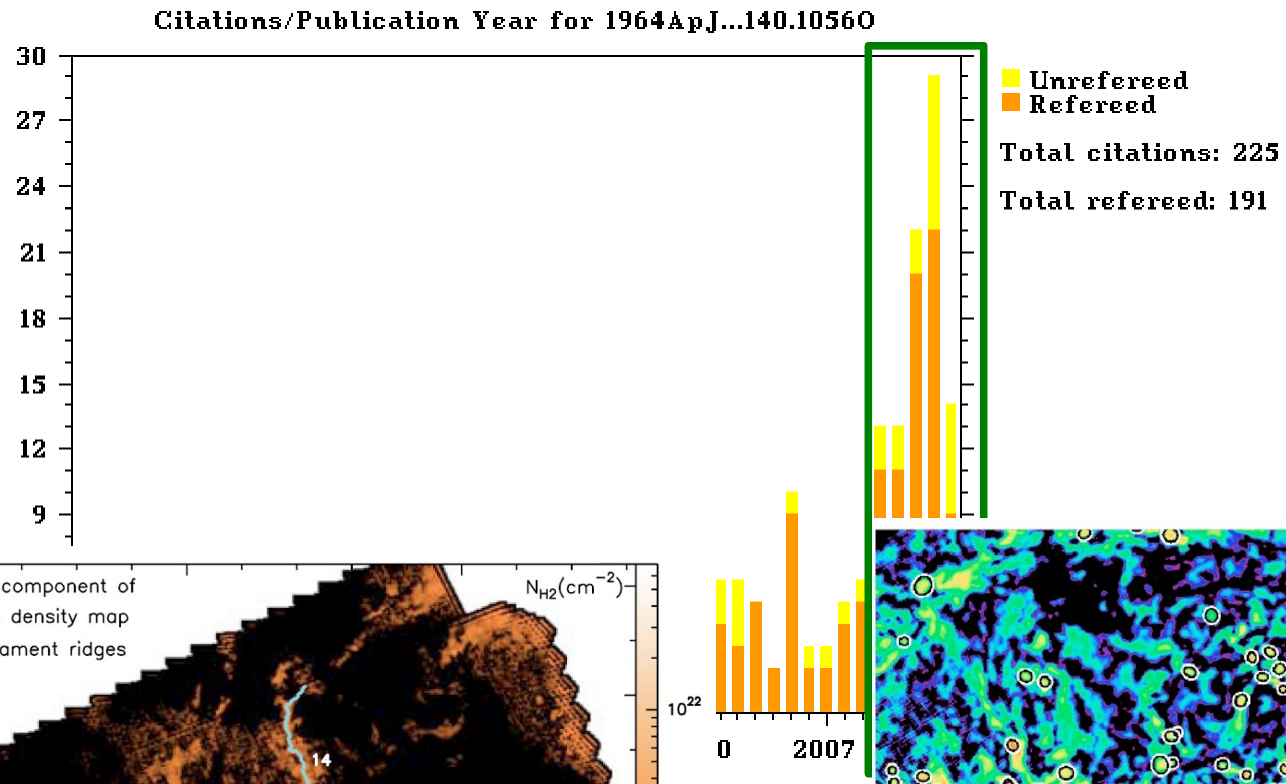
Received 31 March 2010 / Accepted 4 May 2010

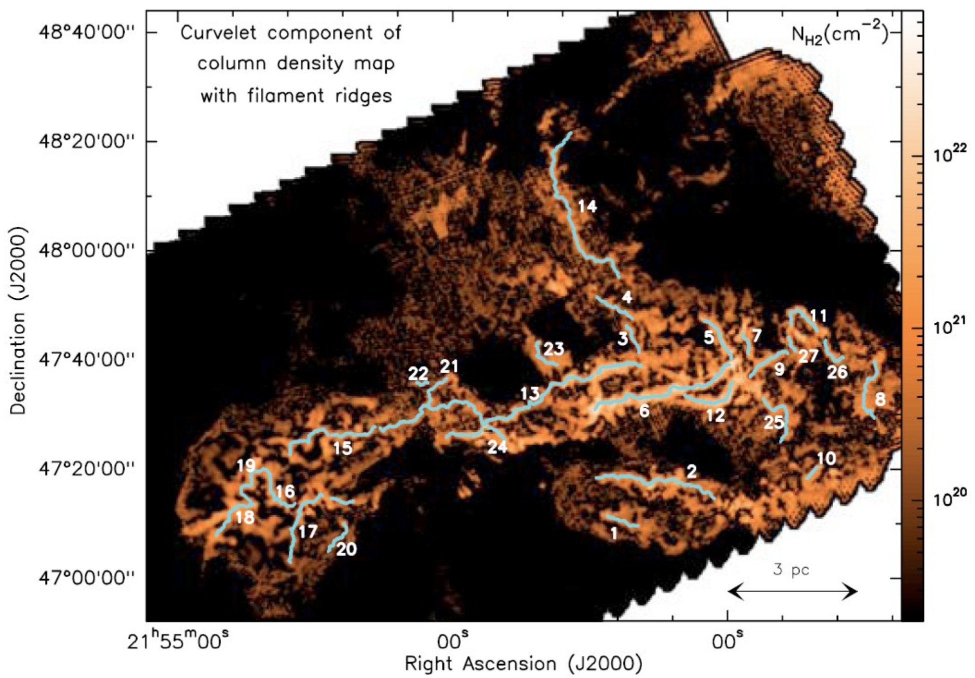
ABSTRACT

We summarize the first results from the Gould Belt Survey, obtained toward the Aquila rift and Polaris Flare regions during the science demonstration phase of *Herschel*. Our 70–500 μm images taken in parallel mode with the SPIRE and PACS cameras reveal a wealth of filamentary structure, as well as numerous dense cores embedded in the filaments. Between ~350 and 500 prestellar cores and ~45–60 Class 0 protostars can be identified in the Aquila field, while ~300 unbound starless cores and no protostars are observed in the Polaris field. The prestellar core mass function (CMF) derived for the Aquila region bears a strong resemblance to the stellar initial mass function (IMF), already confirming the close connection between the CMF and the IMF with much better statistics than earlier studies. Comparing and contrasting our *Herschel* results in Aquila and Polaris, we propose an observationally-driven scenario for core formation according to which complex networks of long, thin filaments form first within molecular clouds, and then the densest filaments fragment into a number of prestellar cores via gravitational instability.

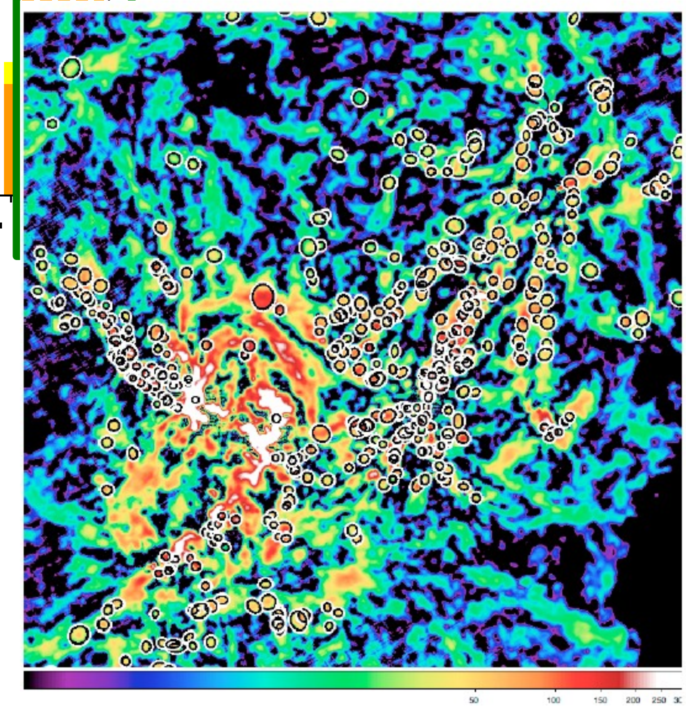


Citations/Publication Year for 1964ApJ...140.1056O
30
27
24
Unrefereed
Refereed
Total citations: 225
Total refereed: 191
10^{21} $N_{H2}(cm^{-2})$ 10^{22}
10^{20} $N_{H2}(cm^{-2})$ 10^{21} 10^{22}
Stability parameter
$M_{Line}/M_{Line,Crit}$
1
10^{-1}
10^{-2}
3 pc
1.5 pc
Declination (J2000)
Galactic Latitude (deg)
Galactic Longitude (deg)
124.5
124
123.5
123
26.2
26
25.8
25.6
25.4
25.2
25
24.8





Azroumainian et. al. 2011



Men’shchikov et. al. 2011

Setting the scene

Observations of filaments are often interpreted on the light of the classical model of Ostriker (1964). Working hypotheses:

- Cylinders are isothermal
- Cylinders are non-rotating
- Support against gravity comes solely from thermal pressure
- Cylinders are isolated
- Filaments are cylinders of circular section and infinite length

Based on these assumption, Ostriker deduced that the radial density profile of equilibrium filaments is given by:

$$\rho = \frac{\rho_c}{\left[1 + \left(r/H\right)^2\right]^2} \qquad H = \sqrt{\frac{2\,c_s^{\,2}}{\pi\, G\, \rho_c}}$$

The linear mass corresponding to this profile is ~1.66*T $M_{\odot}$/pc.

Setting the scene

Based on this theoretical results, astronomers usually assume that

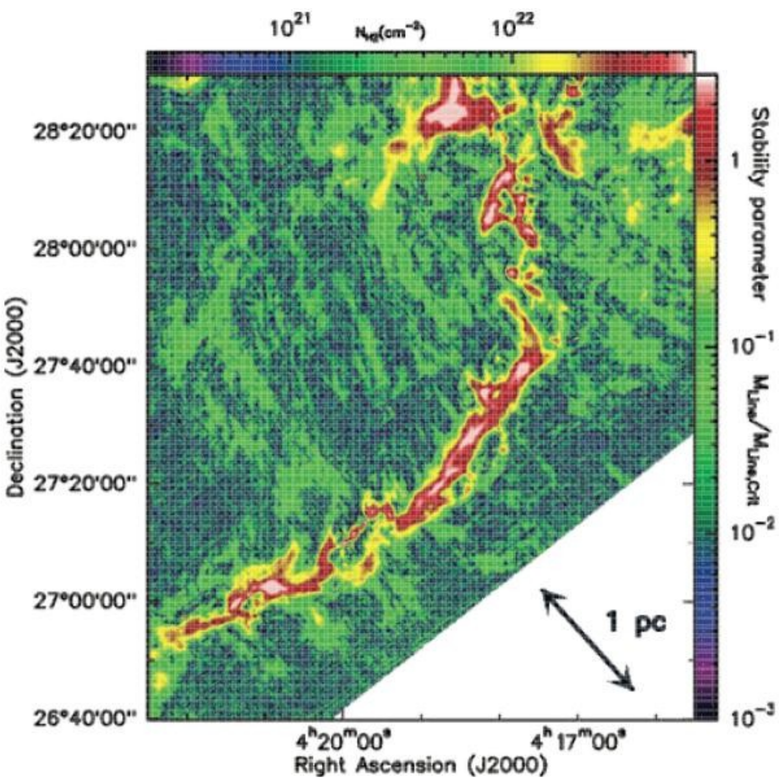
• Filaments are stable only if their linear mass does not exceed ~ 16.6 $M_{\odot}$/pc. (Here, a temperature of 10 K is assumed). This is the $M_{\mathrm{Line,\ Crit}}$ of Andre' et al.'s paper. More massive filaments are assumed to be unstable and thus prone to star formation

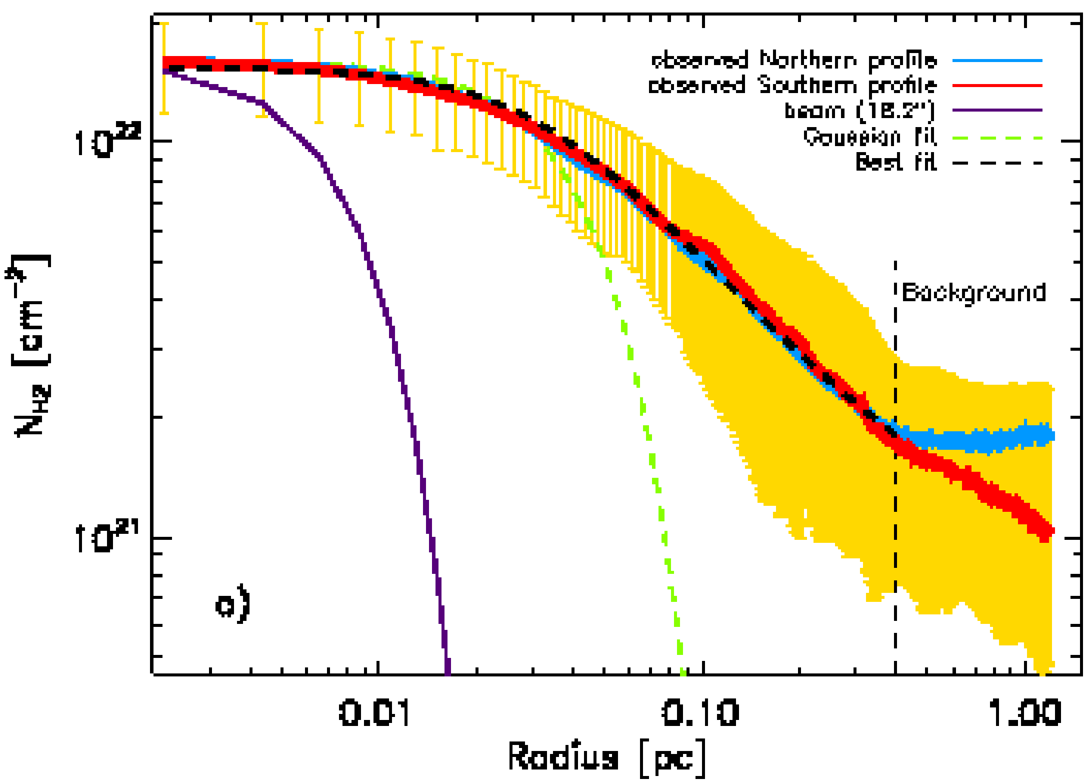
• The profile at large radii of a stable filament should decrease as $\rho \sim r^{-4}$. A flatter profile suggests that the filament is in a phase of collapse.

