

Measuring σ_8 with Weak Lensing of SNe

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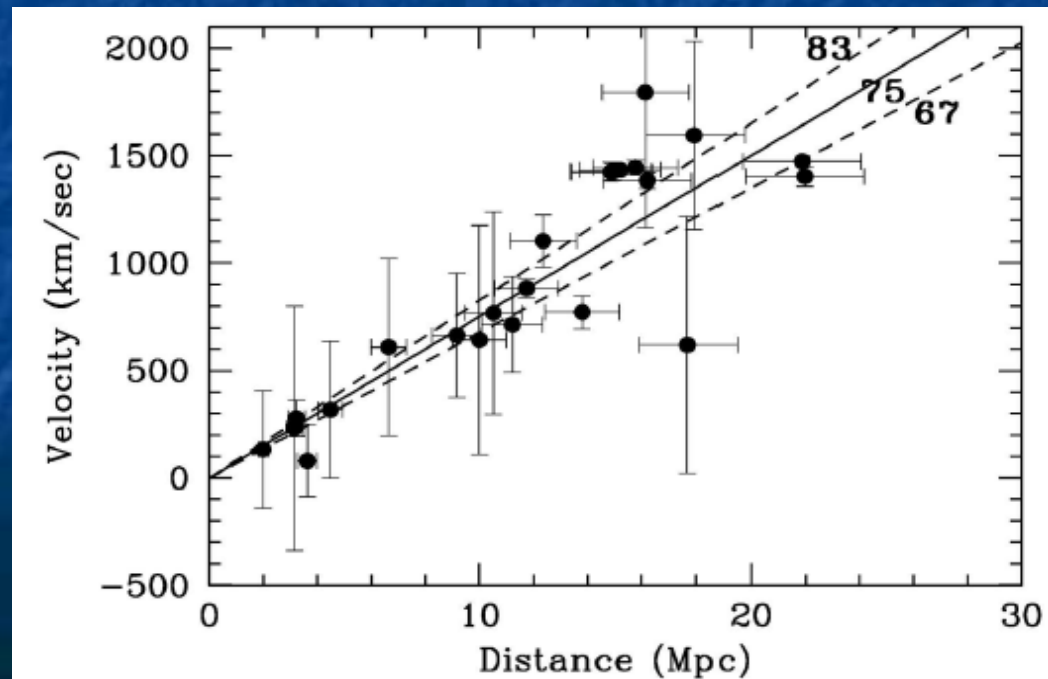
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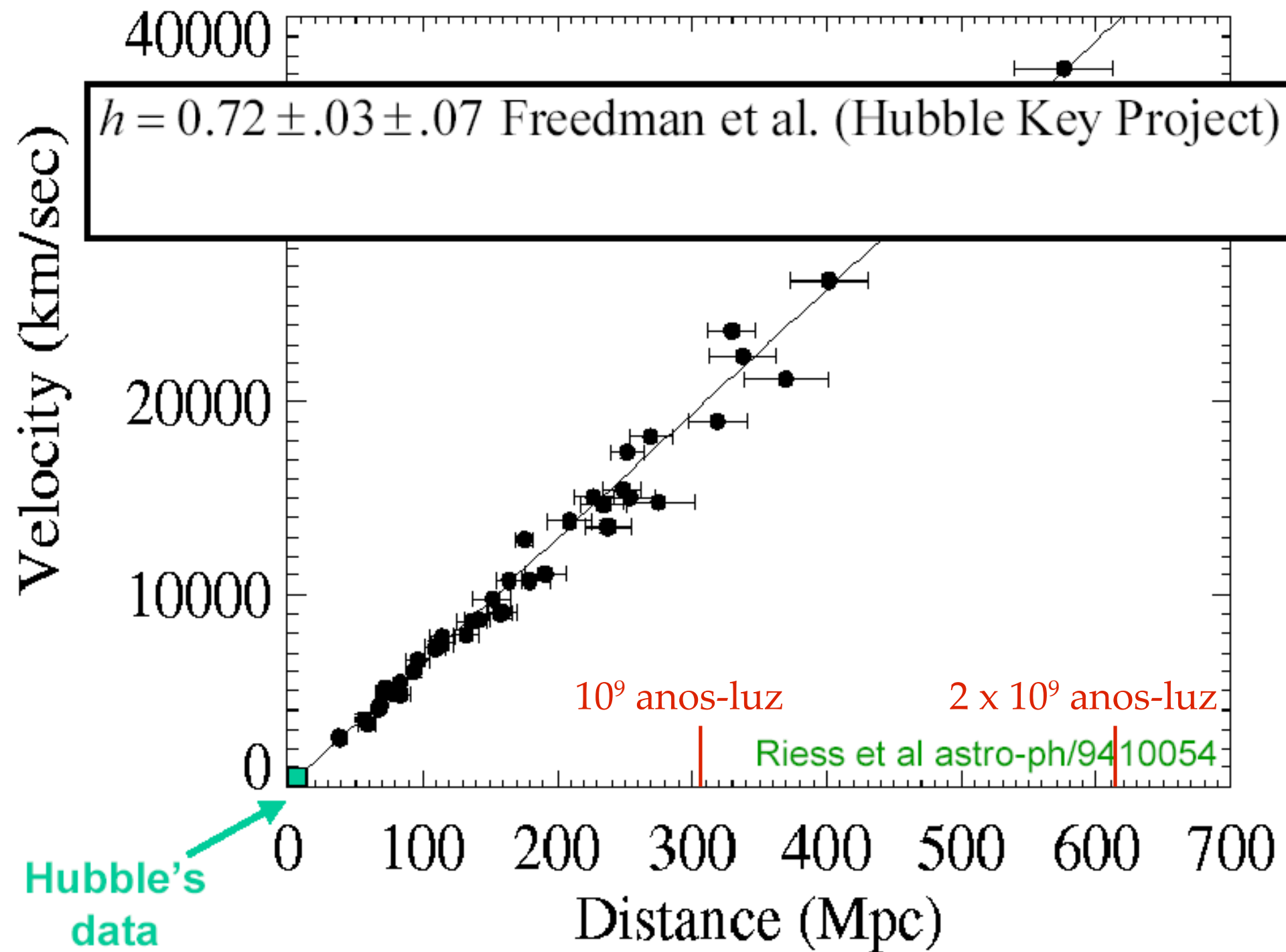
The Hubble's Law

- Lemaître (and later Hubble)* found out that galaxies are, in average, receding from us;
 - The redshift z is linear with distance
 - The velocity is approx. also linear with distance
 - * Stigler's law of eponymy: "No scientific discovery is named after its original discoverer."

$$z = \frac{H_0}{c} D$$

$$v = H_0 D + \mathcal{O}\left(\frac{v}{c}\right)^2$$





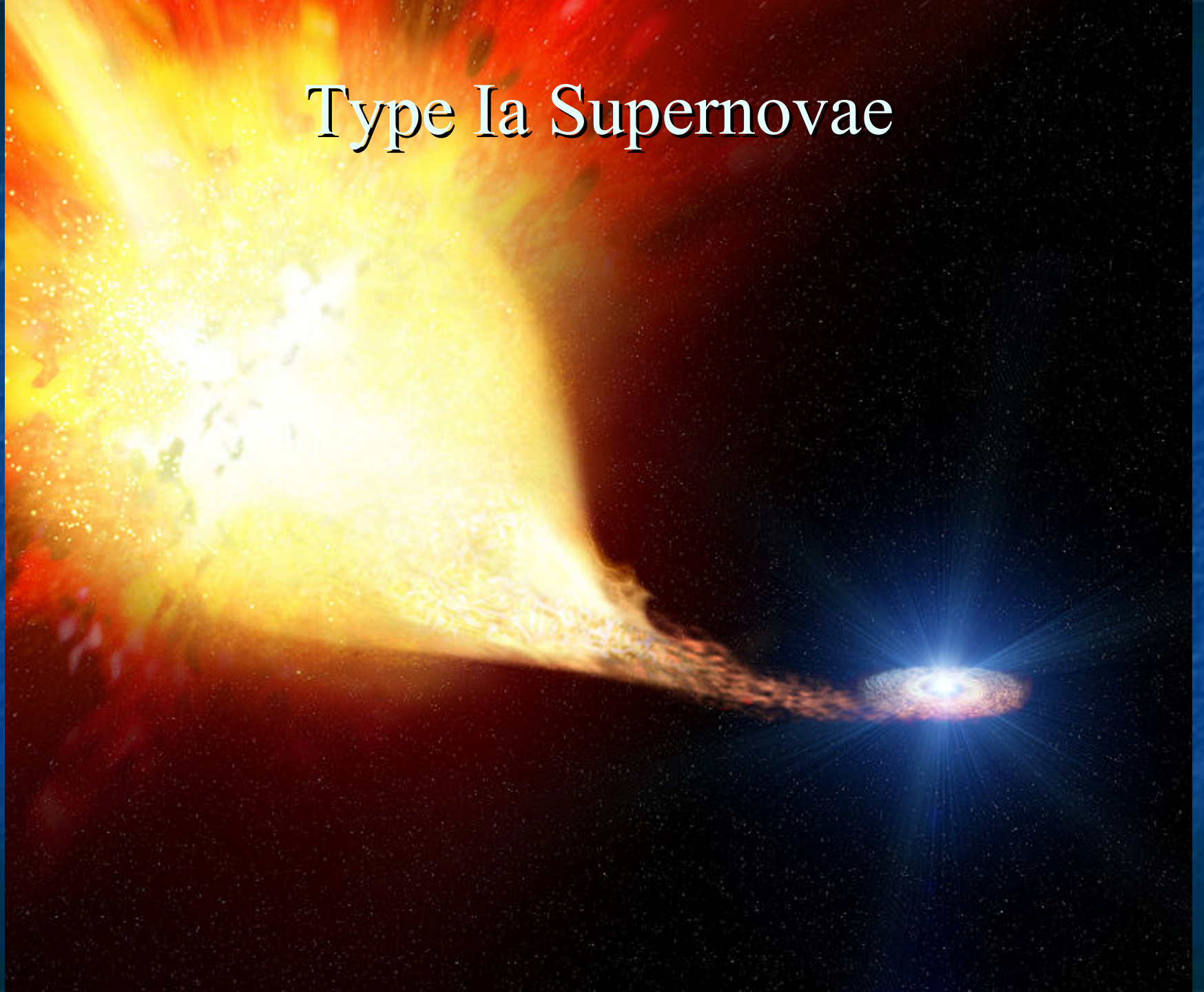
Distances in Cosmology

- Inside the solar system → Laser Ranging
 - Shoot a strong laser at a planet and measure the time it takes to be reflected back to us
- Inside the galaxy → stellar parallax
 - Requires precise astrometry
 - Maximum distance measured: 500 pc (1600 ly), by the Hipparcos satellite (1989–1993)
 - Dec. 2013 → Gaia satellite launched (2013 – 2019) → parallax up to ~50 kpc
- Compare with:
 - Milky Way → ~15 kpc radius
 - Andromeda → ~1 Mpc

Standard Candles

- A plot of distance vs. z is called a **Hubble Diagram**
- To measure distances at $z > \sim 0.0001$ (~ 0.4 Mpc) we need good standard candles (known intrinsic luminosity)
- There are 2 classic standard (rigorously, *standardizable*) candles in cosmology:
 - Cepheid variable stars ($0 < z < 0.05$) * *Jones et al., 1304.0768*
 - Type Ia Supernovae ($0 < z < 1.91^*$)
- Both classes have **intrinsic variability**, but there are empirical relations that allow us to calibrate and *standardize* them

Type Ia Supernovae



Type Ia Supernovae (3)

- Supernovae (SNe) are **very bright** explosions of stars
- There are 2 major kinds of SNe
 - Core-collapse (massive stars which run out of H and He)
 - Collapse by mass accretion in binary systems (**type Ia**)
 - White dwarf + red giant companion (single degenerate)
 - White dwarf + White dwarf (double degenerate)
 - Type Ia SNe explosion → ~ standard energy release
 - Chandrasekar limit on white dwarf mass: $M_{\text{max}} = 1.44 M_{\text{sun}}$
 - Beyond this → instability → explosion
 - SNe Ia → less intrinsic scatter + strong correlation between brightness & duration