Setting the scene

Example: B211/3

- It has a linear mass of ~ 54 $M_{\odot}$/pc.
- The density decreases as $\rho \sim r^{-2}$.
- Author's conclusion: the filament is collapsing

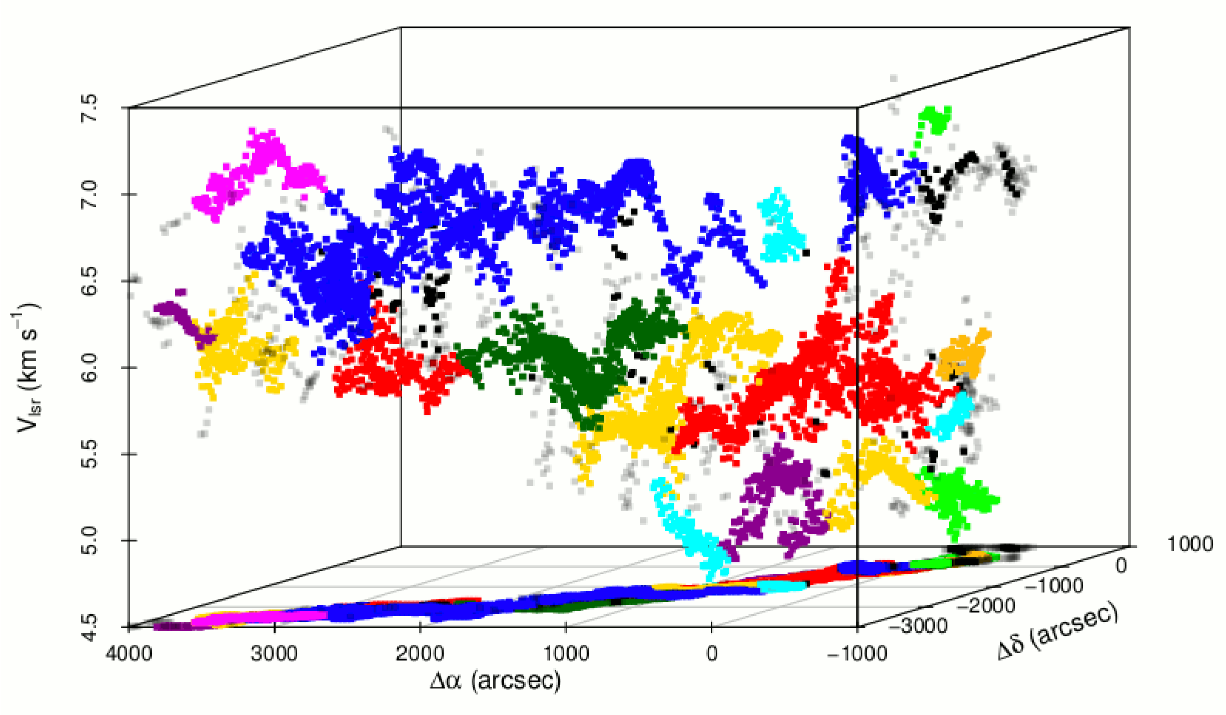




Palmeirim et. al. 2013

Setting the scene

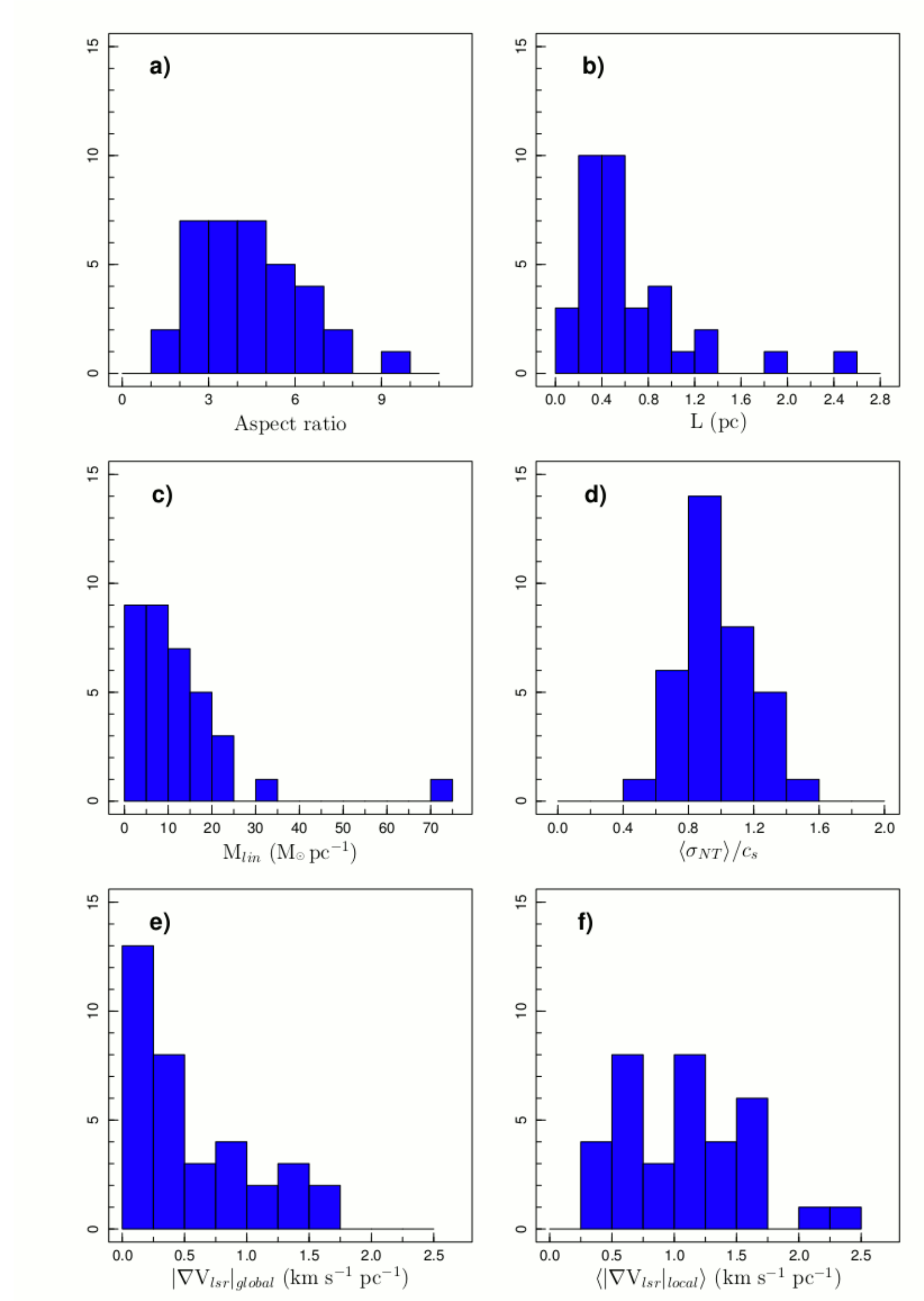
...but note that this filament is indeed composed of many bundled sub-filaments, with their own physical properties

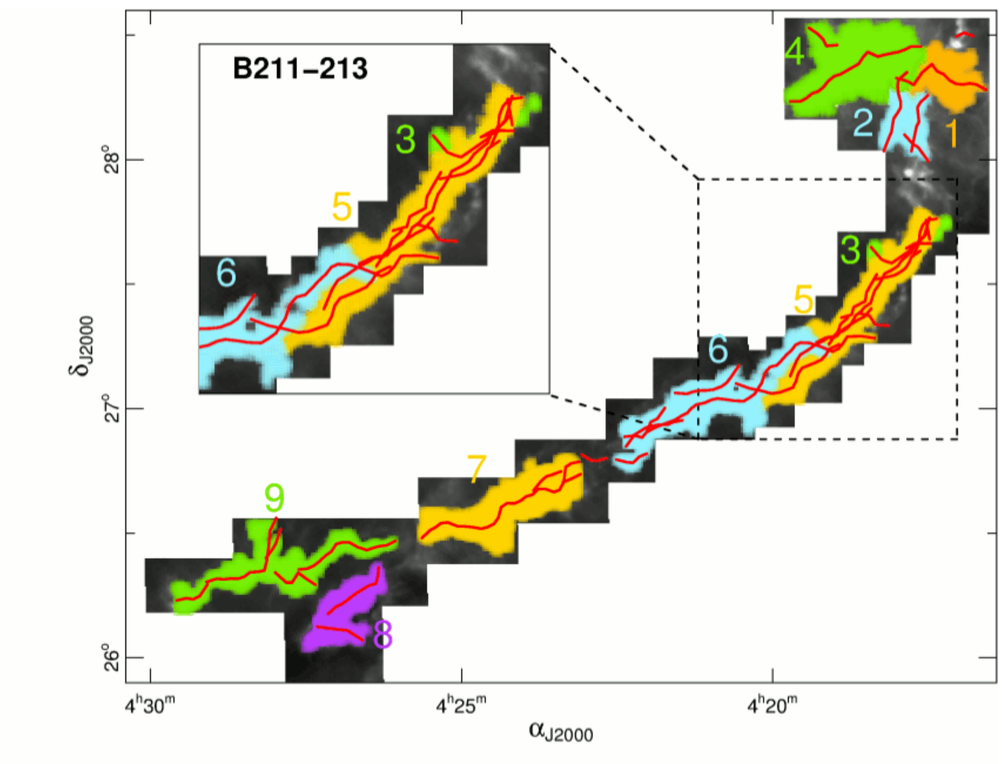


Hacar et. al. 2013

Setting the scene

...but note that this filament is indeed composed of many bundled sub-filaments, with their own physical properties



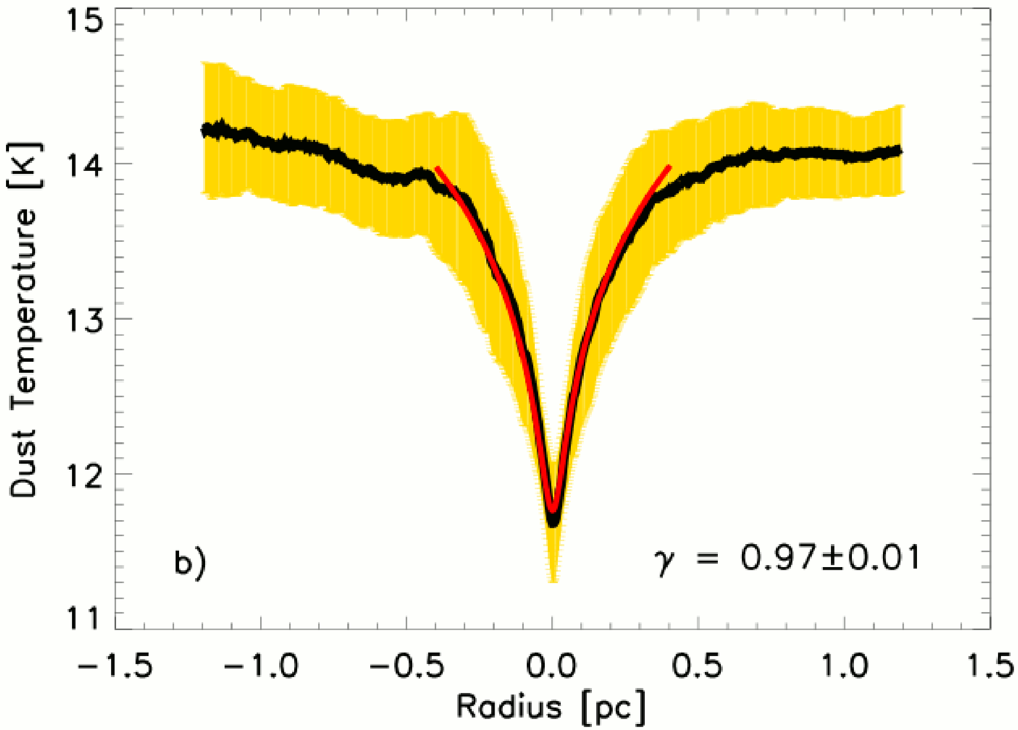


Hacar et. al. 2013

Setting the scene

- **Cylinders are isothermal**
- Cylinders are non-rotating
- Support against gravity comes solely from thermal pressure
- Cylinders are isolated
- Filaments are cylinders of circular section and infinite length

Or are they?