Type Ia Supernovae (4)

- SNe Ia are so far the only proven standard(izable) candles for cosmology
- With good measurements → scatter < 0.15 mag in the Hubble diagram
- But arguably they are subject to more systematic effects than BAO (baryon acoustic oscillations) & CMB
 - Systematic errors already the dominant part ($N_{\text{SNe}} \sim 1000$)
 - In the next ~10 years → statistics will increase by 100x
 - Huge effort to improve understanding of systematics

Howell, 1011.0441 (review of SNe)

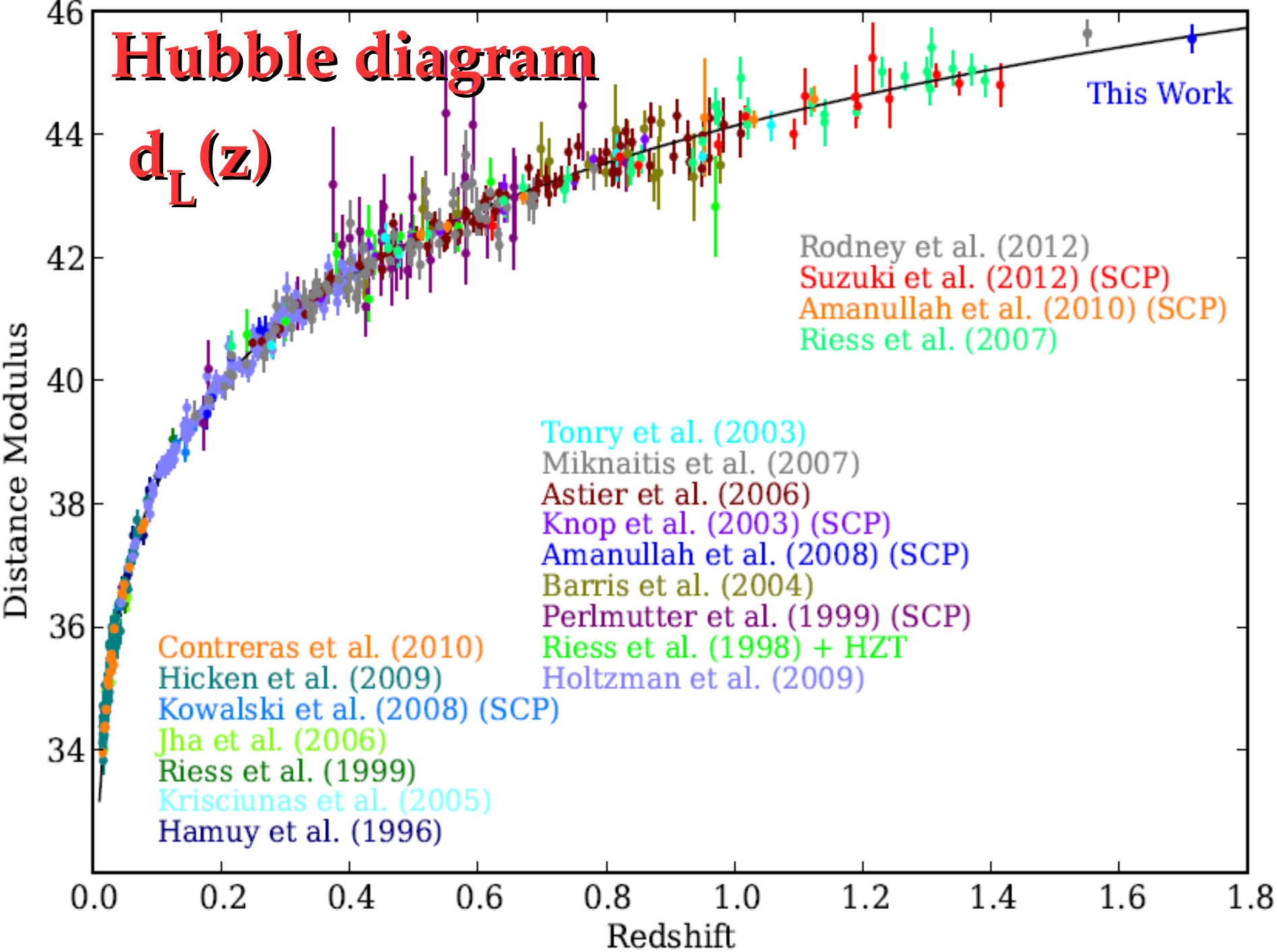
SNe Systematics

Systematic	SNLS3 ¹⁴³	CfA ²⁷ /ESSENCE ⁴⁴	SDSS-II ²⁶	SCP ²⁸
Best fit w (assuming flatness)	...	-0.987	-0.96	-0.997
Statistical error	...	0.067	0.06	0.052
Total stat+systematic error	...	0.13	0.13	0.08
Systematic error breakdown				
Flux reference	0.053	0.02	0.02	0.042
Experiment zero points	0.01	0.04	0.030	0.037
Low-z photometry	0.02	0.005
Landolt bandpasses	0.01	...	0.008	...
Local flows	0.014	...	0.03	...
Experiment bandpasses	0.01	...	0.016	...
Malmquist bias model	0.01	0.02	...	0.026
Dust/Color-luminosity (β)	0.02	0.08	0.013	0.026
SN Ia Evolution	...	0.02
Restframe U band	0.104	0.010
Contamination	0.021
Galactic Extinction	0.022	0.012

Table 1: **Best-fit values of $\langle w \rangle$ and error estimates.** For the CfA3/ESSENCE column

Hubble diagram

$$d_L(z)$$



Supernova Lensing

- Standard SNe analysis → geodesics in FLRW
- Real universe → structure (filaments & voids) → weak-lensing (WL) → **very skewed PDF** (Probab. Distr. Function!)
 - Most SNe → demagnified a little (light-path in voids)
 - A few → magnified “a lot” (path near large structures)
- The lensing PDF is the **key quantity**
 - Hard to measure → need many more SNe
 - Can be computed: ray-tracing in N-body simulations
 - See: *Takahashi et al. 1106.3823*
Hilbert et al. astro-ph/0703803
 - N-body → too expensive to do **likelihoods** → many parameter values (many Ω_{m0} , σ_8 , w_{DE} , etc.)

Supernova Lensing (2)

- Supernova light travels huge distances
 - Lensing → *on average* → no magnification (photon # conser.)
- Important quantity → magnification PDF
 - Zero mean; very skewed (most objects de-magnified)
 - Adds **non-gaussian dispersion** to the Hubble diagram

$$\bar{\mu} \equiv \text{magn} = \frac{1}{(1 - \kappa)^2 - \gamma^2} \simeq \frac{1}{(1 - \kappa)^2}$$

$$\kappa(z_s) = \int_0^{r_s} dr \rho_{M0} G(r, r_s) \delta_M(r, t(r))$$

Function of three $d_A(z)$

Supernova Lensing (3)

- Note that the N-body approach might not be appropriate
 - Supernovae light bundles form a very thin (< 1 AU) pencil
 - N-body simulations coarse grained in scales $\ggg 1$ AU
 - Relativistic effects (e.g. Ricci + Weyl focusing) might be important

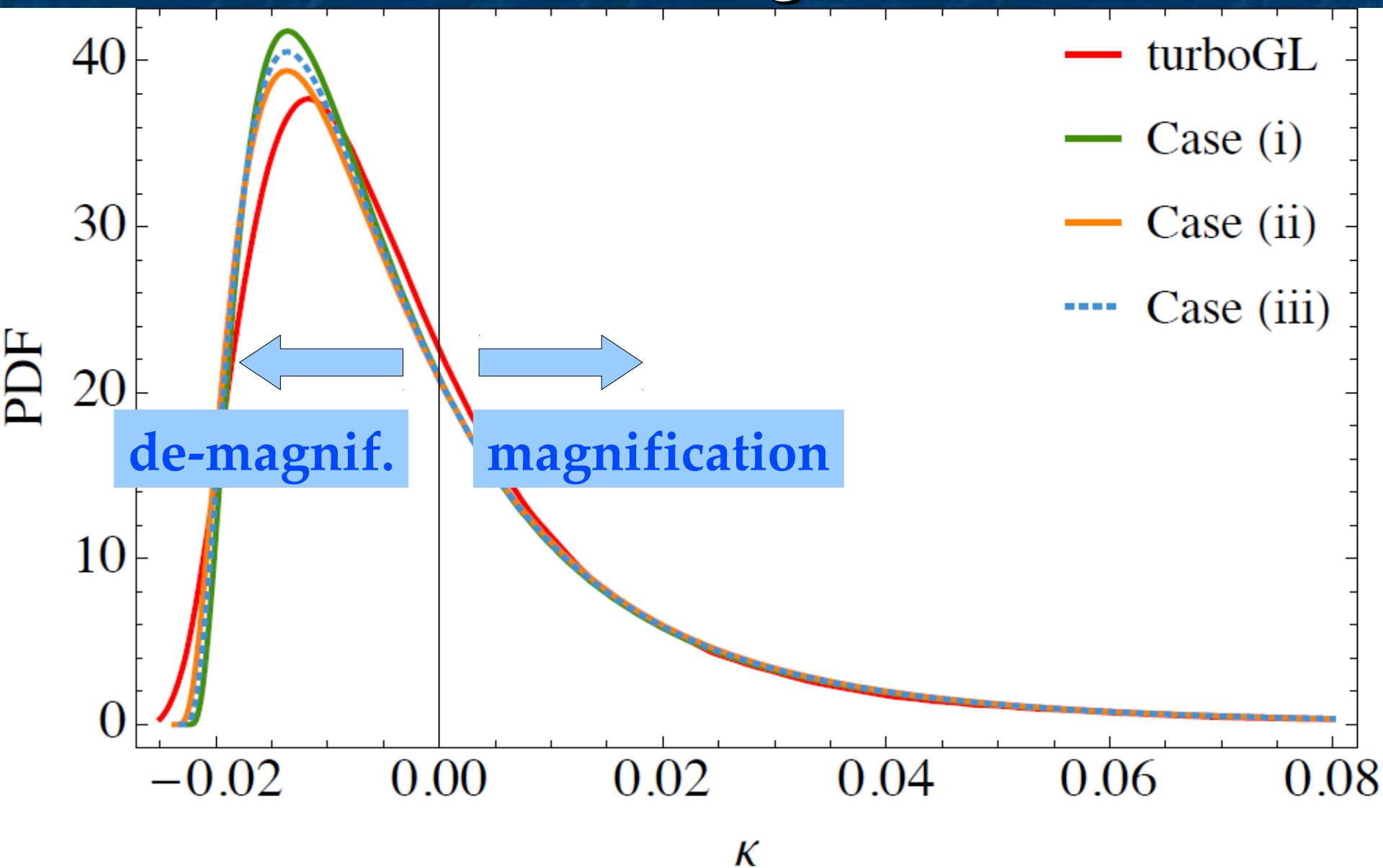
*Clarkson, Ellis, Faltenbacher, Maartens, Umeh, Uzan
(1109.2484, MNRAS)*

- There are also corrections due to a neglected Doppler term

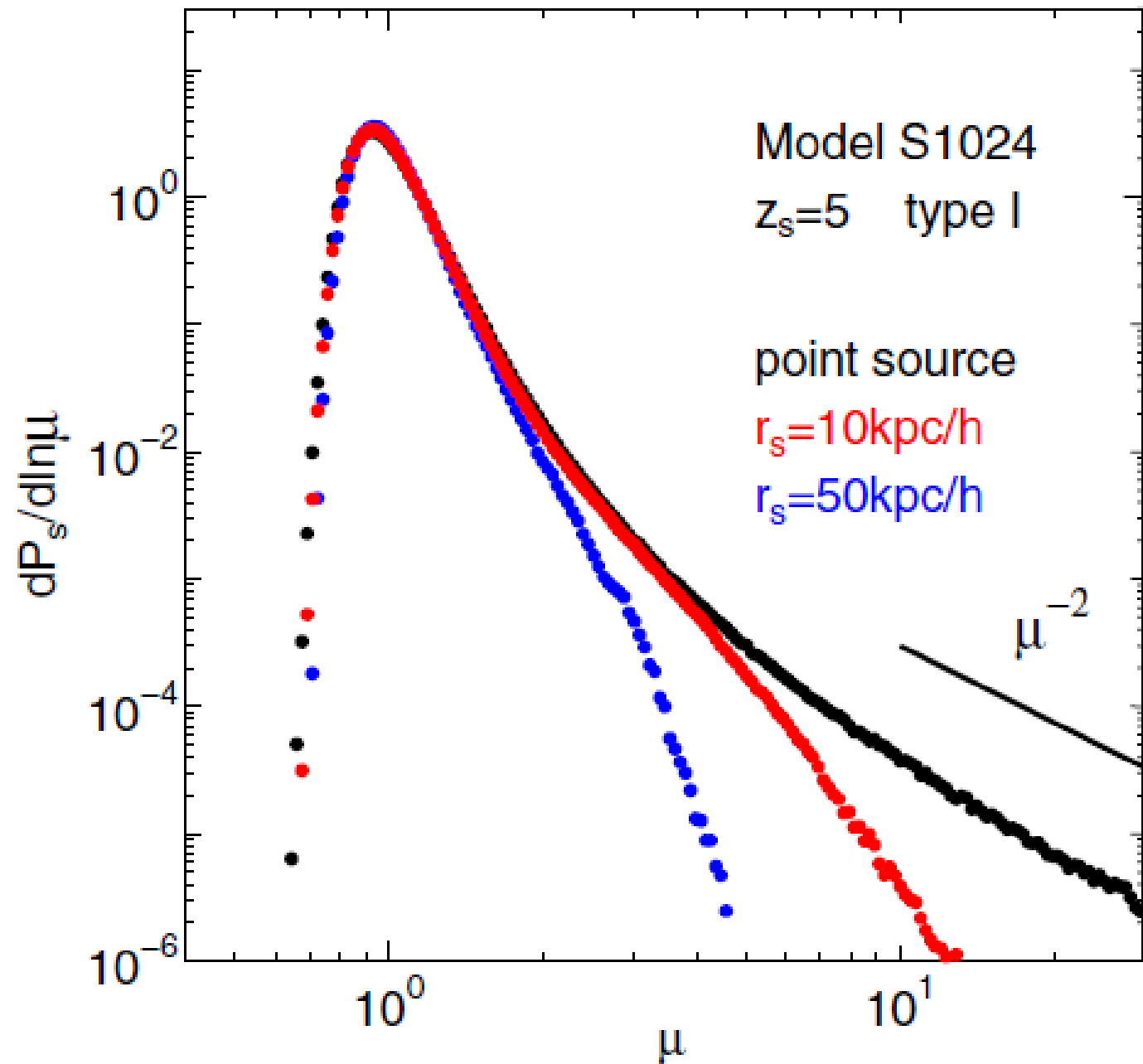
*Bolejko, Clarkson, Maartens, Bacon, Meures, Beynon
(1209.3142, PRL)*

- We neglect these corrections here

The Lensing PDF

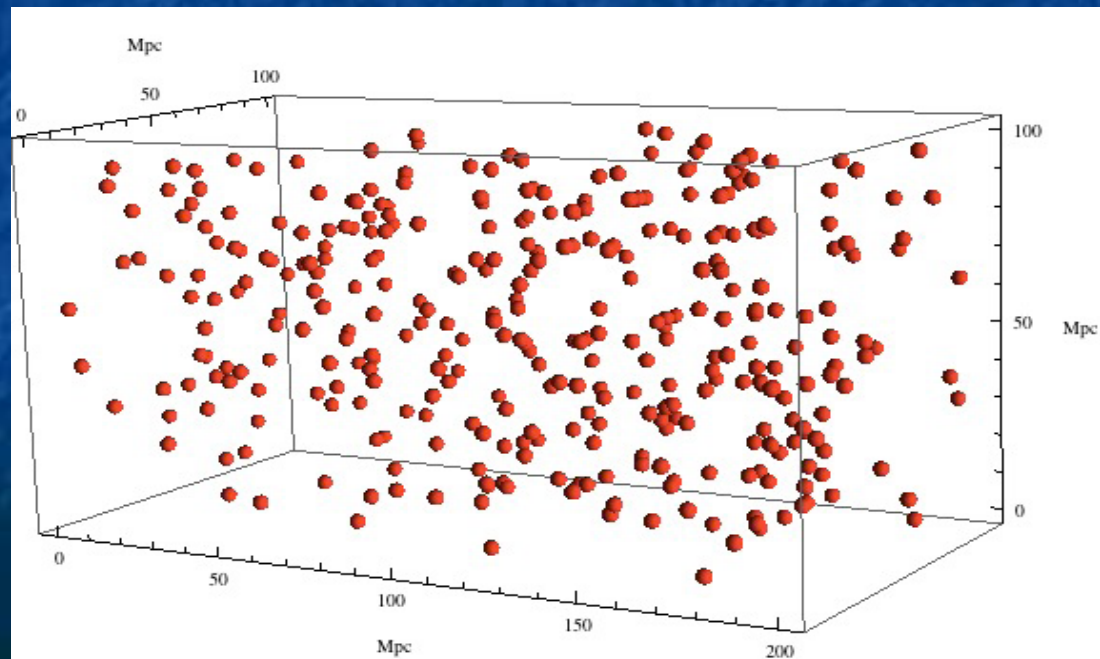


Finite sources



A New Method

- We need something faster → **stochastic GL analysis (sGL)**
 - Populate the universe with NFW halos → Halo Model
 - need prescriptions for mass fun. & concentration param.
 - In a given direction, draw nearby distribution of halos
 - **Bin** in distance & impact parameter
 - compute the convergence (**fast**)

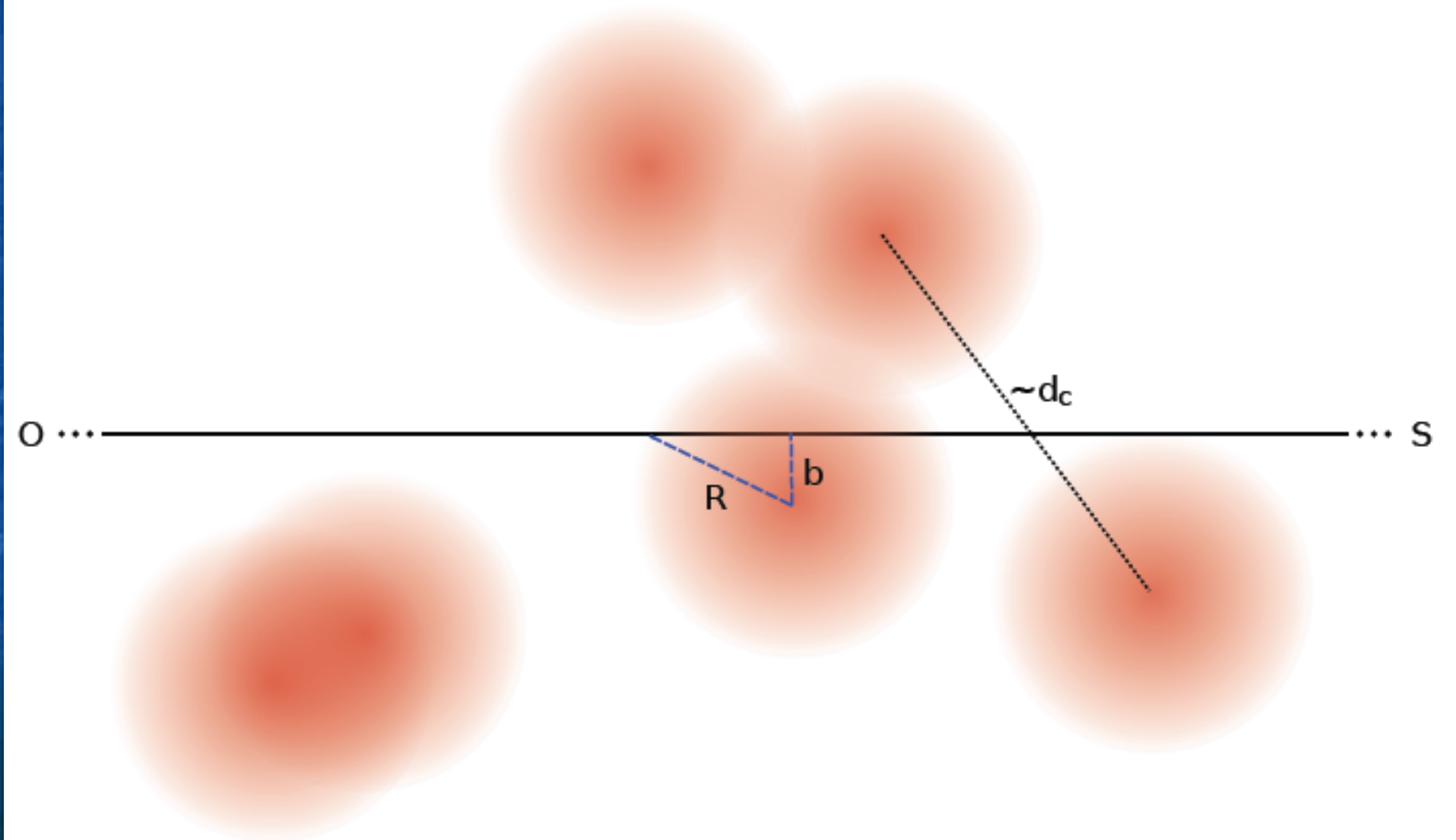


K. Kainulainen & V. Marra

0906.3871 (PRD)

0909.0822 (PRD)

A New Method (2)



NFW Profile

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}$$



Supernova Lensing (4)

- sGL → **fast way** to compute the κ PDF
 - accurate when compared to N-body simulations
 - many redshift bins; **different cosmological** parameters
 - fast enough to be used on likelihood analysis
 - Mathematica code available at www.turbogl.org
- We computed the κ PDF for a broad parameter range
 - PDF is well parametrized by the *first 3 central moments*
 μ_2, μ_3, μ_4
 - Lensing depends mostly on Ω_{m0} & σ_8
 - Very weak dependence on: $w, h, \Omega_{k0}, n_s, w, \dots$

Marra, Quartin & Amendola 1304.7689 (PRD)