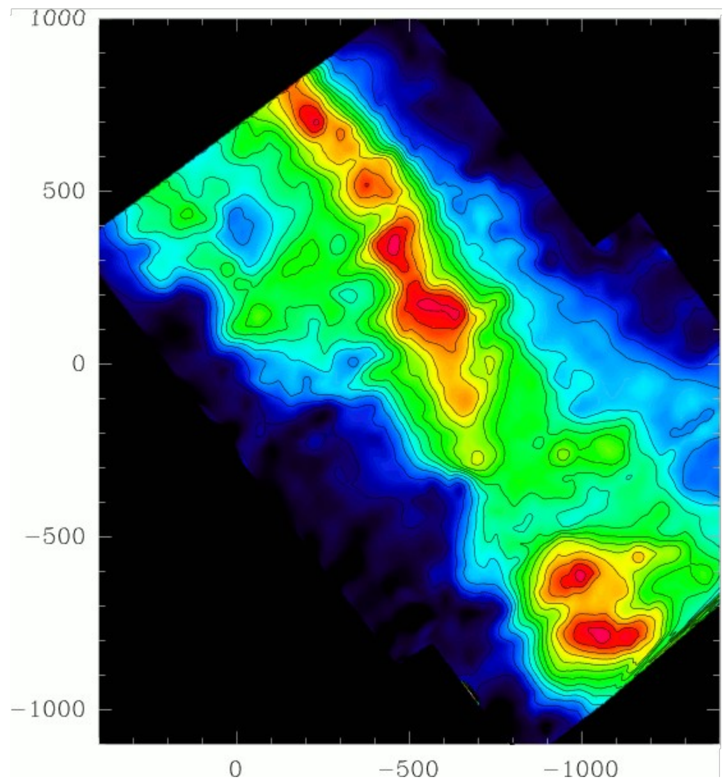


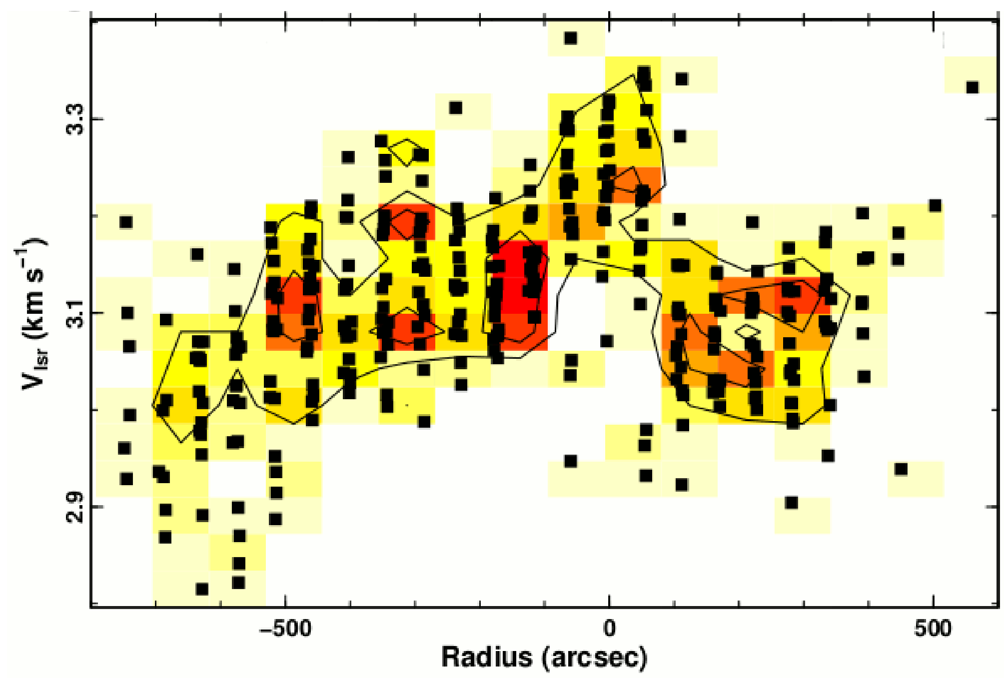
Palmeirim et. al. 2013

Setting the scene

- Cylinders are isothermal
- **Cylinders are non-rotating**
- Support against gravity comes solely from thermal pressure
- Cylinders are isolated
- Filaments are cylinders of circular section and infinite length

Or aren't they?



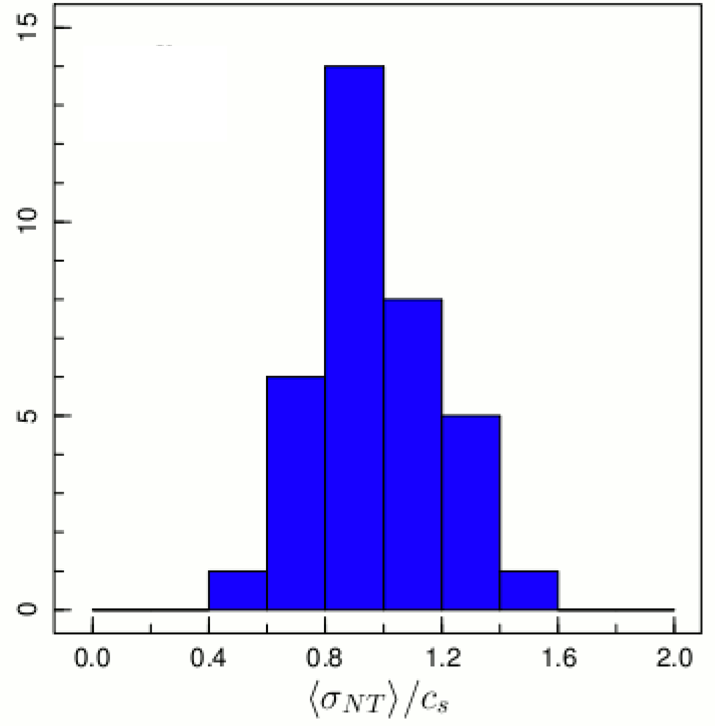


Hacar et. al. in prep.

Setting the scene

- Cylinders are isothermal
- Cylinders are non-rotating
- **Support against gravity comes solely from thermal pressure**
- Cylinders are isolated
- Filaments are cylinders of circular section and infinite length

Or does it?



Hacar et. al. 2013

Setting the scene

- Cylinders are isothermal
- Cylinders are non-rotating
- Support against gravity comes solely from thermal pressure
- **Cylinders are isolated**
- Filaments are cylinders of circular section and infinite length

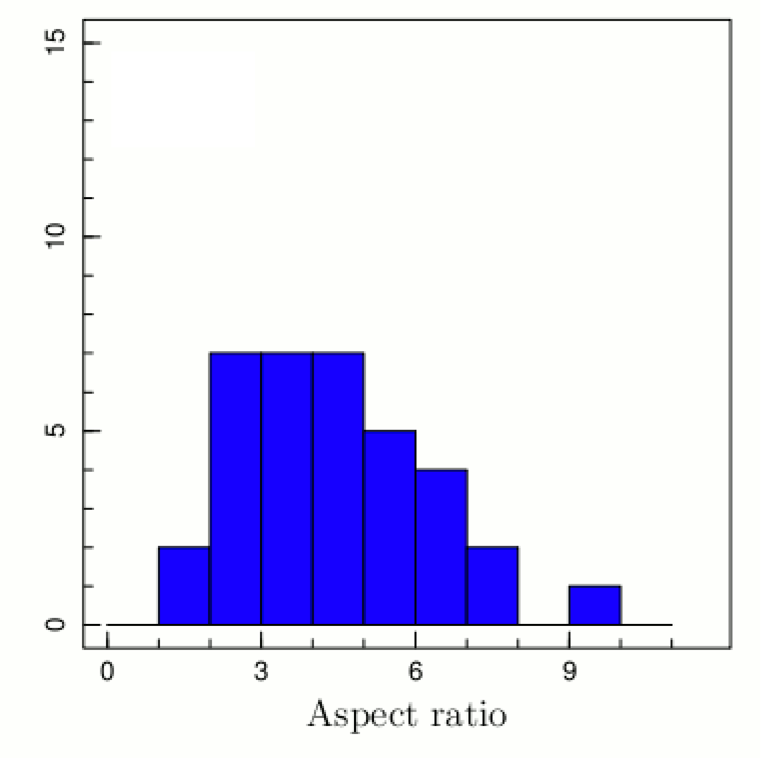
Or are they?

ESO press release, T. Stanke

Setting the scene

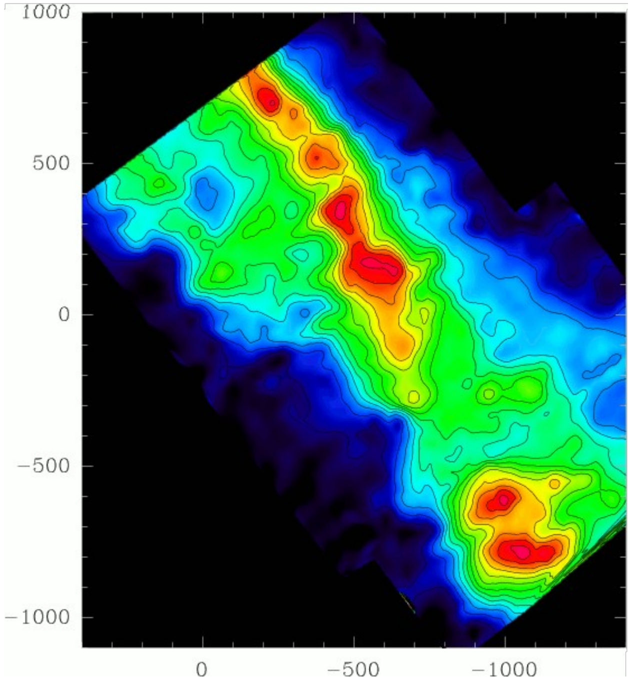
- Cylinders are isothermal
- Cylinders are non-rotating
- Support against gravity comes solely from thermal pressure
- Cylinders are isolated
- **Filaments are cylinders of circular section and infinite length**

Or are they?



Hacar et. al. 2013

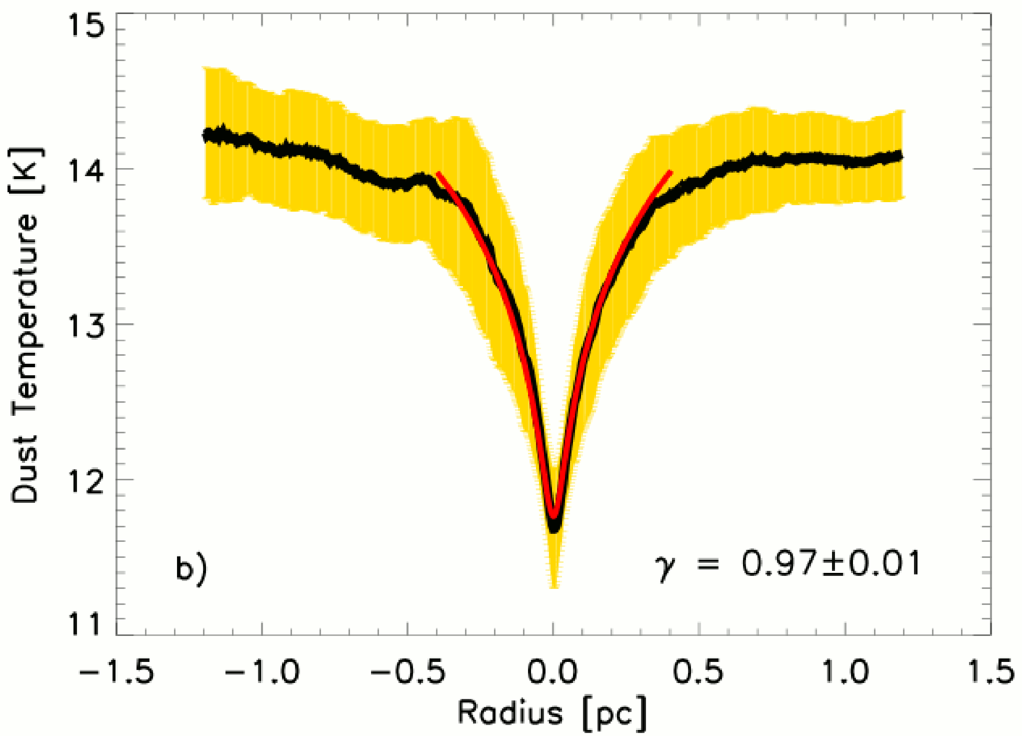
Filaments - ID card



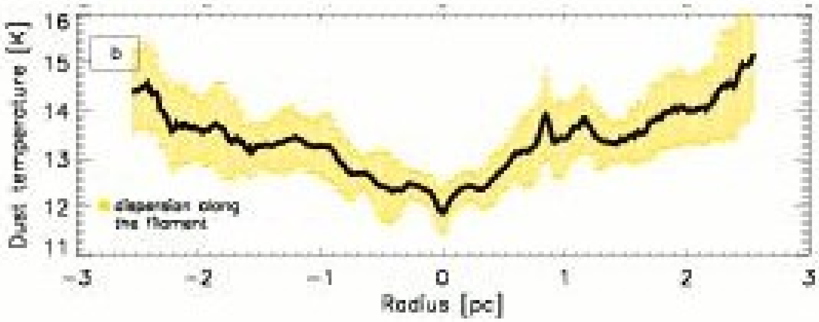
- Elongated structures, prominent in dust continuum and in molecular (CO, N_2H^+, SO, SiO...) emission
- Might or might not contain young stellar objects (YSOs)
- Radius R_c~0.15-0.2 pc
- Length ~0.2-1 pc
- Temperature ~ 10 K
- Radial velocity gradients ~1-2 km s^{-1} pc^{-1}. If due to rotation: $\omega \sim 6.5*10^{-14}$ s^{-1}.

- **Setting the scene**
- **Non-isothermal filaments**
- **Rotating filaments**
- **Non-isolated filaments**
- **Conclusions and outlook**

Non-isothermal filaments



Palmeirim et. al. 2013



Arzoumanian et. al. 2011

Herschel observations seem to be able to measure temperature profiles within filaments. Idea: use observationally based temperature profiles to build equilibrium density profiles.