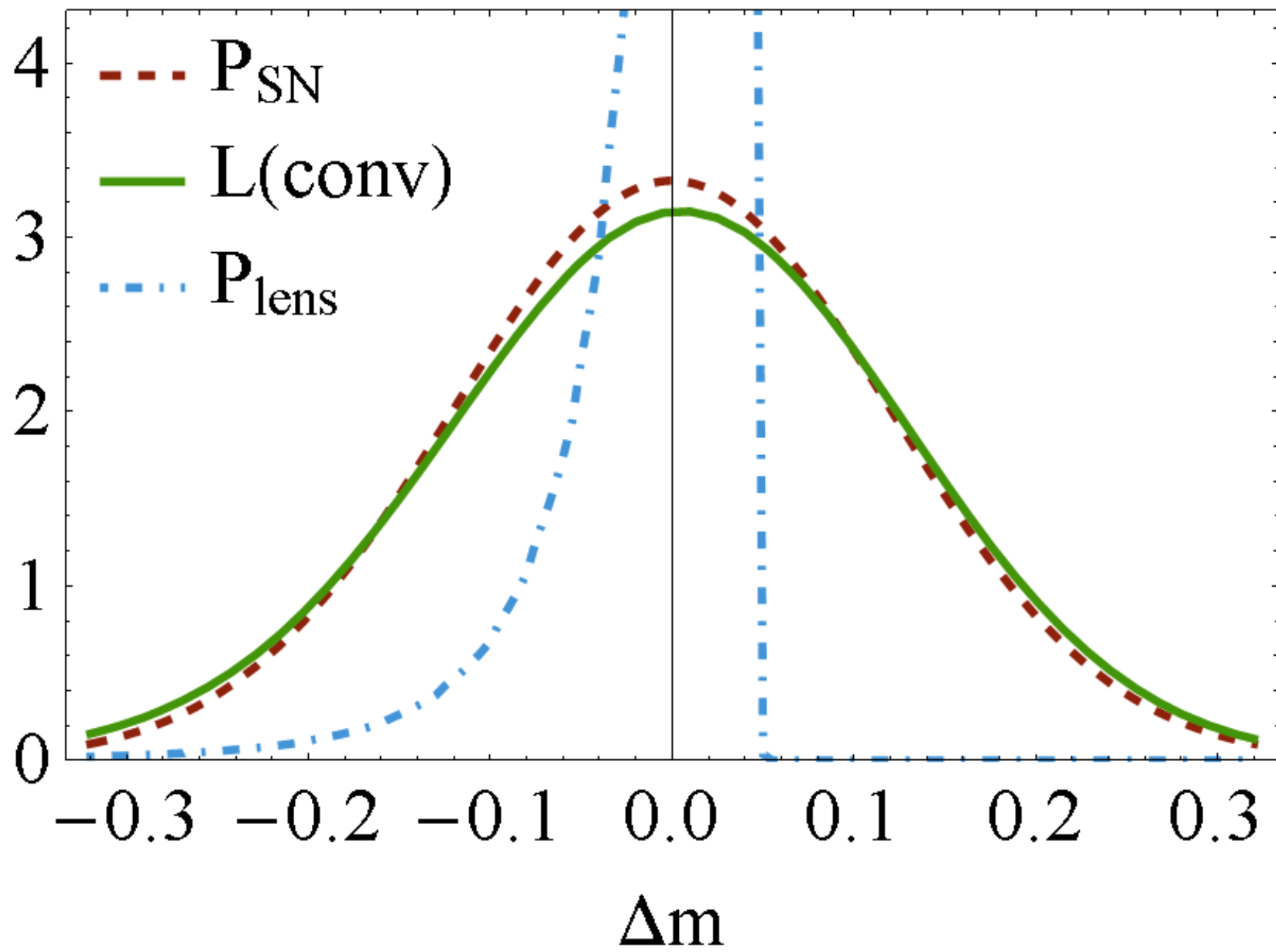
Supernova Lensing (5)

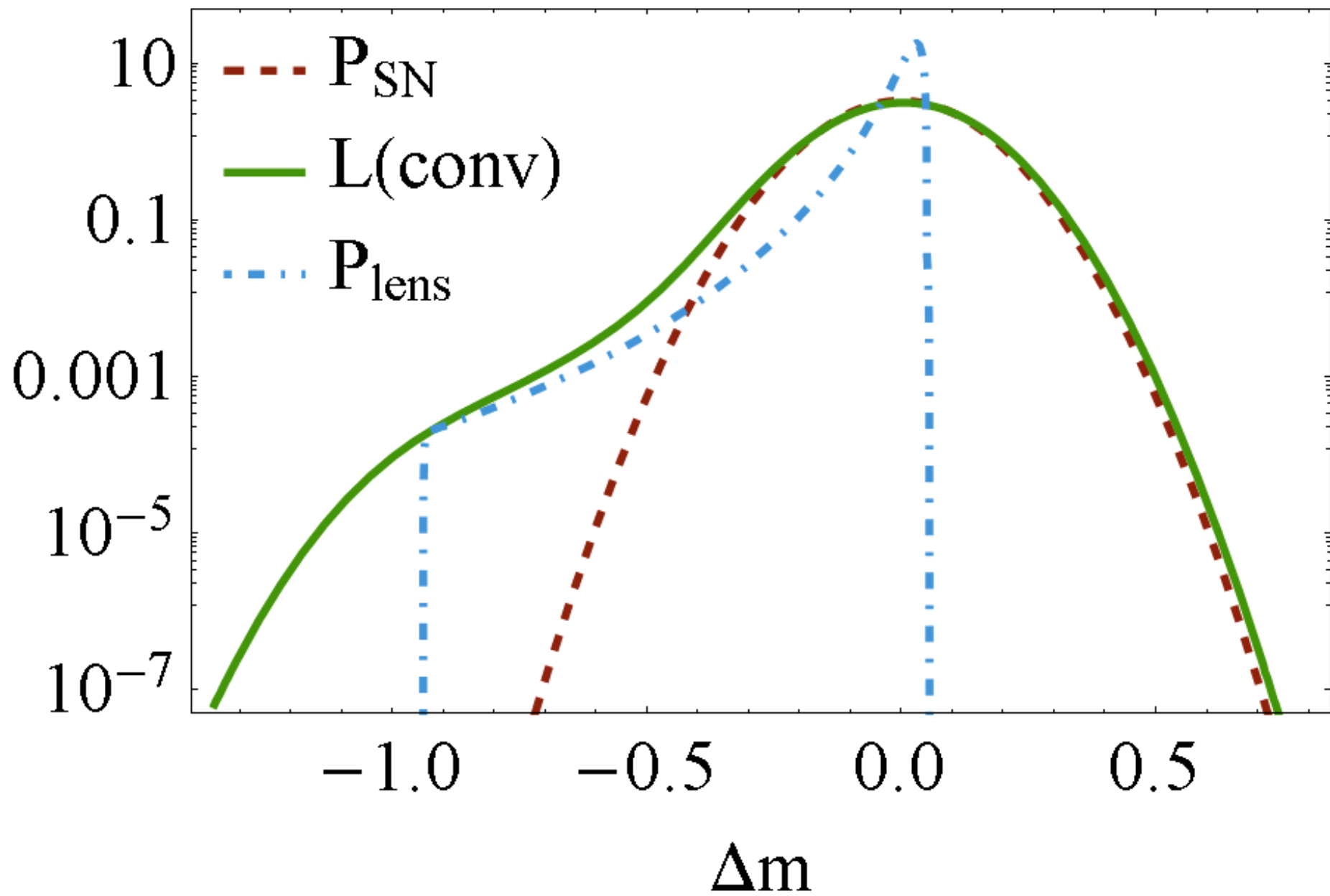
- Likelihood for SNe analysis → convolution of lensing PDF and intrinsic (standard) SNe PDF

$$L(\mu) = \int dy P_{wl}(y, \Omega_{m0}, \sigma_8, \dots) P_{SN}(\Delta m - \mu - y, \sigma)$$

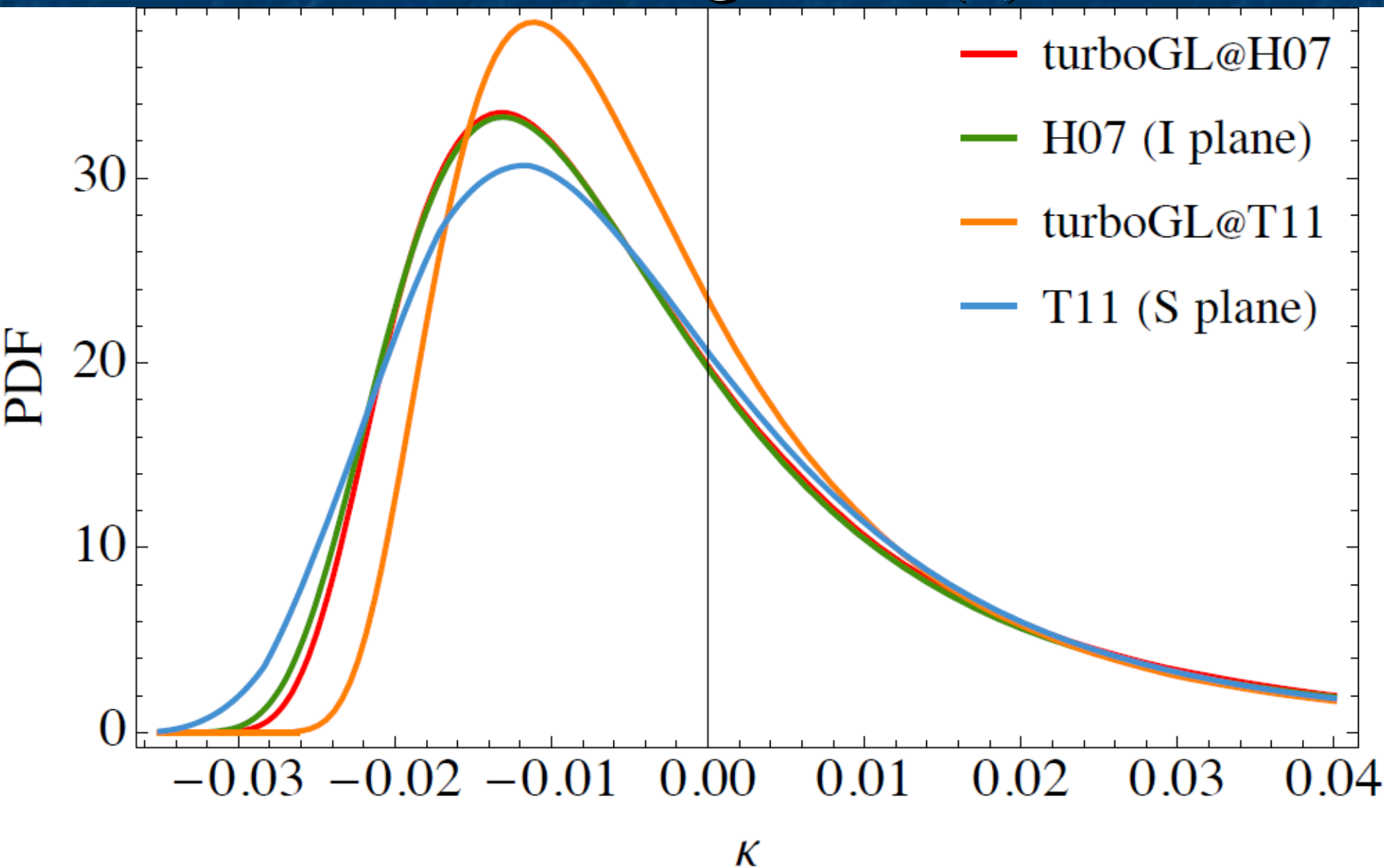
Gaussian!

- It is useful to compute the *first central moments* of the PDF
 - Mean (zero); variance; skewness & kurtosis
 - “Cumulants cumulate”:
 - Convolution variance = lensing var + intrinsic var
 - Convolution skewness = lensing skew + “0”
- We computed the κ PDF for many cosmological params.

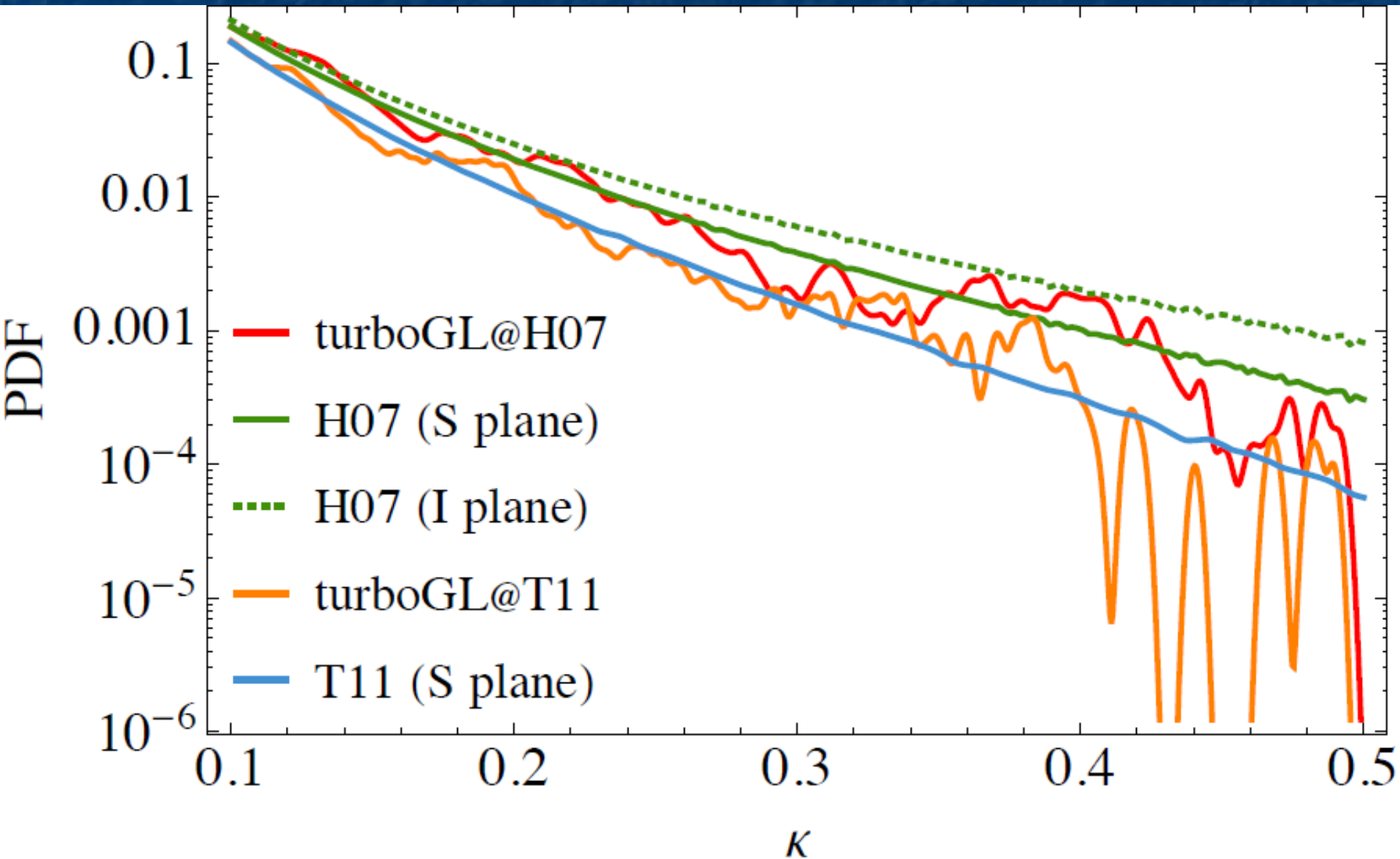




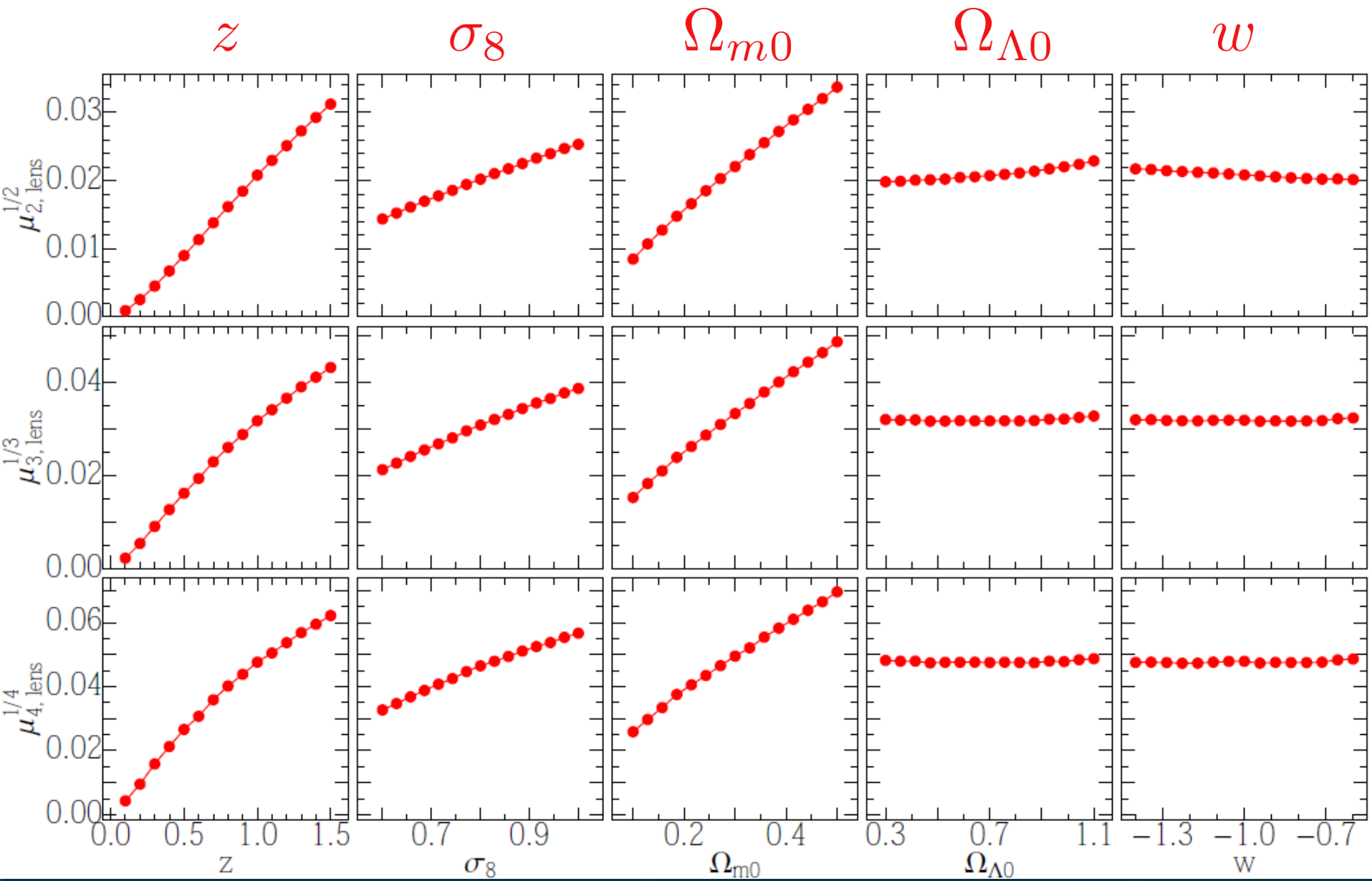
The Lensing PDF (2)



The Lensing PDF (3)



Variance, Skewness & Kurtosis



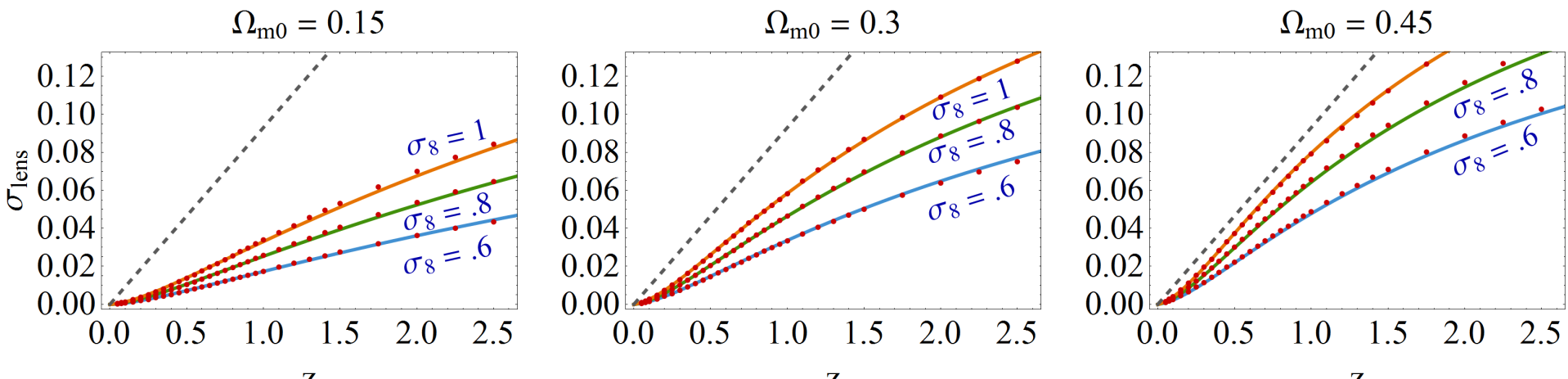
Fitting Functions

- We provide accurate and flexible analytical fits for the variance, skewness & kurtosis
 - Significant improvement upon current HL fit: $\sigma_{\text{lens}}^{\text{HL}} = 0.093z$