Non-isothermal filaments

Assume hydrostatic equilibrium (ΔP=gρ) in the filament. Define non-dimensional variables:

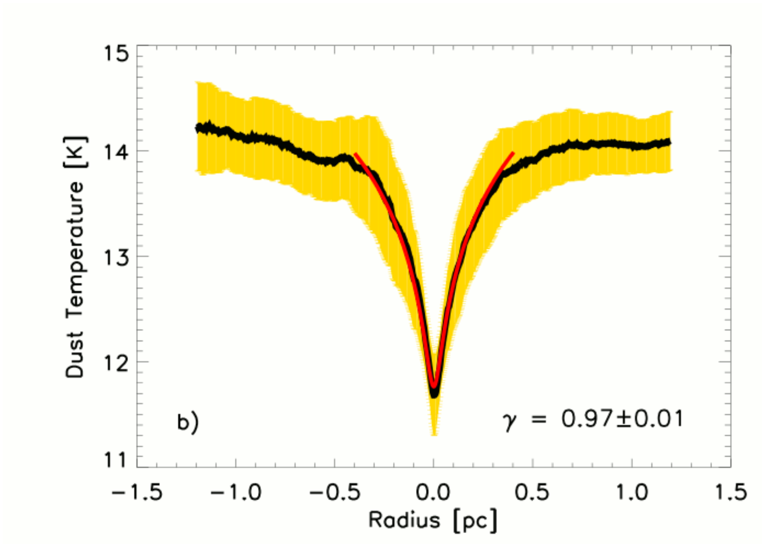
$$\theta=\frac{\rho}{\rho_c}, \quad \tau=\frac{T}{T_c}, \quad x=\frac{r}{H}, \quad H=\sqrt{\frac{2kT_c}{\mu m_H \pi G \rho_c}}$$

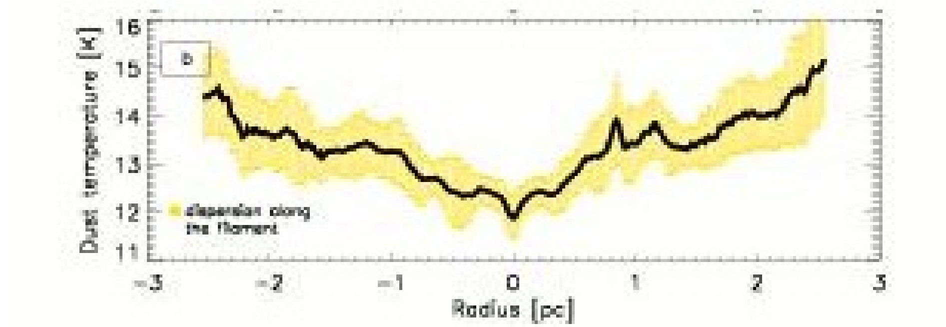
A manipulation of the hydrostatic equilibrium equation leads to:

$$\theta''=\frac{(\theta')^2}{\theta}-\theta'\left(\frac{\tau'}{\tau}+\frac{1}{x}\right)-\theta\left(\frac{\tau''}{\tau}+\frac{\tau'}{\tau x}\right)-8\frac{\theta^2}{\tau}$$

For τ=Const., this equation is satisfied by the “Ostriker” profile. In this formulation we are free to choose the function τ=τ(x) in order to match the observations.

Non-isothermal filaments





How can we approximate these temperature profiles?

A: LINEAR

$$\tau = 1 + A\,x$$

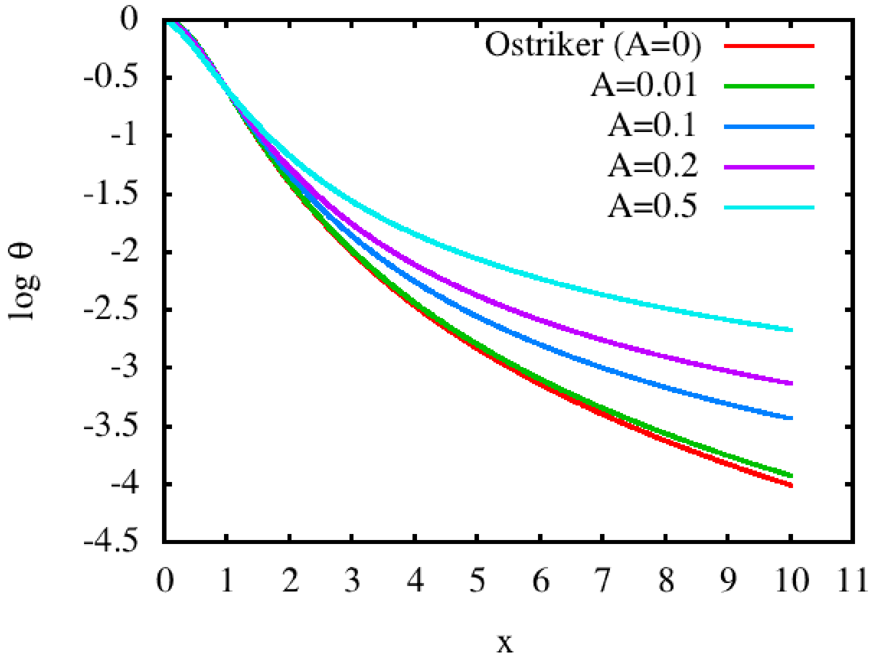
B: ASYMPTOTICALLY CONSTANT

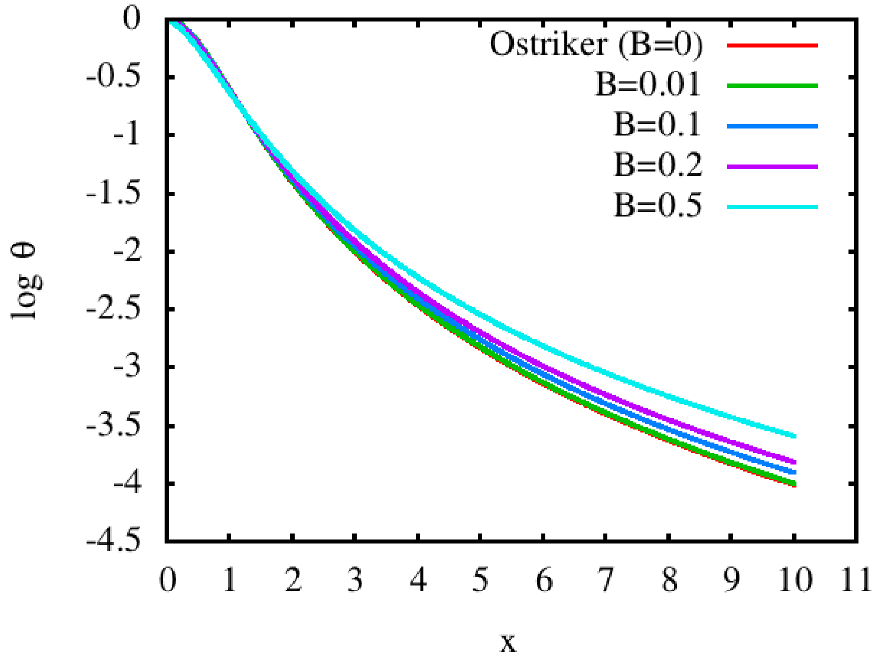
$$\tau = \frac{1 + (B+1)\,x}{1 + x}$$

A, B: adjustable parameters (slopes of the temperature profile at x=0).

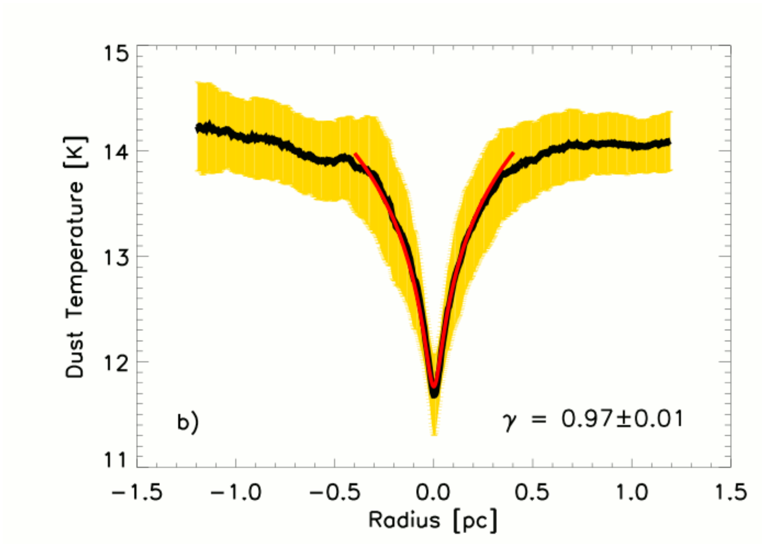
Non-isothermal filaments - profiles

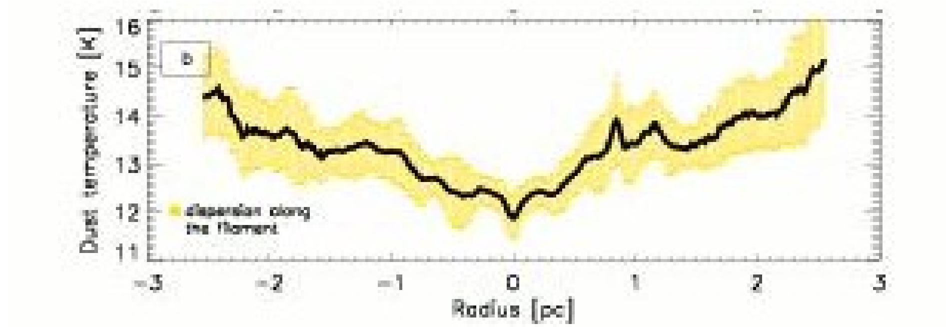
Numerical solutions as a function of A and B: density profiles





Non-isothermal filaments - profiles

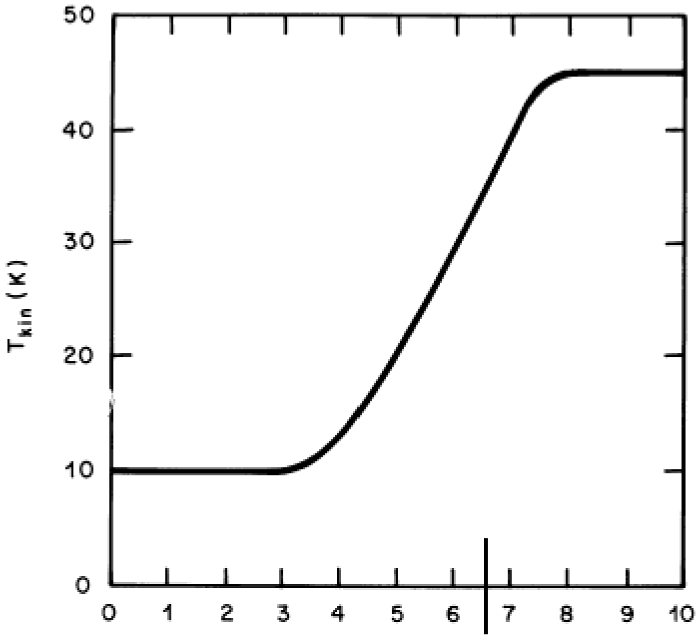




Take only the linear case. For a typical central density of $5*10^4$ cm^{-3} and T=10, H≈0.045 pc. At face value the data suggest thus A≈0.025.

Remember, though, that this is just the dust temperature, that need not be equal to the gas temperature! Actually, the gas temperature is considerably higher in the external layers of the cylinder because of photo-electric and UV heating of molecules are more efficient at lower densities.

Non-isothermal filaments - profiles

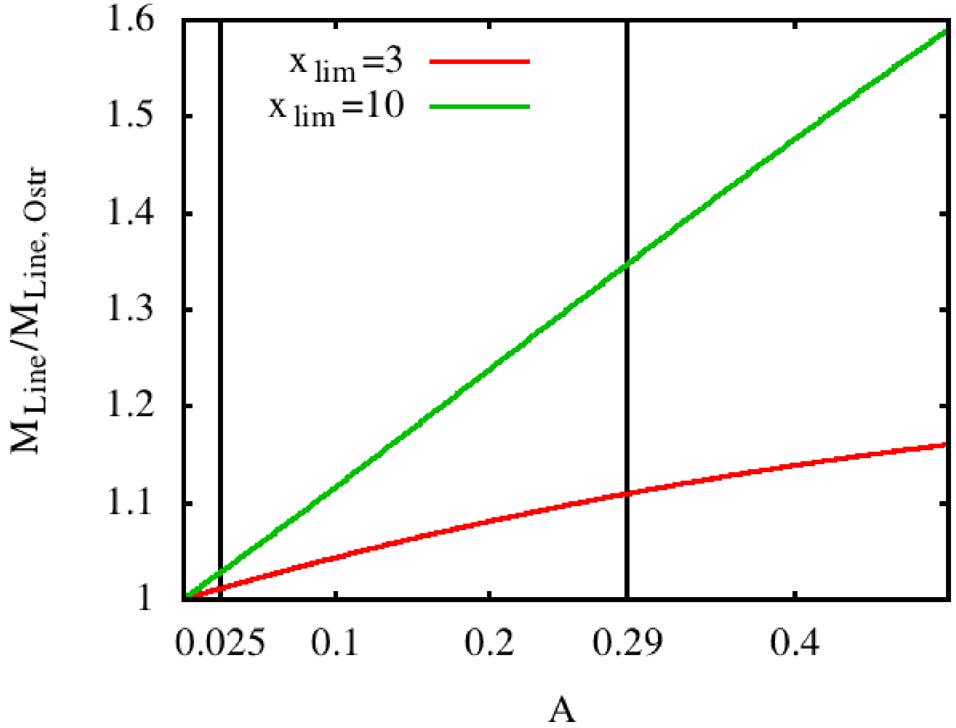


x=10

Frerking et. al. 1987

This specific example (B335) suggests that A might be higher (A≈0.29 for H≈0.045 pc)

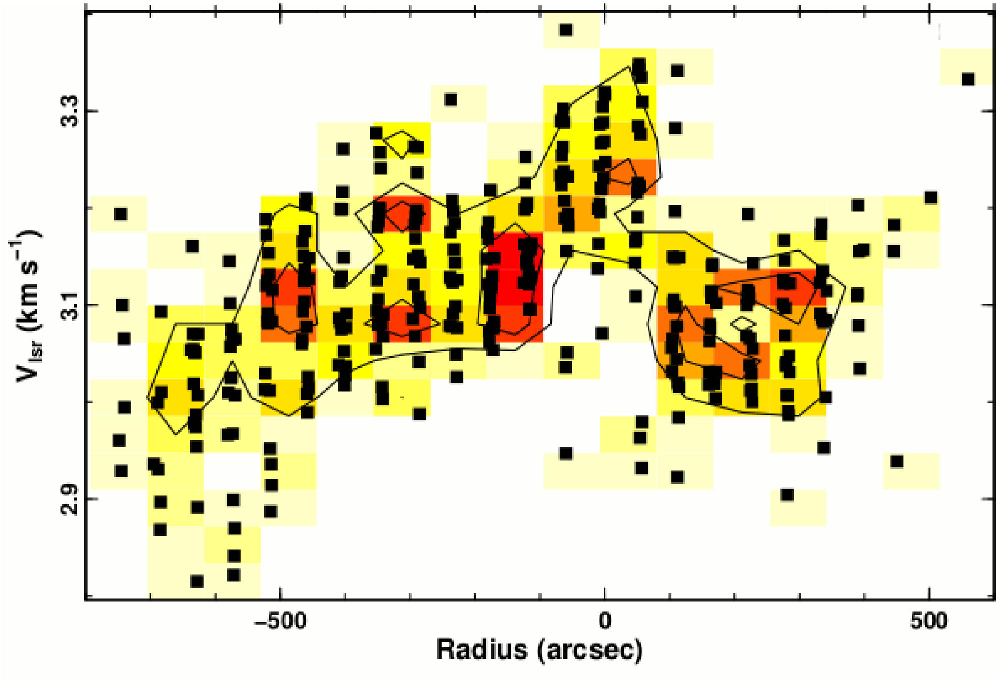
Non-isothermal filaments - profiles



The non-isothermal filament in equilibrium sustains more mass than the Ostriker one. The increase in critical linear mass is negligible for A=0.025 (1.011 for x_{lim}=3, 1.028 for x_{lim}=10) but becomes significant for A=0.29 (1.11 for x_{lim}=3, 1.347 for x_{lim}=10). This last figure corresponds to 22.1 $M_{\odot}$ pc^{-1} for T=10 K.

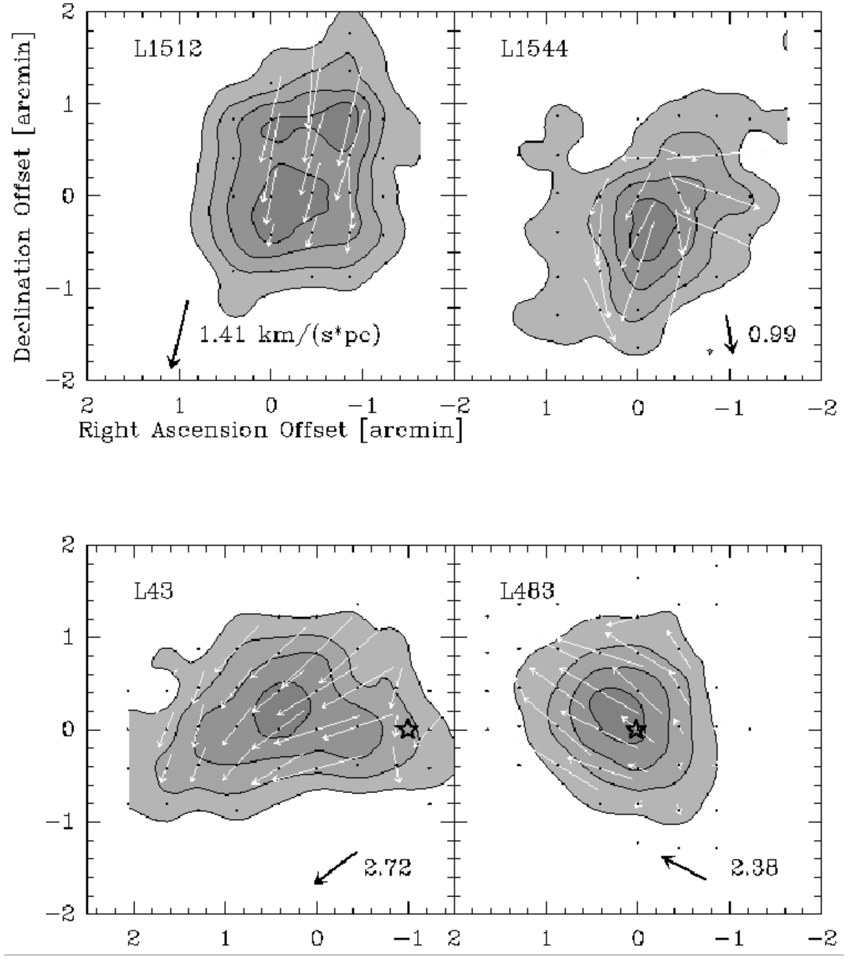
- **Setting the scene**
- **Non-isothermal filaments**
- **Rotating filaments**
- **Non-isolated filaments**
- **Conclusions and outlook**

Rotating filaments



Examples of clearly rotating filaments are unfortunately rare.

Rotating filaments



Caselli et. al. 2002

Examples of clearly rotating filaments are unfortunately rare. It is easier to find clearly rotating dense cores (notice that dense cores have the same radius as the filaments – no spin-up is involved here!) Gradients are again of the order of 1-2 km s^{-1} pc^{-1}.

Rotating filaments

Order-of-magnitude estimate:

- Total kinetic energy per unit length due to rotation $T \sim 1/2\ \omega^2 R_c^{\ 2} M_{lin}$.
- Total gravitational energy per unit length $W = GM_{lin}^{\ 2}$.

$$\frac{T}{W} \simeq 0.65 \left(\frac{\omega}{6.5 \cdot 10^{-14}} \right)^2 \left(\frac{R_c}{0.15\,\mathrm{pc}} \right)^2 \left(\frac{M_{lin}}{16.6\, M_{sun}\,\mathrm{pc}^{-1}} \right)^{-1}$$

For nominal values of ω, R_c, M_{lin}, rotation can not be neglected.

Rotating filaments

In what follows we will solve the hydrostatic equilibrium equation with rotation

$$\nabla P = \rho \left(g + \omega^2 r \right)$$

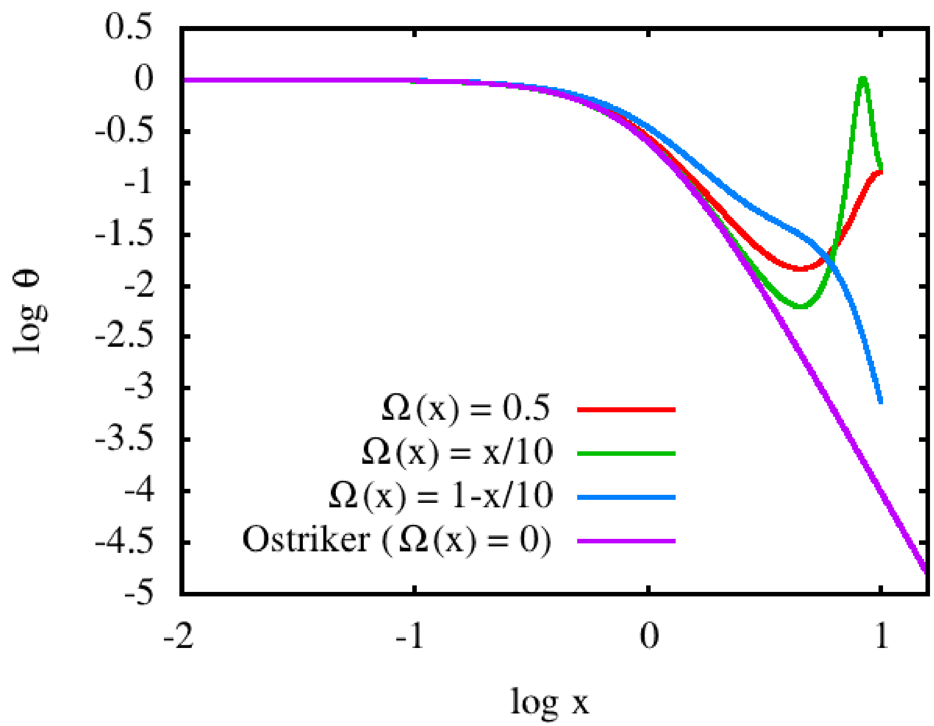
The employed normalizations are now

$$\theta = \frac{\rho}{\rho_c}, \quad \tau = \frac{T}{T_c}, \quad x = \frac{r}{H}, \quad \Omega = \sqrt{\frac{2}{\pi G \rho_c}} \; \omega$$

As before, we will assume temperature profiles based on observations. We will mostly focus on the linear case τ=1+Ax. Given our ignorance, we are free to choose Ω=Ω(x) (taking into account however that for nominal values of ρ_c and ω, Ω, must be of the order of 0.5). We will mostly consider

$$\Omega(x) = 0.5, \quad \Omega(x) = \frac{x}{10}, \quad \Omega(x) = 1 - \frac{x}{10}$$

Rotating filaments



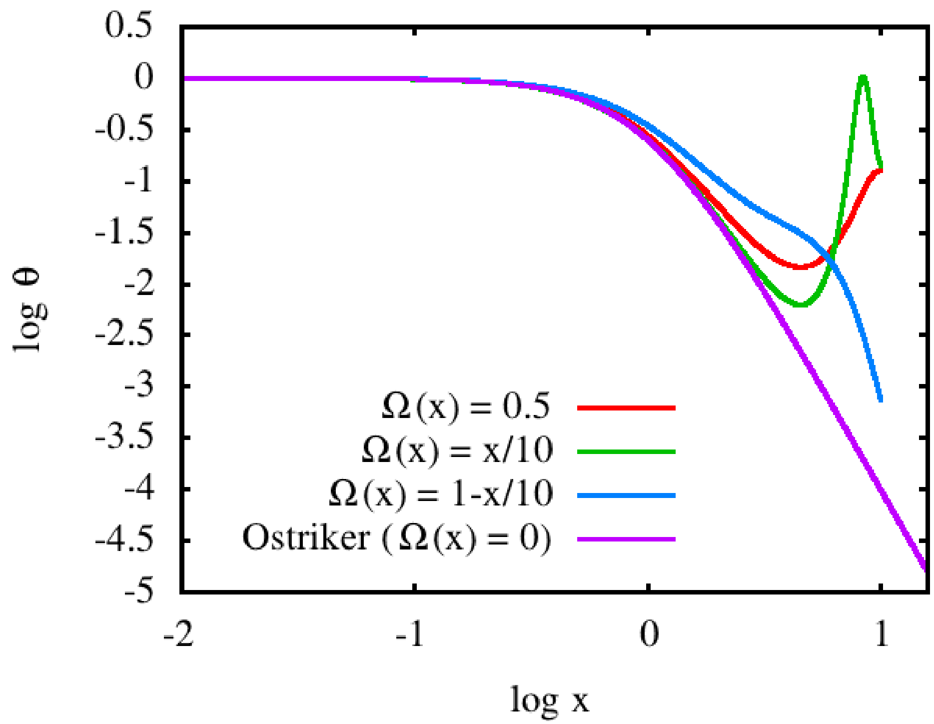
Models with A=0. Things to notice:

• The density inversions are dynamically unstable

• Also the profile 1-x/10 is unstable after x~6.7 because it does not satisfy the Rayleigh criterion (angular momentum decreases outwards)

• Until x~3 all profiles are stable and flatter than the Ostriker profile

Rotating filaments

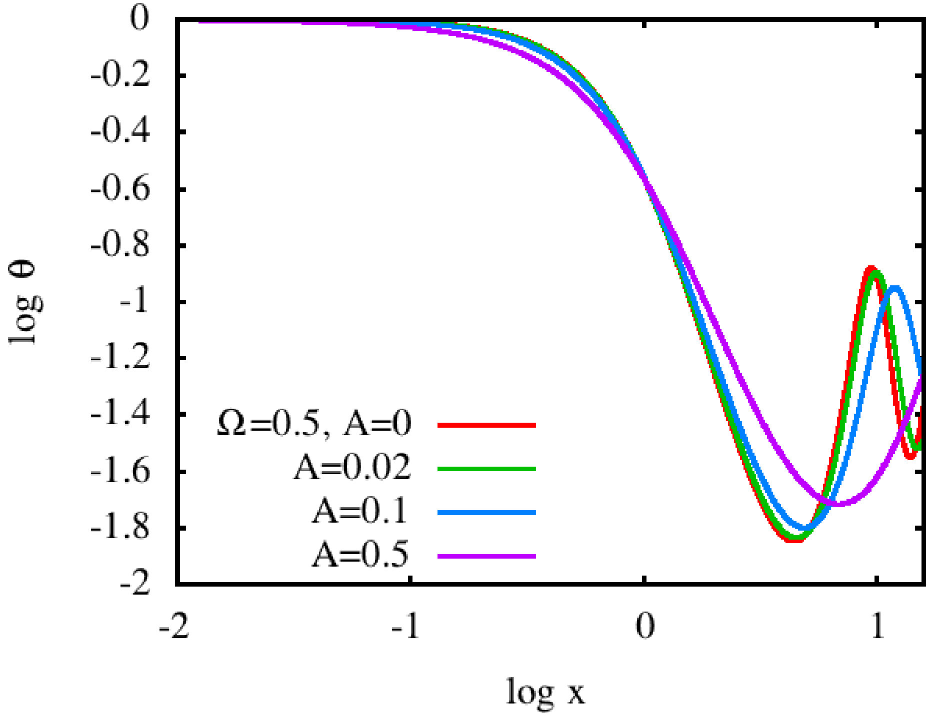


$$\Omega(x) = 0.5: \quad \frac{M_{line}}{M_{Line,Ostr}} = 7.02, \quad \frac{M_{line}(x < 4.42)}{M_{Line,Ostr}(x < 4.42)} = 1.23$$

$$\Omega(x) = \frac{x}{10}: \quad \frac{M_{line}}{M_{Line,Ostr}} = 23.9, \quad \frac{M_{line}(x < 4.45)}{M_{Line,Ostr}(x < 4.45)} = 1$$