$$0 \leq z \leq 3$$

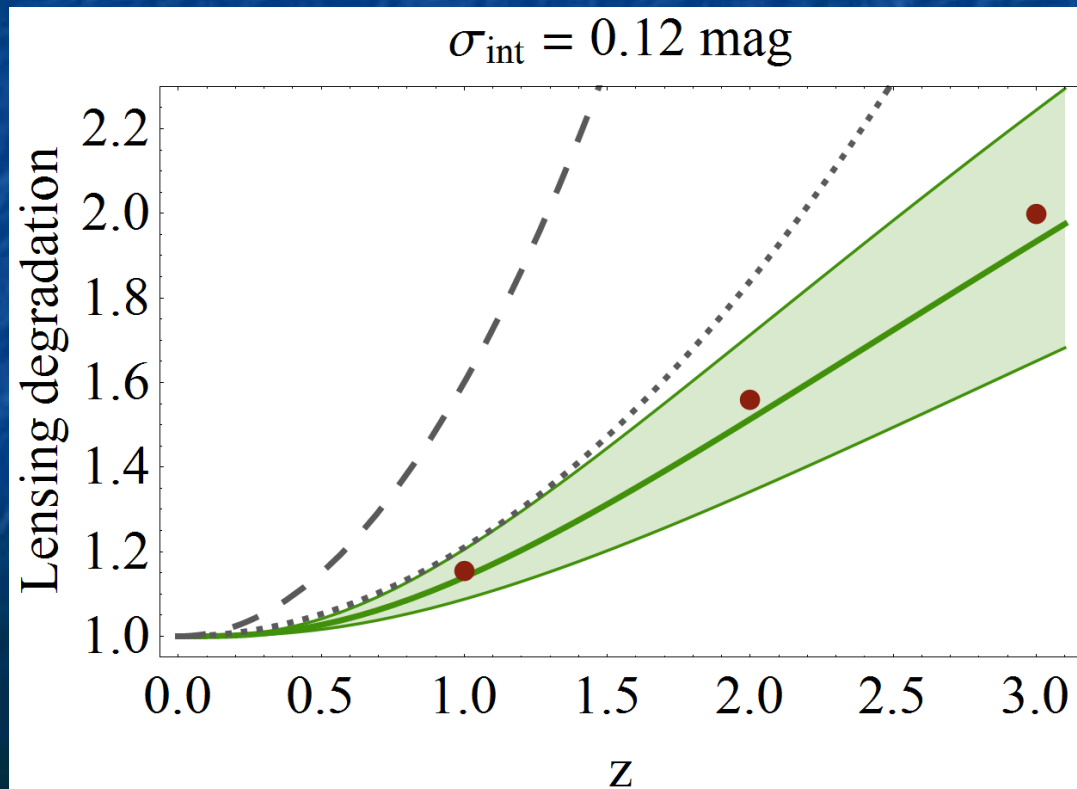
$$0.35 \leq \sigma_8 \leq 1.25$$

$$0.1 \leq \Omega_{m0} \leq 0.52$$



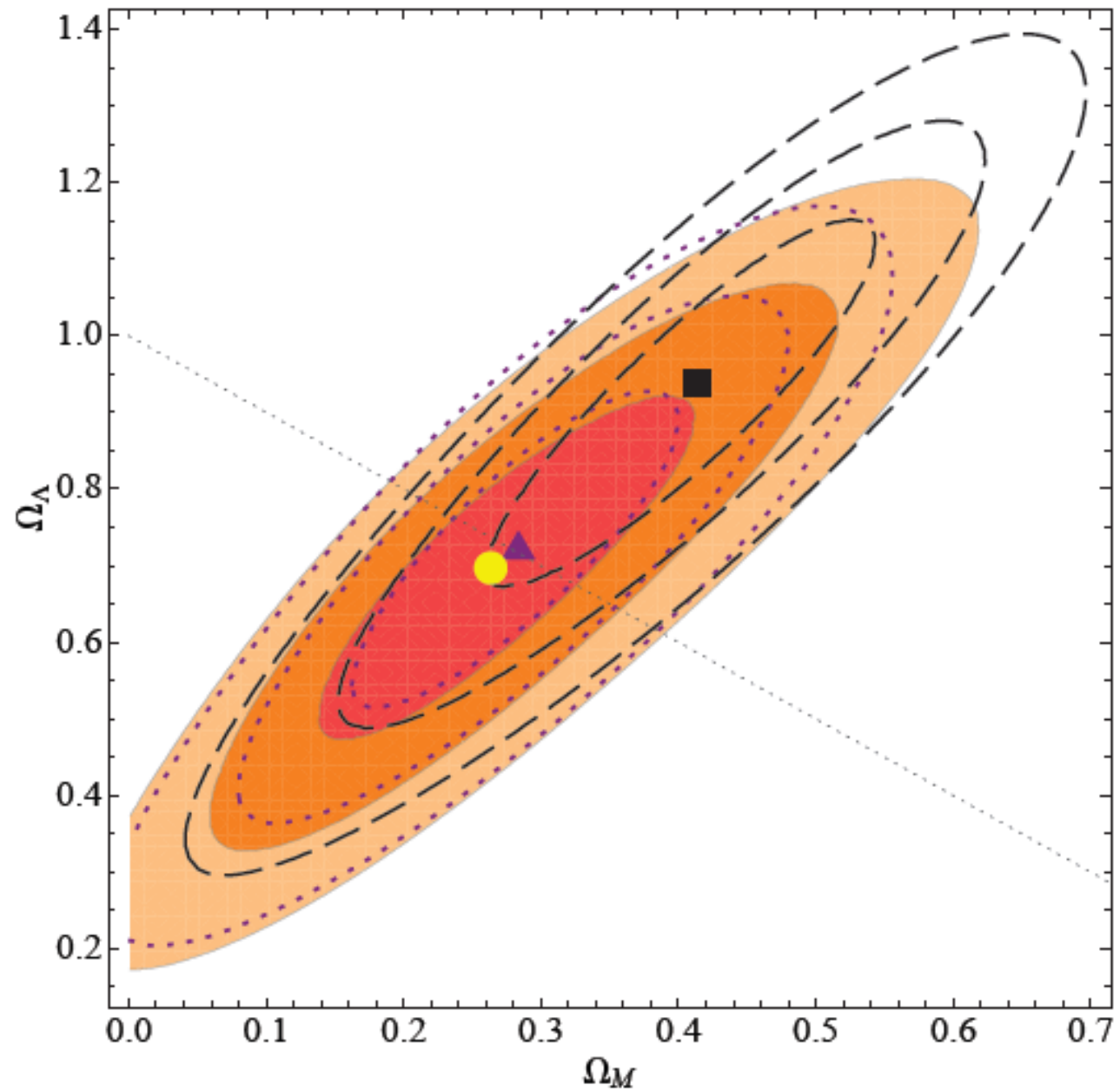
Fitting Functions (2)

- We find that the variance is $\sim 2x$ smaller than some previous estimates *D. Holz & E. Linder 0412173 (ApJ)*
 - But are in better agreement w/ SNLS *Jonsson et al. 1002.1374 (MNRAS)*
- Conclusion \rightarrow high- z supernovae are **more useful** than sometimes thought
- Lensing bias less of a problem



Lensing bias

Exaggerated
effect

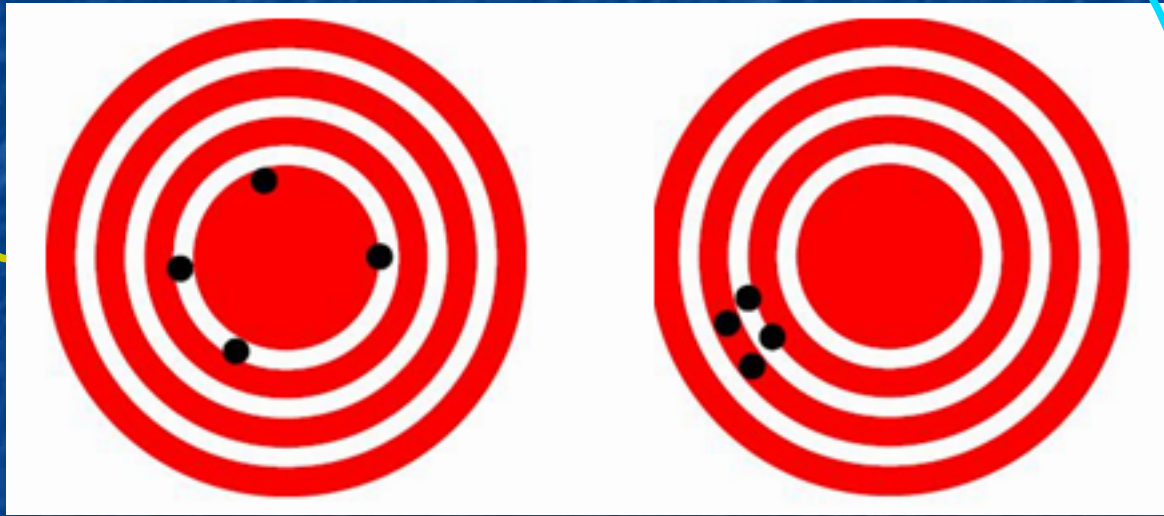


The Inverse Lensing Problem

- Can we turn Noise into Signal?
 - Can we learn about cosmology from the scatter of supernovae in the Hubble diagram? *Dodelson & Vallinotto*
 - Answer: **YES!** We can constrain σ_8 ! *(astro-ph/0511086)*
 - Caveat 1: no revolutionary precision
 - need $\sim 10^4$ SNe to get to $\sim 10\%$, $\sim 10^6$ to get to $\sim 1\%$
 - LSST will give us $\sim 10^6$
 - Caveat 2: need to assume halo profiles: e.g. NFW
 - It is a very good **cross-check**
 - It is a new observable

Quartin, Marra & Amendola 1307.1155 (PRD)

Precision Cosmology vs. Accuracy Cosmology



- Precise parameter estimation in Λ CDM **not enough**
 - Very important to cross-check **observations**
 - Rule-out systematics
 - Very important to cross-check **theoretical assumptions**
 - e.g.: homogeneity, isotropy

What is σ_8 ?

- σ_8 is the amplitude of the matter fluctuations at the scale of 8 Mpc/h

$$\Delta^2(r, z) \equiv \left(\frac{\delta M}{M} \right)^2 = \int_0^\infty \frac{dk}{k} \Delta^2(k, z) W^2(kr)$$

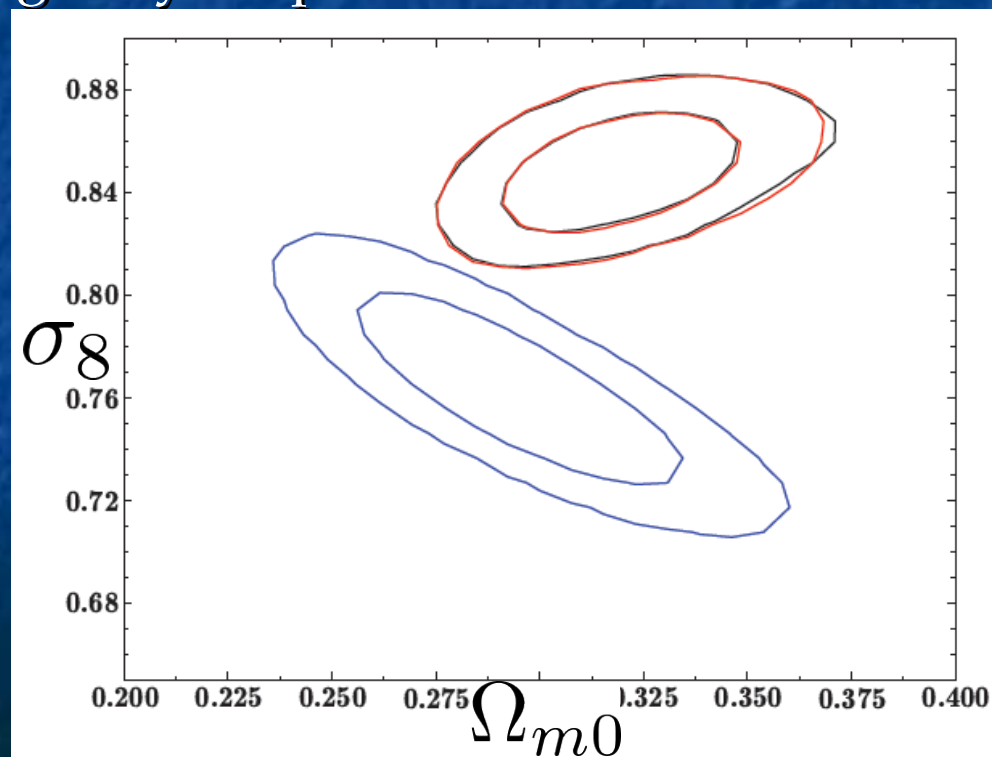
$$\Delta^2(k, z) \equiv \frac{k^3}{2\pi^2} P(k, z) = \delta_{H_0}^2 \left(\frac{ck}{a_0 H_0} \right)^{3+n_s} T^2(k/a_0) D^2(z)$$

$$\Delta(r = 8 \text{ Mpc}/h, z = 0) = \sigma_8$$

What is σ_8 ? (2)

- σ_8 measurements are usually done in either of 3 ways:
 - CMB \rightarrow measure fluctuations at $z = 1090$ and propagate them to $z = 0$
 - Cosmic Shear \rightarrow requires galaxy shapes
 - Cluster abundance
- Some tension between these measurements
 - Cross-check important!

Planck XX (1303.5080)

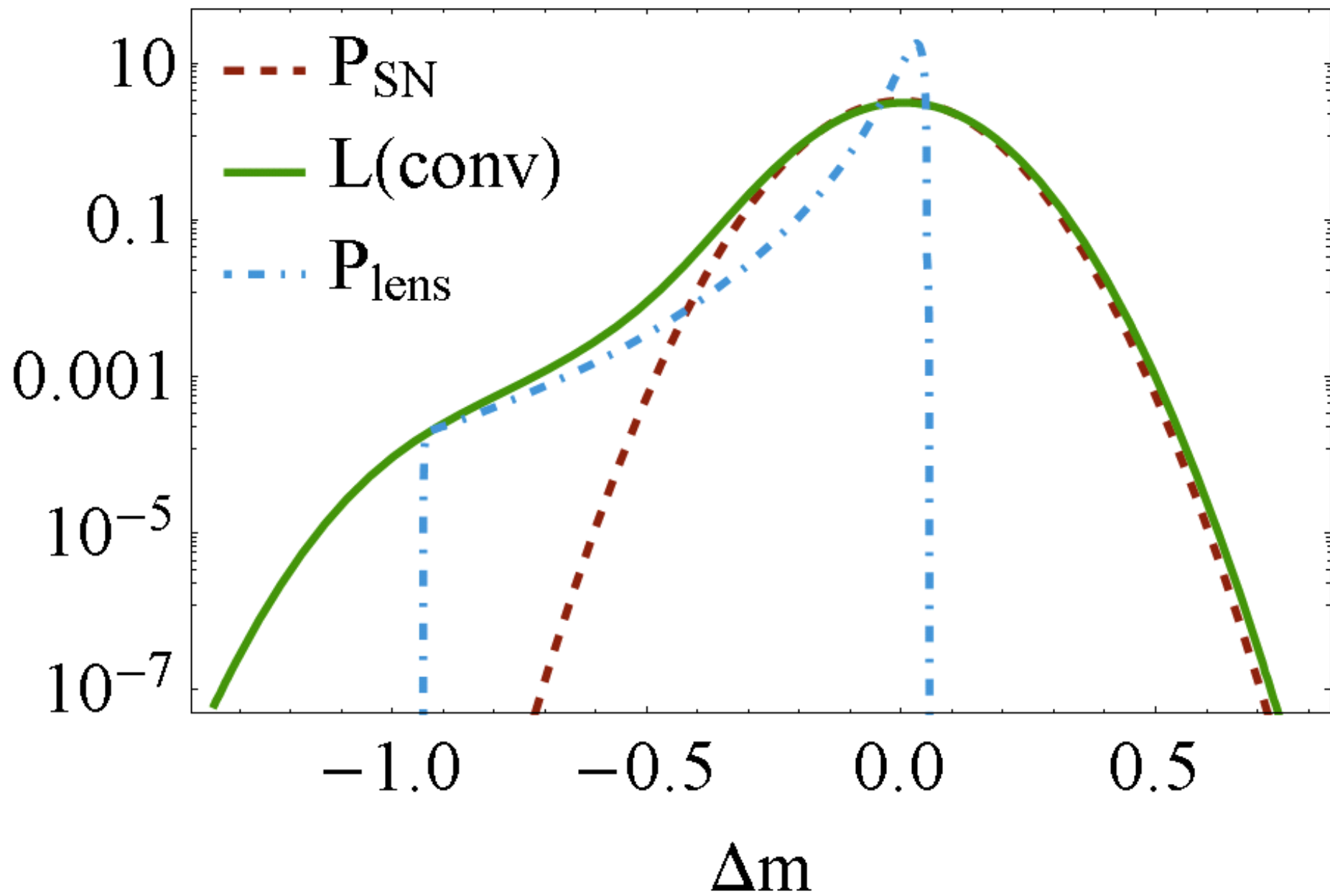


The Inverse Lensing Problem (2)