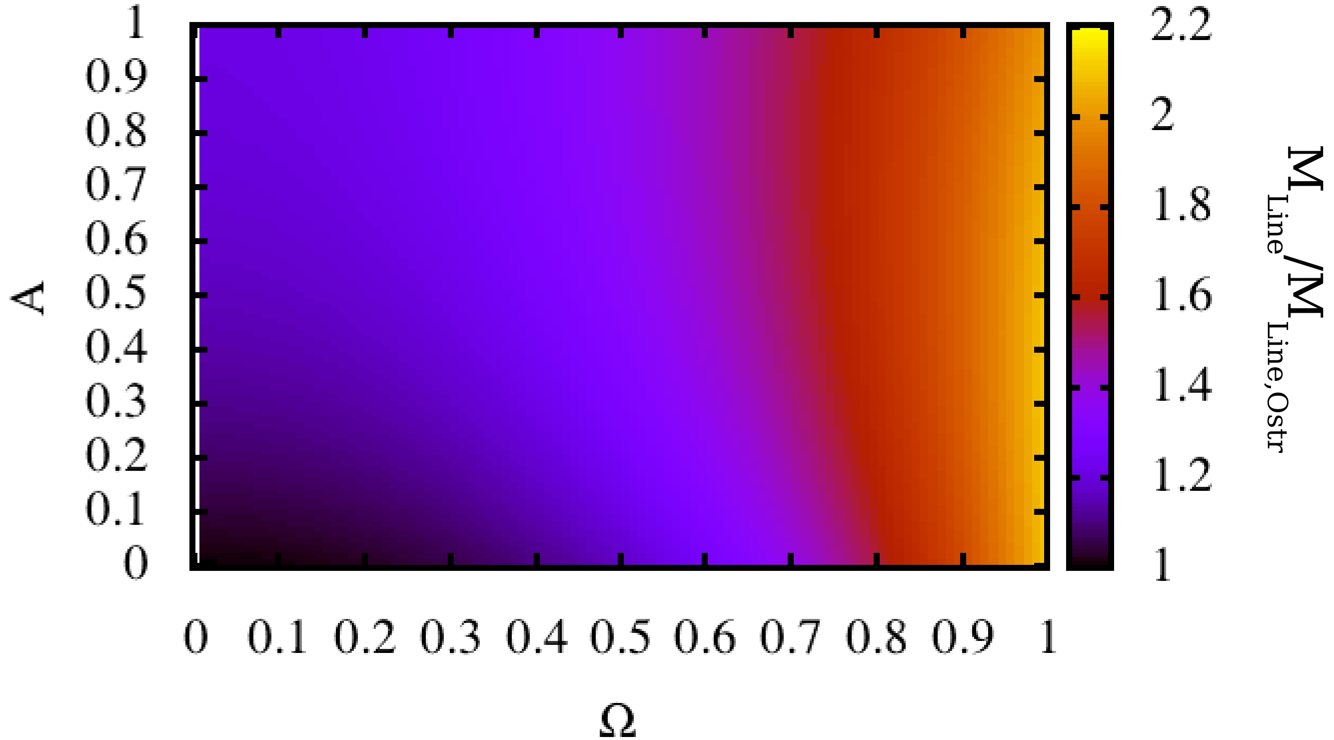
$$\Omega(x) = 1 - \frac{x}{10}: \quad \frac{M_{line}}{M_{Line,Ostr}} = 2.58, \quad \frac{M_{line}(x < 6.67)}{M_{Line,Ostr}(x < 6.67)} = 2.39$$

Rotating, non-isothermal filaments



Temperature gradients tend to smooth out density oscillations. The density profiles at small x tend to be flatter.

Rotating, non-isothermal filaments



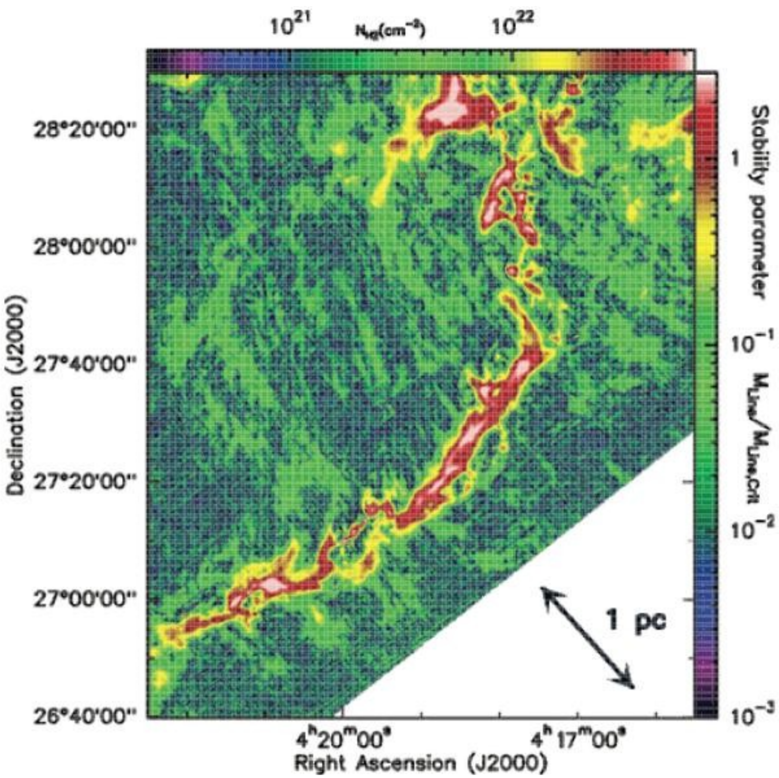
Increase of linear mass compared to the Ostriker profile. Filaments are truncated at x=3. Even at this modest radius, the combined effect of rotation and temperature gradient can be significant

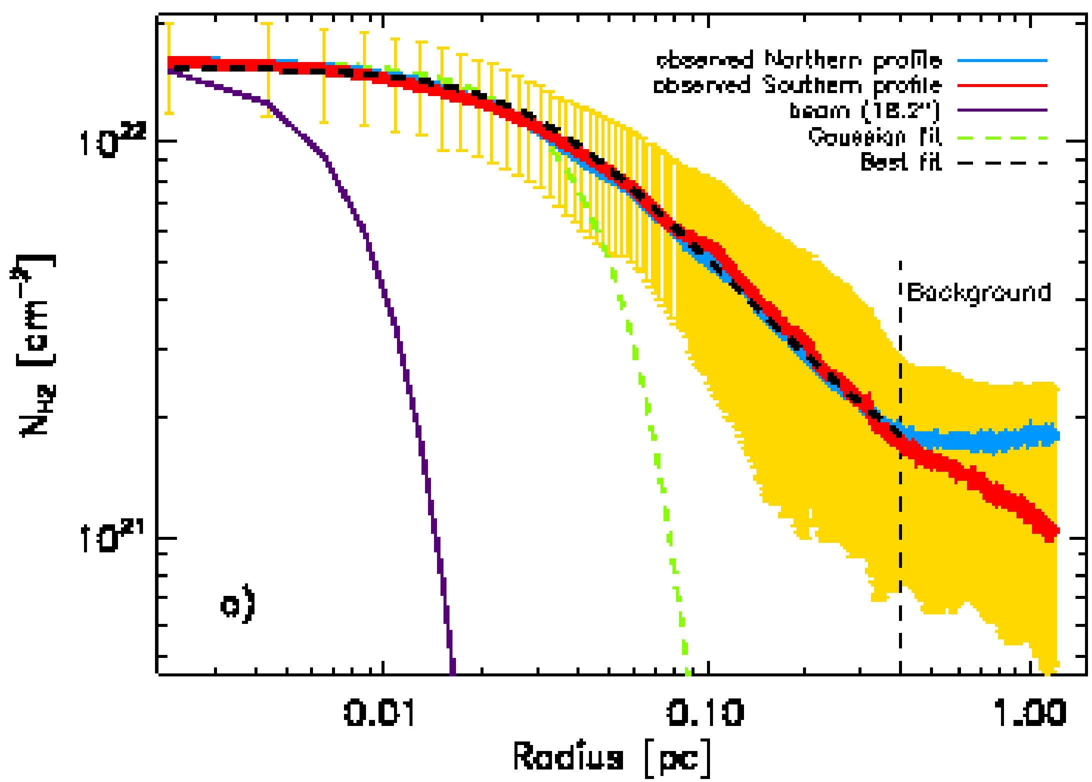
- For Ω=0.5 and A=0.025, $M_{\mathrm{Line}}/M_{\mathrm{Line,Ostr}}$=1.181
- For Ω=0.5 and A=0.29, $M_{\mathrm{Line}}/M_{\mathrm{Line,Ostr}}$=1.282

Density profiles and the role of envelopes

Envelopes surround filaments and their presence modifies the flux and hence the recovered density along the line of sight. Notice that the best fit corresponds to the model:

$$\rho = \frac{\rho_c}{\left[1 + \left(r/H\right)^2\right]^{p/2}} \qquad \text{with } p = 2$$

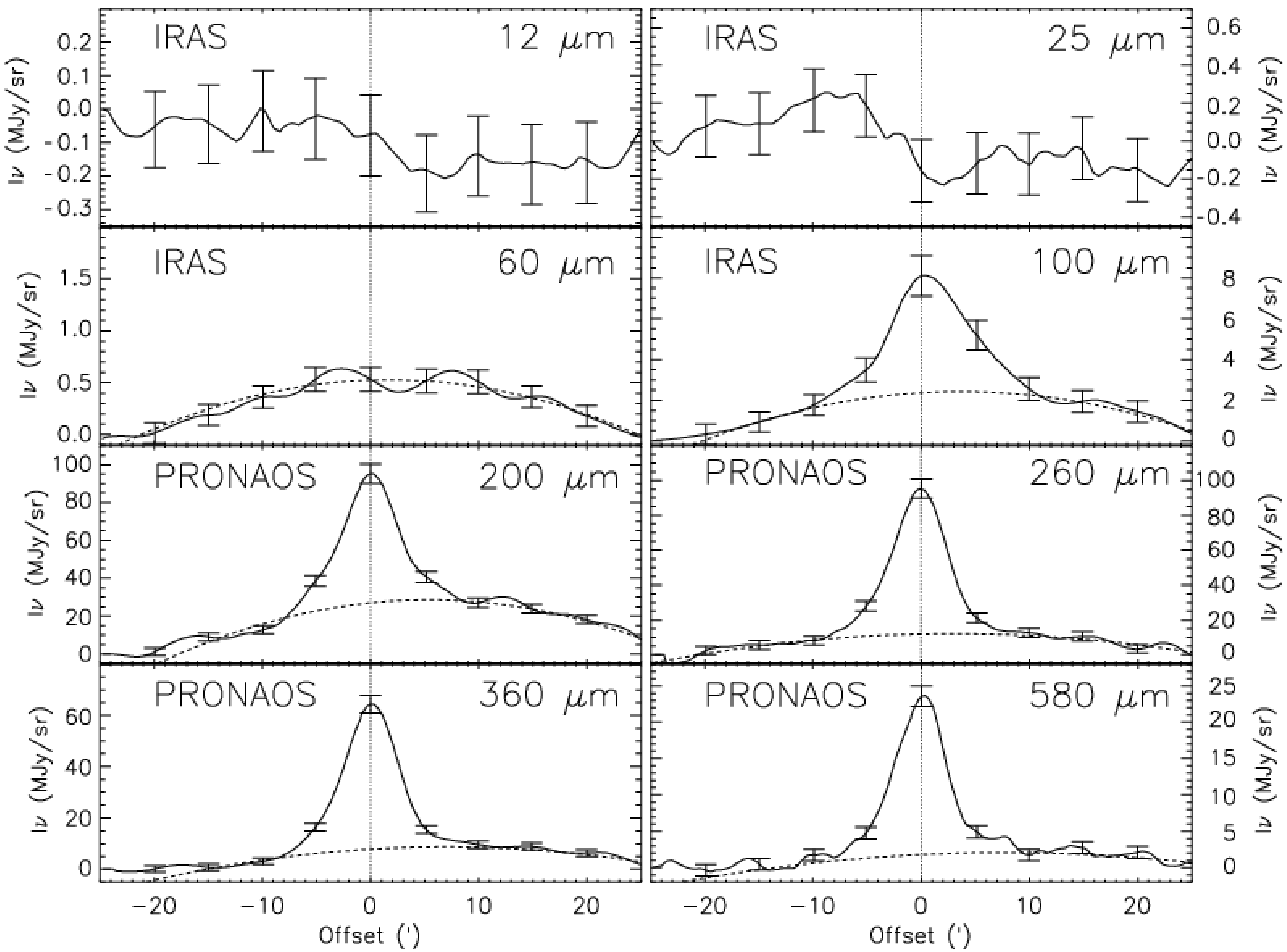




Palmeirim et. al. 2013

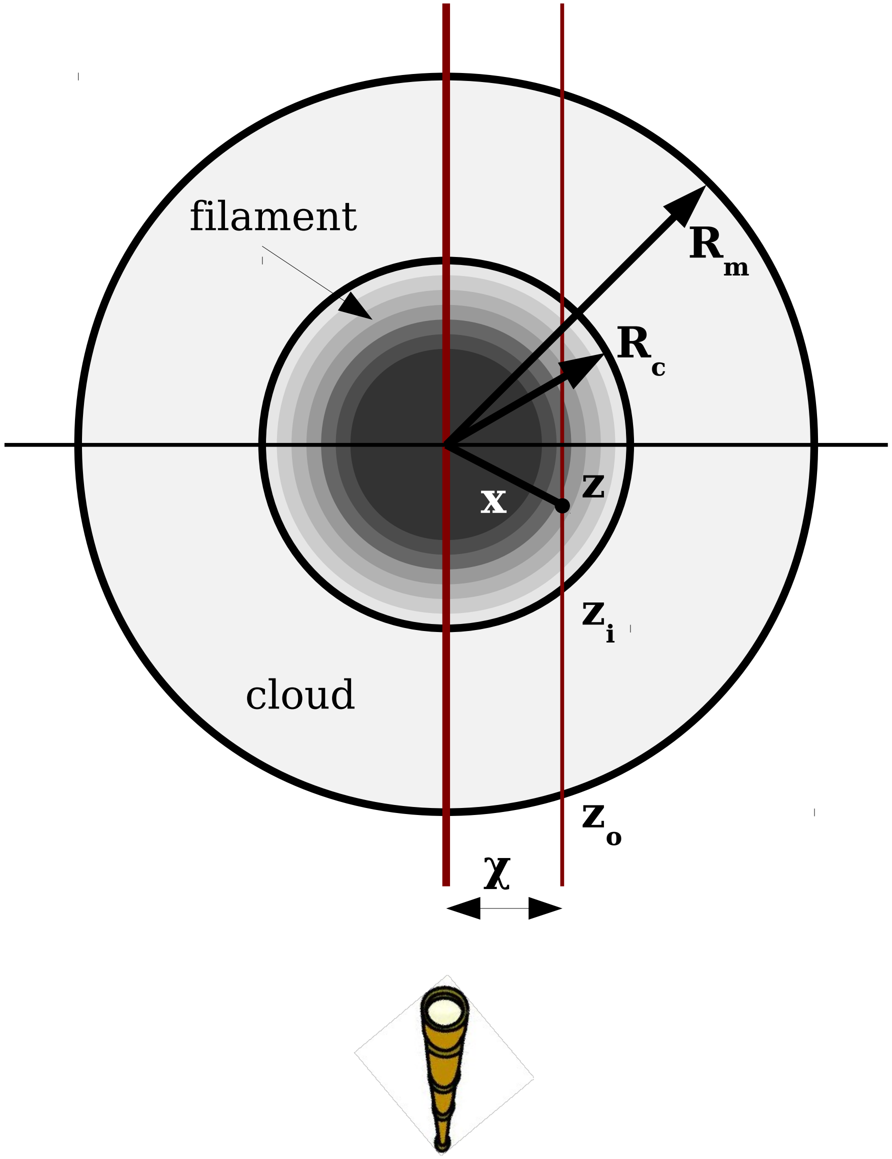
Density profiles and the role of envelopes

Envelopes seem to extend to more than three times the filament's radius



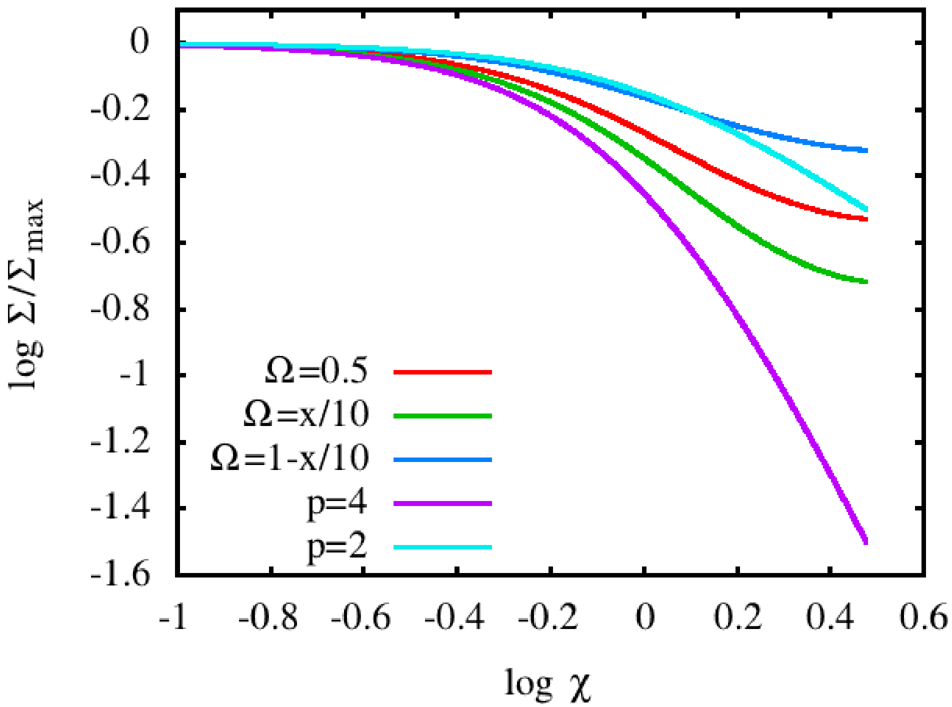
Stepnik et. al. 2003

Density profiles and the role of envelopes



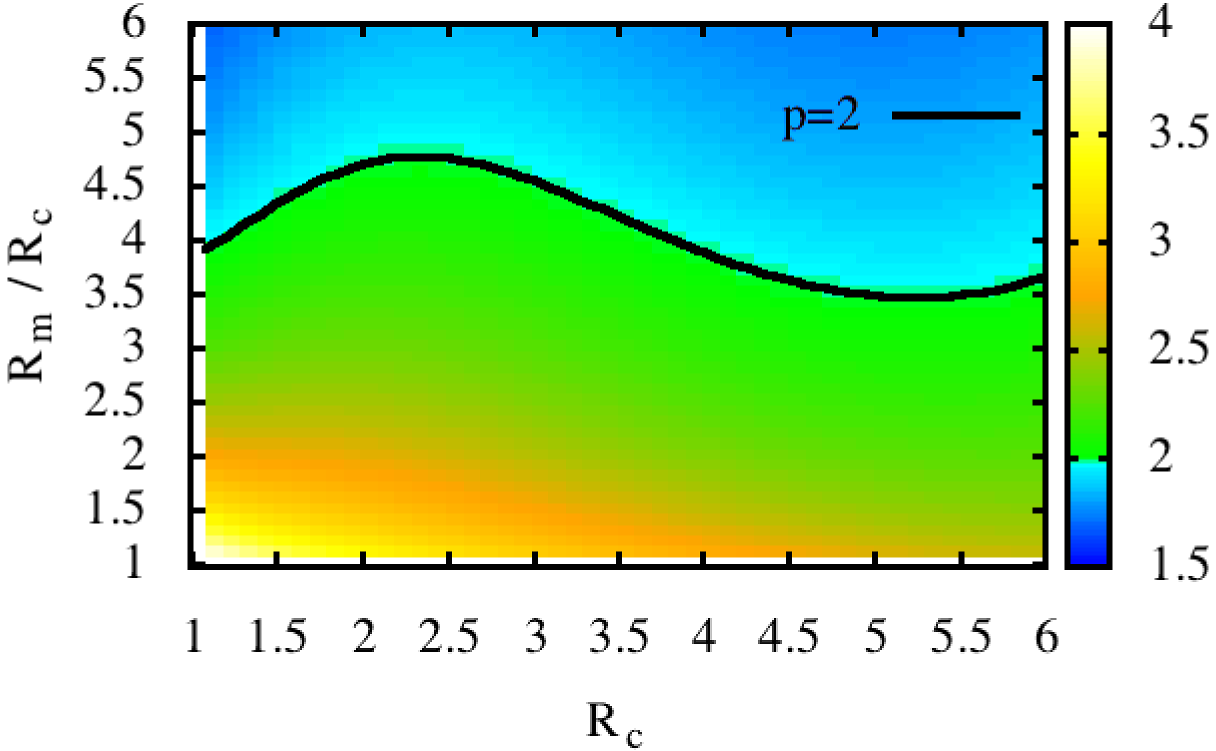
We assume that the enveloping molecular cloud material is also a cylinder with the same axis. The density is assumed to be constant and assume to match the external density of the filament. The impact parameter χ is assumed to vary from 0 to R_c. Envelopes in the form of slabs have been also considered.

Density profiles and the role of envelopes



Here the cylinder is truncated at x=3 and the molecular envelope extends until x=15. A=0. The model with p=4 is the Ostriker filament; the model with p=2 is the best fit to the B211/3 filament according to Palmeirim et al.

Density profiles and the role of envelopes

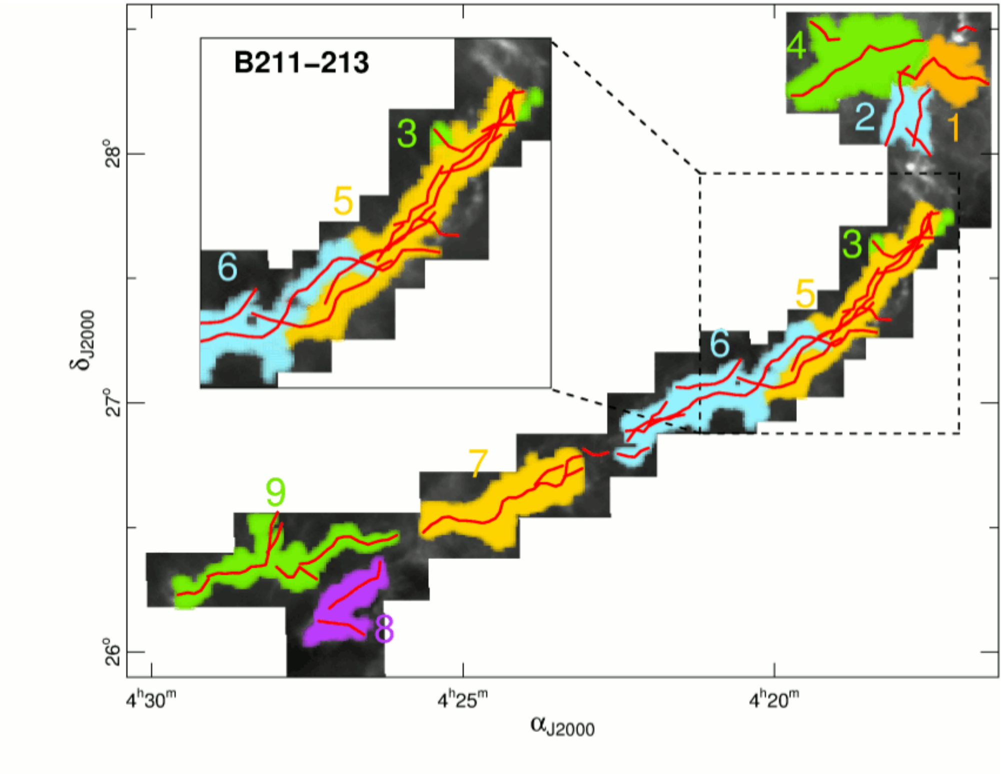


Average slopes of various models. Here A=0 and Ω=1-x/10. Clearly, for envelopes 3-4 times larger than the radius of the filament, the average slope can be close to the p=2 obtained by Palmeirim et al.

- Setting the scene
- Non-isothermal filaments
- Rotating filaments
- **Non-isolated filaments**
- Conclusions and outlook

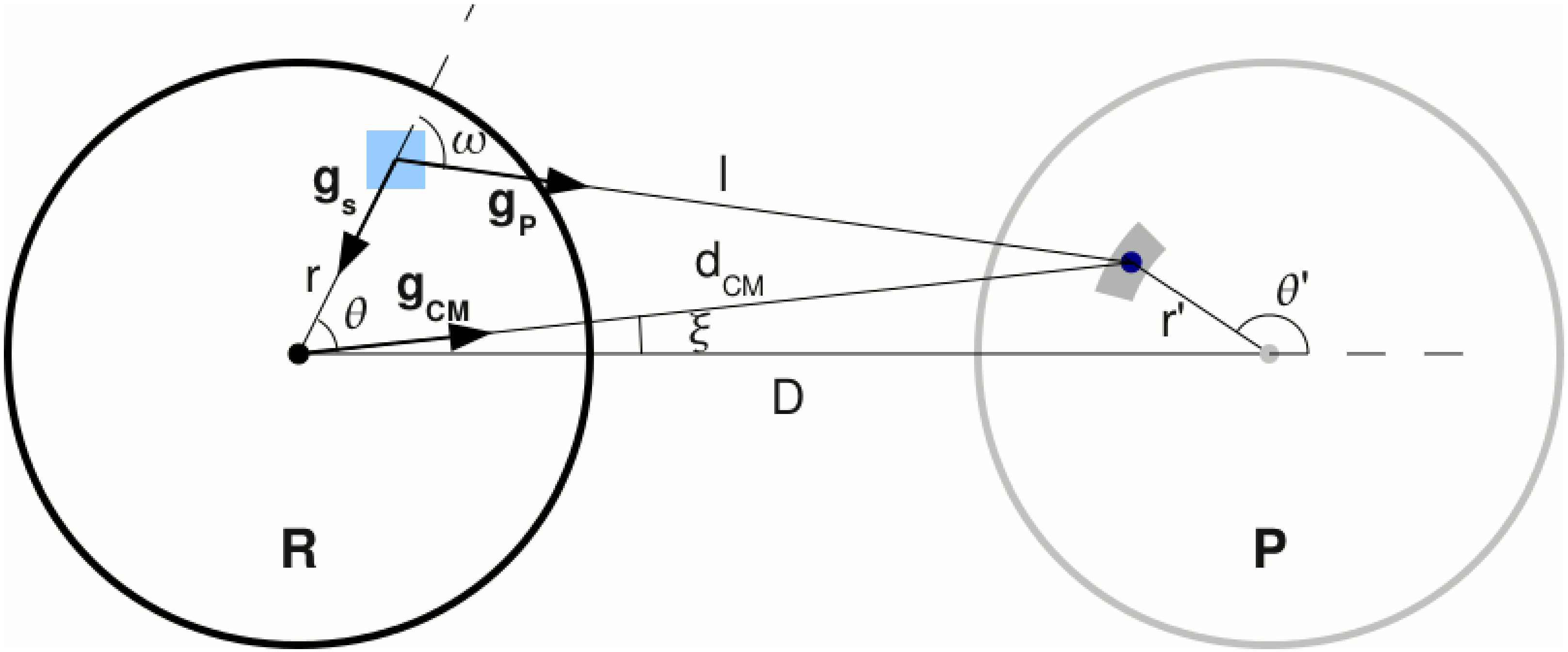
Non-isolated filaments

A structure like B211/3 is composed of many individual filaments. Do the presence of neighbor filaments affect the stability properties and the density profiles of filaments?



Non-isolated filaments

Idea: take two parallel, pressure-truncated filaments and calculate their mutual gravity.