- Information from lensing \leftrightarrow full, lensing-dependent likelihood:

$$L(\mu) = \int dy P_{wl}(y, \Omega_{m0}, \sigma_8, \dots) P_{SN}(\Delta m - \mu - y, \sigma)$$

- It works BUT there is a faster & more interesting method: the Method of the Moments (MeMo)
 - Instead of the full lensing PDF we just use the first 3 **central moments**
 - *Advantages*: faster; directly related to observations \rightarrow simpler to control systematics step-by-step
 - *Disadvantage*: more involved equations



The MeMo Likelihood

- Using the first 4 moments, we write:

$$L_{\text{MeMo}}(\Omega_{m0}, \sigma_8, \{\sigma_{\text{int},j}\}) = \exp \left(-\frac{1}{2} \sum_j^{\text{bins}} \chi_j^2 \right),$$

$$\chi_j^2 = (\boldsymbol{\mu} - \boldsymbol{\mu}_{\text{fid}})^t \boldsymbol{\Sigma}_j^{-1} (\boldsymbol{\mu} - \boldsymbol{\mu}_{\text{fid}}),$$

$$\boldsymbol{\mu} = \{\mu'_1, \mu_2, \mu_3, \mu_4\},$$

- Very complicated covariance matrix: if the PDFs were gaussian, it would be:

$$\boldsymbol{\Sigma}_{\text{gau},j} = \frac{1}{N_j} \begin{pmatrix} \sigma_j^2 & 0 & 0 & 0 \\ 0 & 2\sigma_j^4 & 0 & 12\sigma_j^6 \\ 0 & 0 & 6\sigma_j^6 & 0 \\ 0 & 12\sigma_j^6 & 0 & 96\sigma_j^8 \end{pmatrix}$$

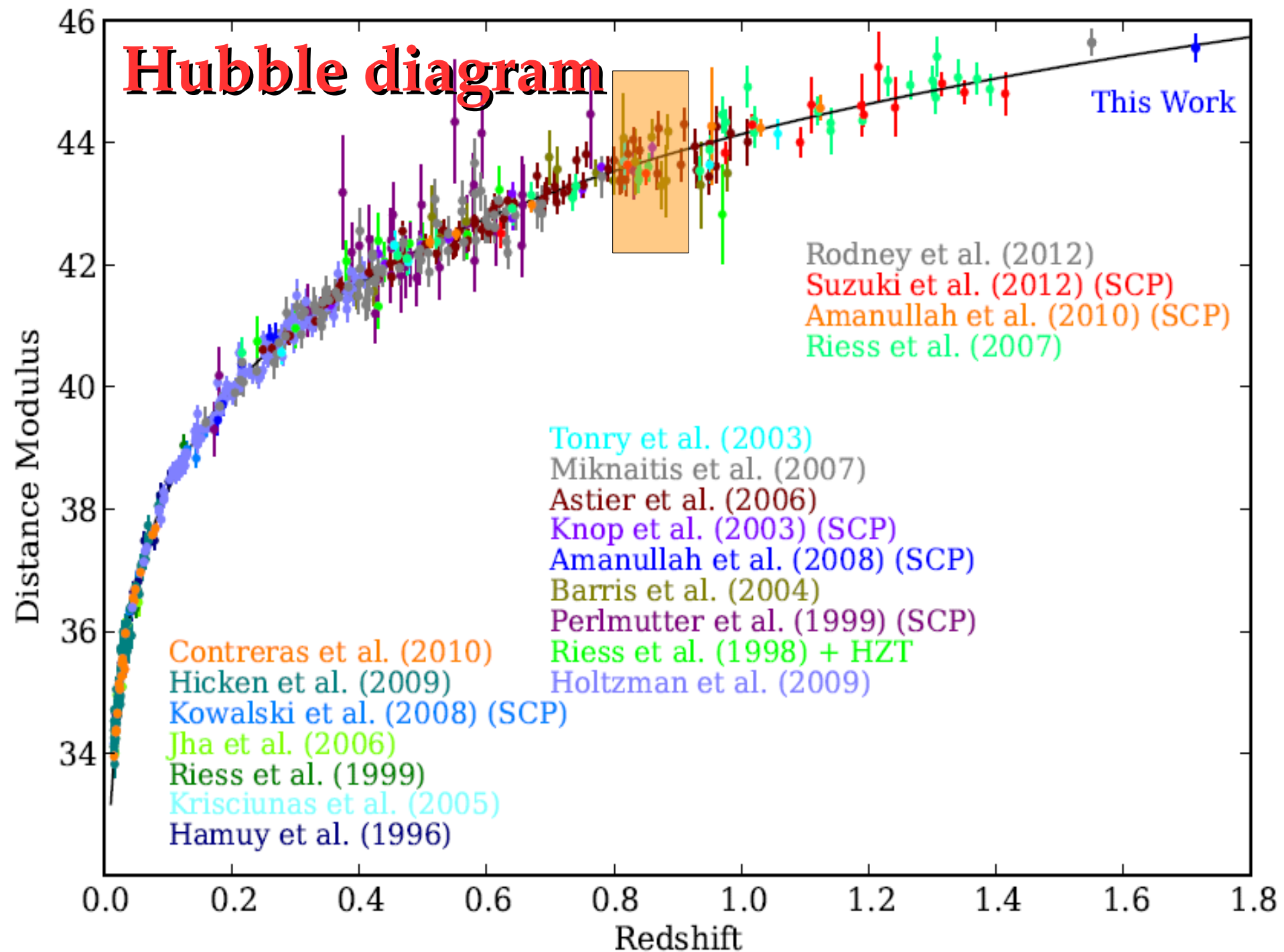
Scary Movie

- The full covariance matrix is very complicated.
 - Variance of the variance
 - Variance of the skewness
 - Variance of the kurtosis
 - Covariance terms...
- Must actually use the **sample central moments** (not the true central moments)
 - But could be done with a little help from Mathematica...

$$\Sigma_j = \frac{1}{N_j} \times$$

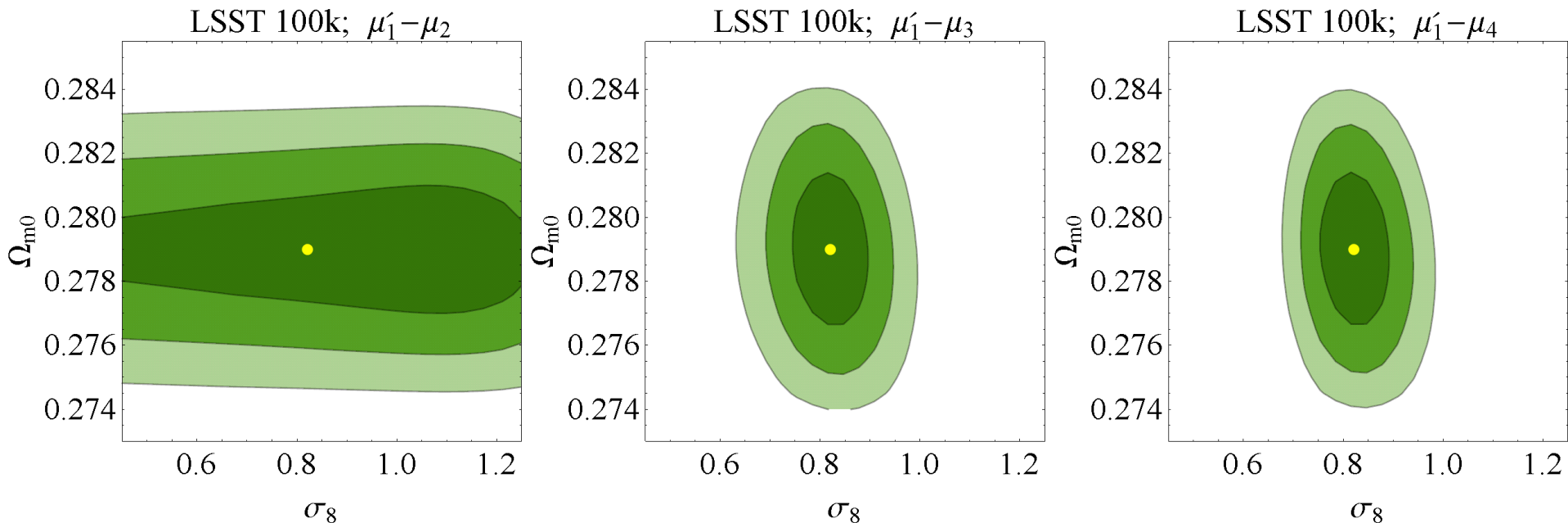
$$\begin{pmatrix} K_2 & K_3 & K_4 & 6K_2K_3 + K_5 \\ - & 2K_2^2 + K_4 & 6K_2K_3 + K_5 & 12K_2^3 + 14K_4K_2 + 6K_3^2 + K_6 \\ - & - & 6K_2^3 + 9K_4K_2 + 9K_3^2 + K_6 & 72K_3K_2^2 + 18K_5K_2 + 30K_3K_4 + K_7 \\ - & - & - & 96K_2^4 + 204K_4K_2^2 + 216K_3^2K_2 + 28K_6K_2 + 34K_4^2 + 48K_3K_5 + K_8 \end{pmatrix}$$

Hubble diagram



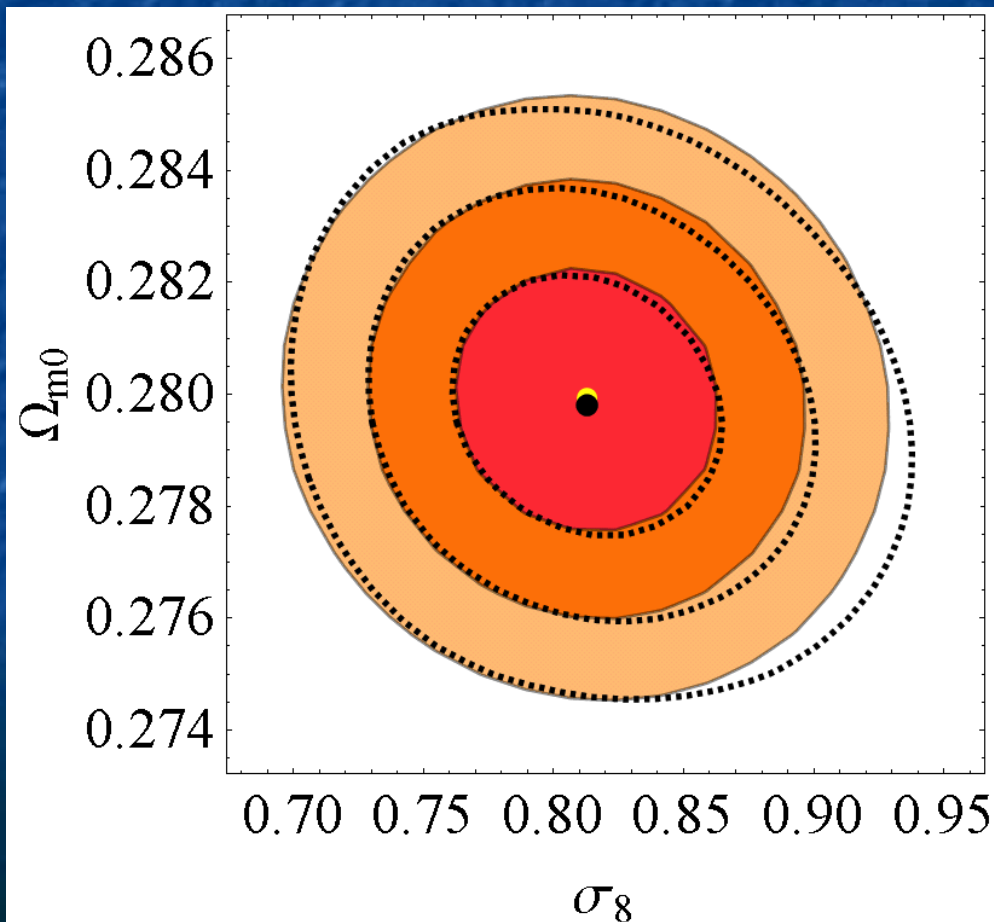
The MeMo Likelihood (2)

- How many moments are needed?
 - More moments \rightarrow more information
 - With first 3 we already have $\sim 90\%$ of the information
 - With first 4, we have close to 100%.