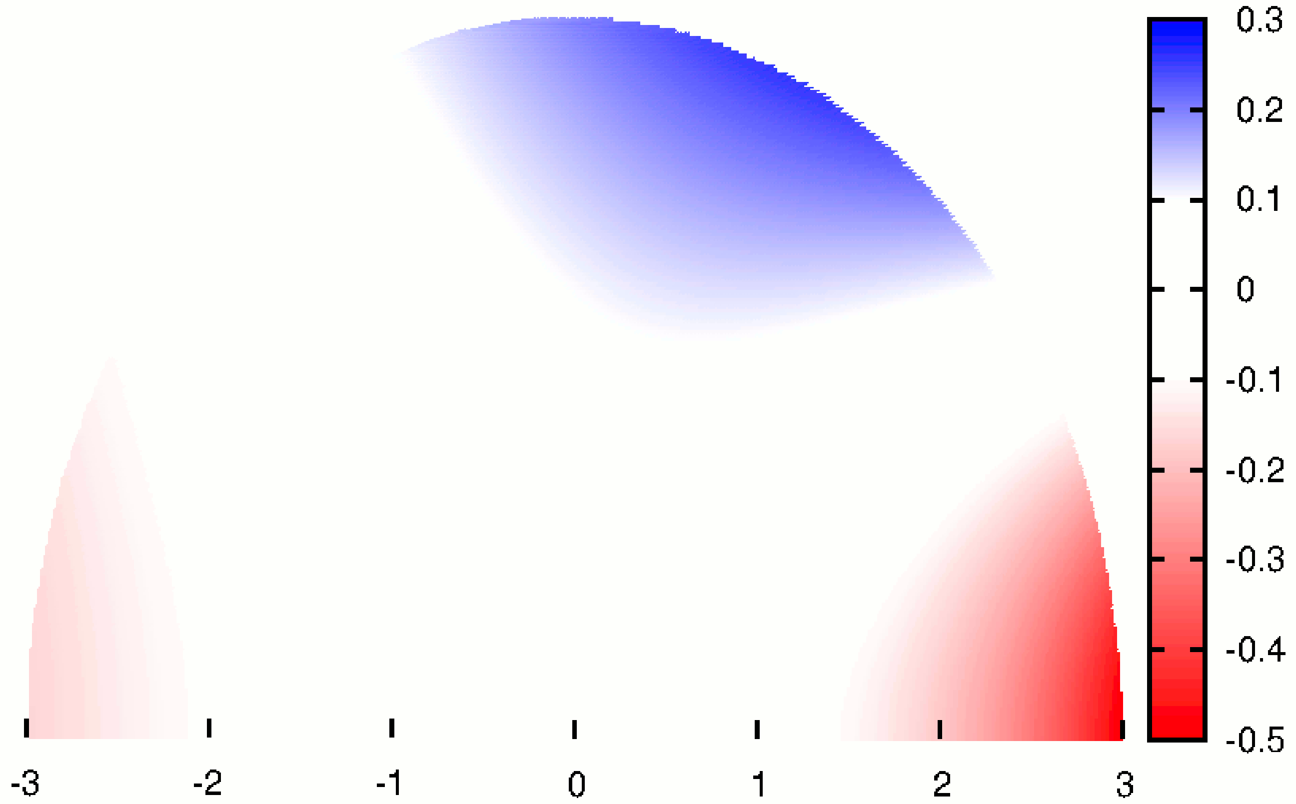


For each fluid element in the reference cylinder, calculate the gravity due to the perturber. Calculate the radial component of $\mathbf{g}_{\mathbf{CM}}$-$\mathbf{g}_{\mathbf{P}}$ and compare it with $\mathbf{g}_{\mathbf{s}}$. If this component is more than 10% of the self-gravity, I assume that the gravity due to the perturber cannot be neglected.

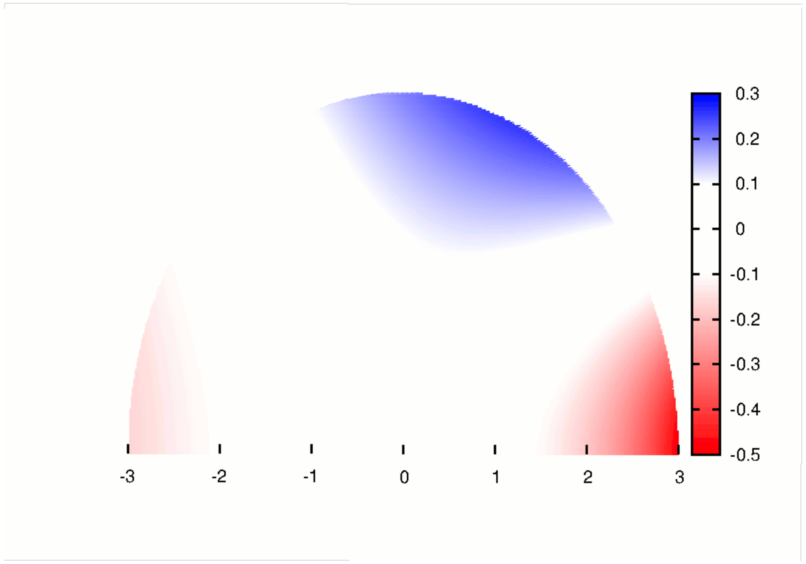
Take (equal) isothermal cylinders.

Non-isolated filaments

Ratio of the radial components of the perturbing and self gravity. D=2.1 R.



Non-isolated filaments



A large fraction (in mass) of the cylinder (white part) is not significantly affected by the perturber. In another part (blue wedge) the tide is compressive and, if the cylinder is unstable, this part would tend to collapse faster than the rest of the cylinder towards the axis. Only for ~11% in mass of the cylinder the perturber opposes a possible collapse. We assume that 89% of the mass of the cylinder is “potentially collapsing”.

Even in this quite extreme case of two very close cylinders, the tidal files seems not to change significantly the stability properties of the cylinder. Notice however that I have consider only the radial component of the gravity: tangential components can be significant and hence the profile of a tidally perturbed cylinder could appear quite distorted.

Non-thermal pressure support

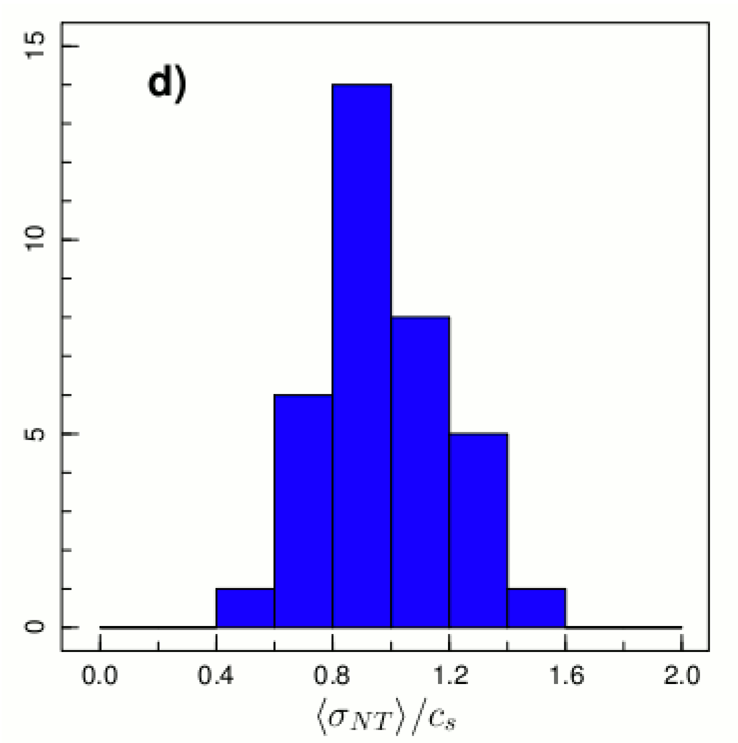
If a component of the pressure is of non-thermal origin, the equilibrium configuration will contain more mass. Suppose that the non-thermal pressure is a fraction β of the thermal one – $P=P_{NT}+P_T=(1+\beta)P_T$. Then the density profile of the equilibrium cylinder remains unaltered and only the scale H changes:

$$H=\sqrt{1+\beta}\,H_T, \quad \rightarrow \quad M_{Line}=(1+\beta)M_{Line,T}$$

Non-thermal pressure support

If a component of the pressure is of non-thermal origin, the equilibrium configuration will contain more mass. Suppose that the non-thermal pressure is a fraction β of the thermal one – $P=P_{NT}+P_T=(1+\beta)P_T$. Then the density profile of the equilibrium cylinder remains unaltered and only the scale H changes:

$$H=\sqrt{1+\beta}\,H_T, \qquad \rightarrow \qquad M_{Line}=(1+\beta)\,M_{Line,T}$$



Filaments can have about twice as much mass as predicted by the Ostriker profile just because there is non-thermal pressure support!

- **Setting the scene**
- **Non-isothermal filaments**
- **Rotating filaments**
- **Non-isolated filaments**
- **Conclusions and outlook**

Conclusions

What we can conclude so far is:

• All considered additional processes (non-isothermality, rotation, tidal interaction, non-thermal support) modify the profile of an equilibrium filament. In general relaxation of the Ostriker's assumptions lead to an increase of the equilibrium linear mass of filaments and to a flattening of the density profiles.

• The most significant effect appears to be the non-thermal pressure support. Non-isothermality and rotation can lead to changes of the order of 10-20%. Tidal interactions appear to be quite unimportant

• Filaments appear to be embedded by envelopes. These envelopes can significantly change the column density profiles. A flat observed column density profile might only indicate that a significant contribution comes from the envelope

Where to go from here?

- More effort is needed observationally to identify rotation patterns in star-forming filaments
- More attention needs to be devoted to the non-thermal pressure support of filaments (Turbulence? Magnetic fields?)
- A full calculation of the tidal perturbation of bundled filaments can be useful

Where to go from here?

• More effort is needed observationally to identify rotation patterns in star-forming filaments

• More attention needs to be devoted to the non-thermal pressure support of filaments (Turbulence? Magnetic fields?)

• A full calculation of the tidal perturbation of bundled filaments can be useful

• One shall not forget that some observational properties of filaments come from dust continuum observations, some from molecular line observations!
-> a two-phase description of the filament is necessary!

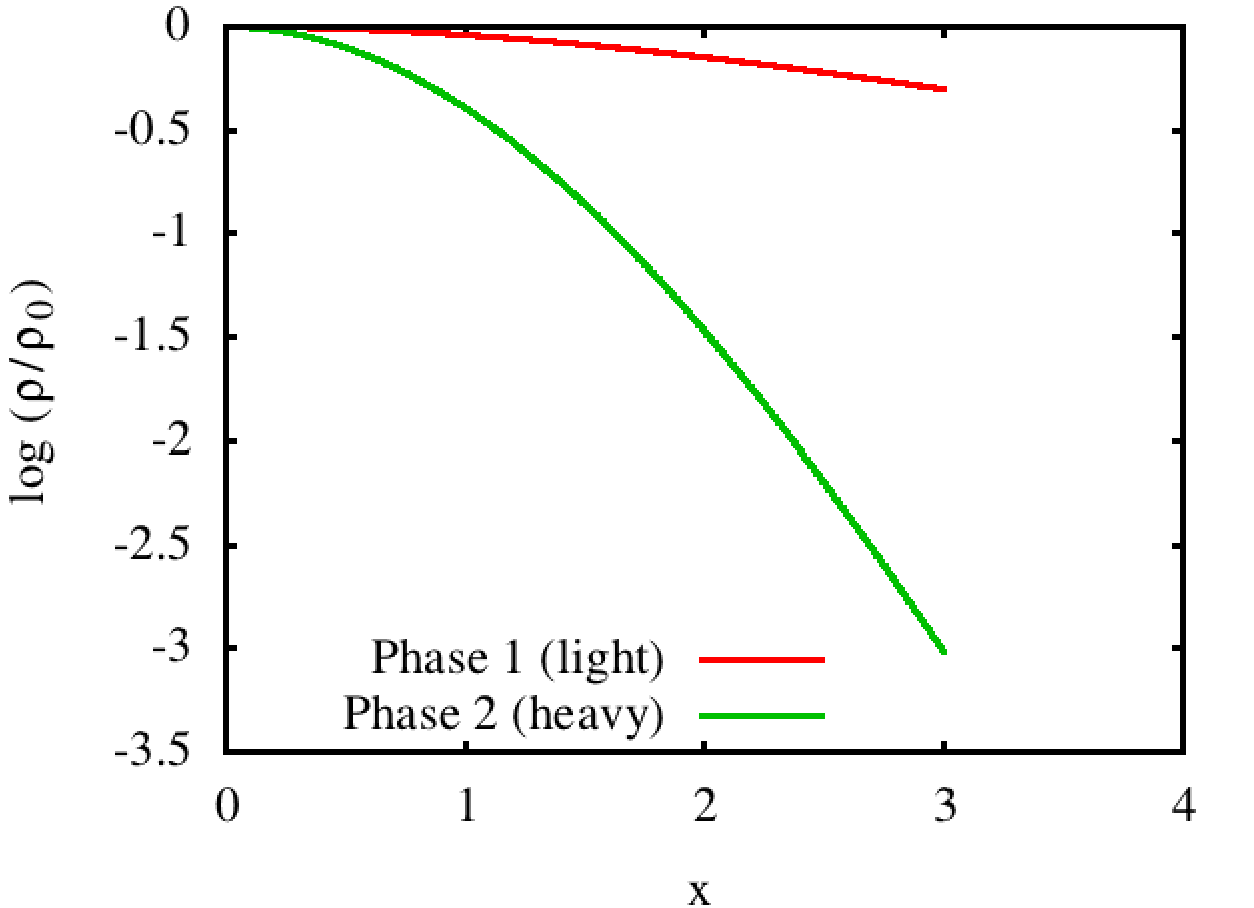
Where to go from here?

• More effort is needed observationally to identify rotation patterns in star-forming filar

• More atten … support of filaments (Tu

• A full calcu … can be useful

• One shall n … ents come from dust co … ervations!
-> a two-pha



Example: phase 2 has a mean molecular weight ten times higher than phase 1 – its distribution is much steeper than the distribution of phase 1.

Where to go from here?

• More effort is needed observationally to identify rotation patterns in star-forming filaments

• More attention needs to be devoted to the non-thermal pressure support of filaments (Turbulence? Magnetic fields?)

• A full calculation of the tidal perturbation of bundled filaments can be useful

• One shall not forget that some observational properties of filaments come from dust continuum observations, some from molecular line observations!
-> a two-phase description of the filament is necessary!

• At some point a full hydrodynamical simulation is necessary..

Thank you for the attention!