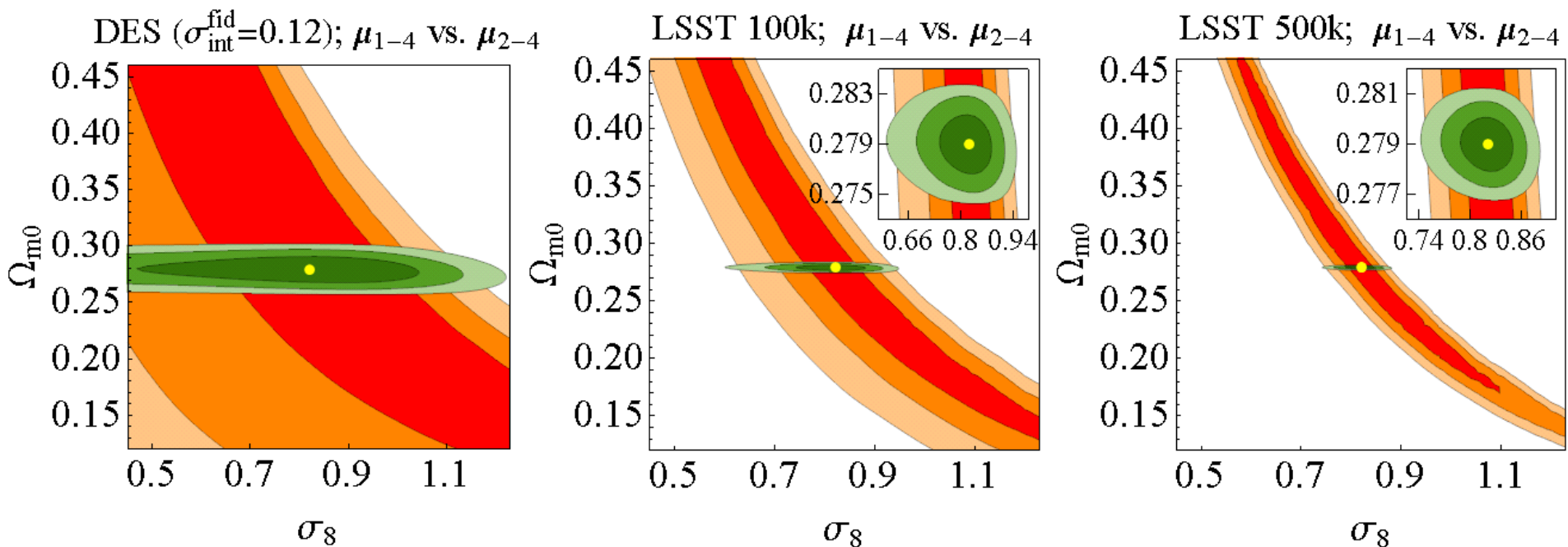
The MeMo Likelihood (3)

- Comparison between MeMo and full likelihood with first 4 moments:



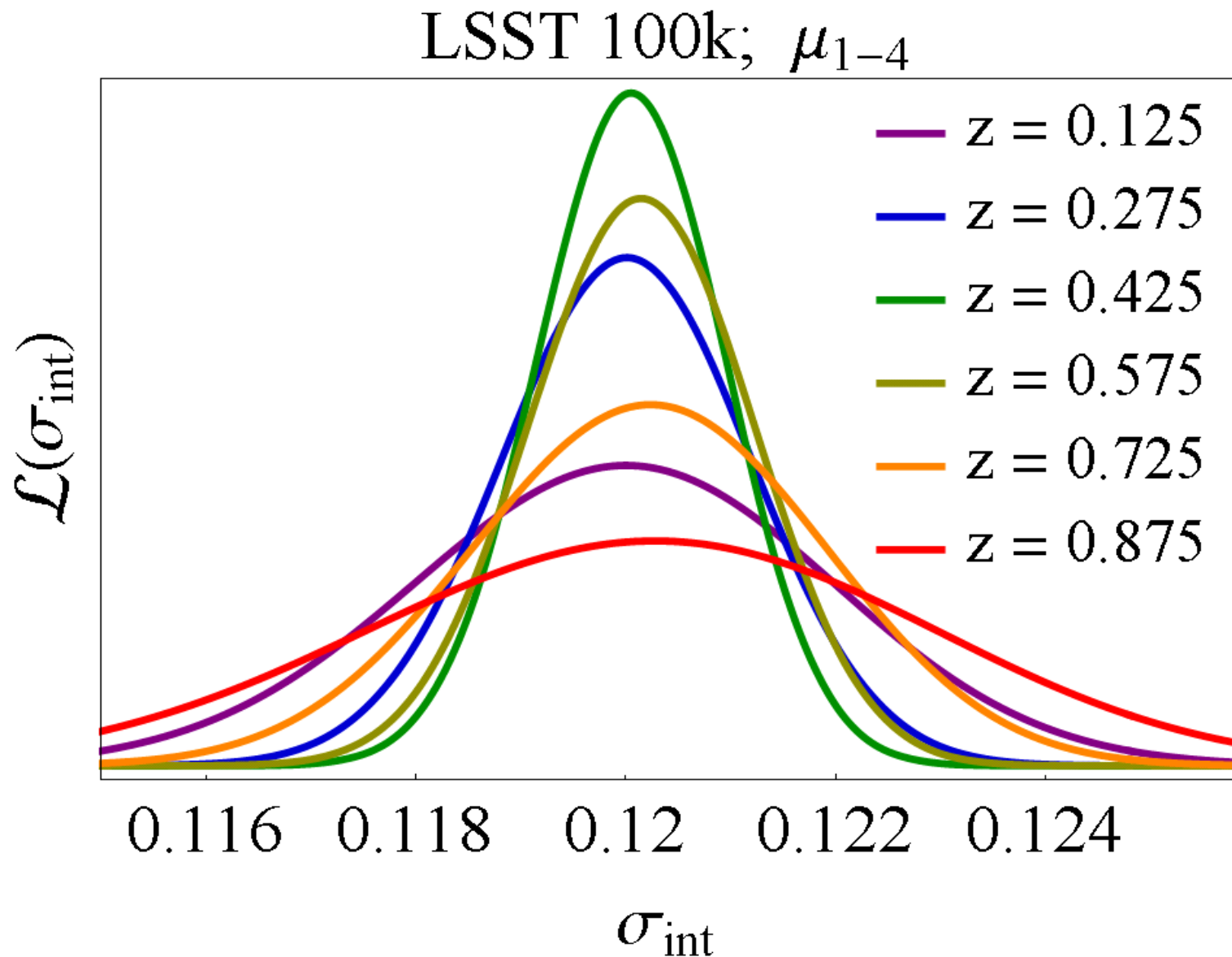
The Inverse Lensing Problem (2)

- The *non-gaussian scatter* of 10^5 supernovae in the Hubble diagram will tell us about σ_8 up to $\sim 7\%$ precision!



Quartin, Marra & Amendola 1307.1155 (PRD)

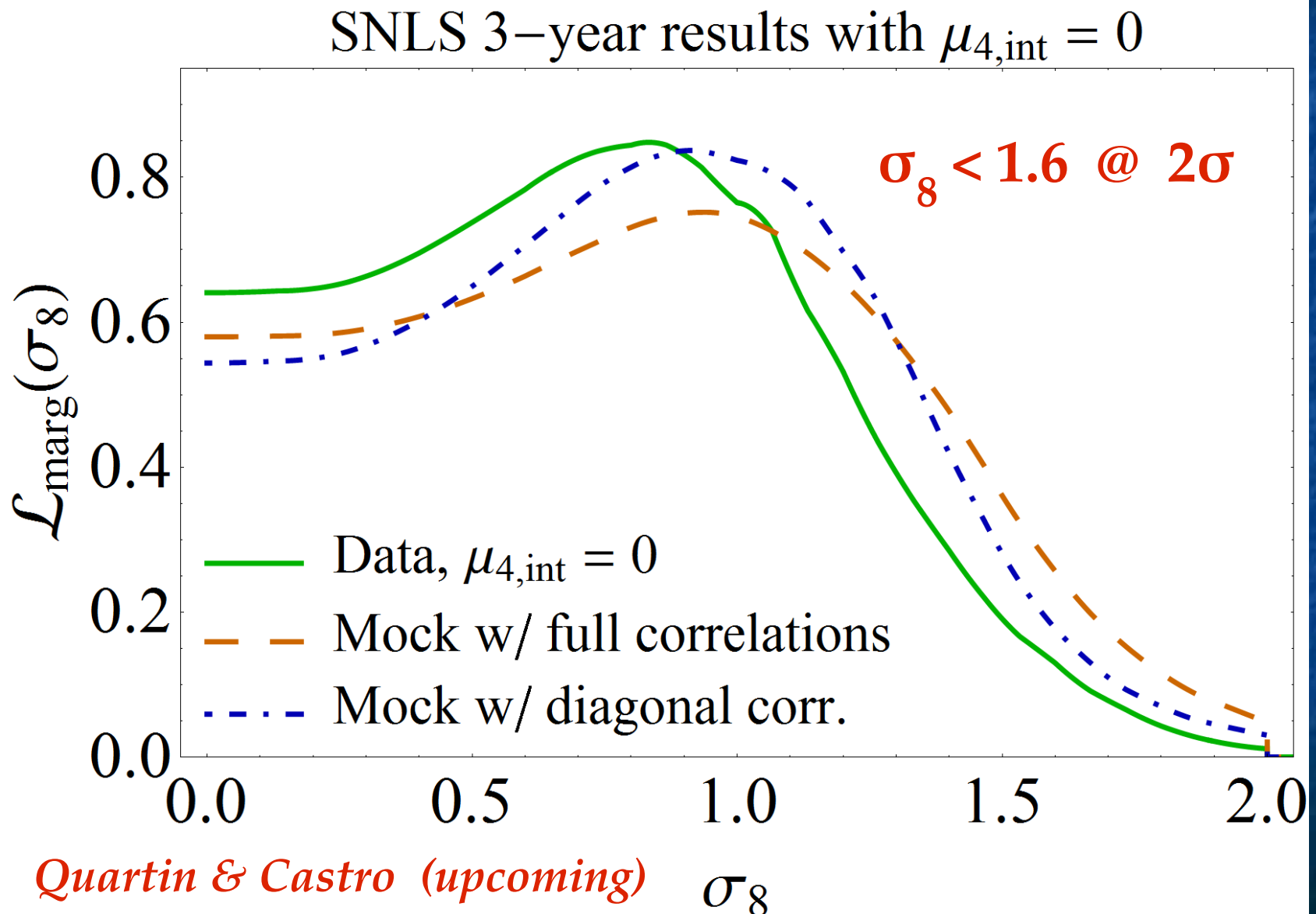
σ_{int} posteriors

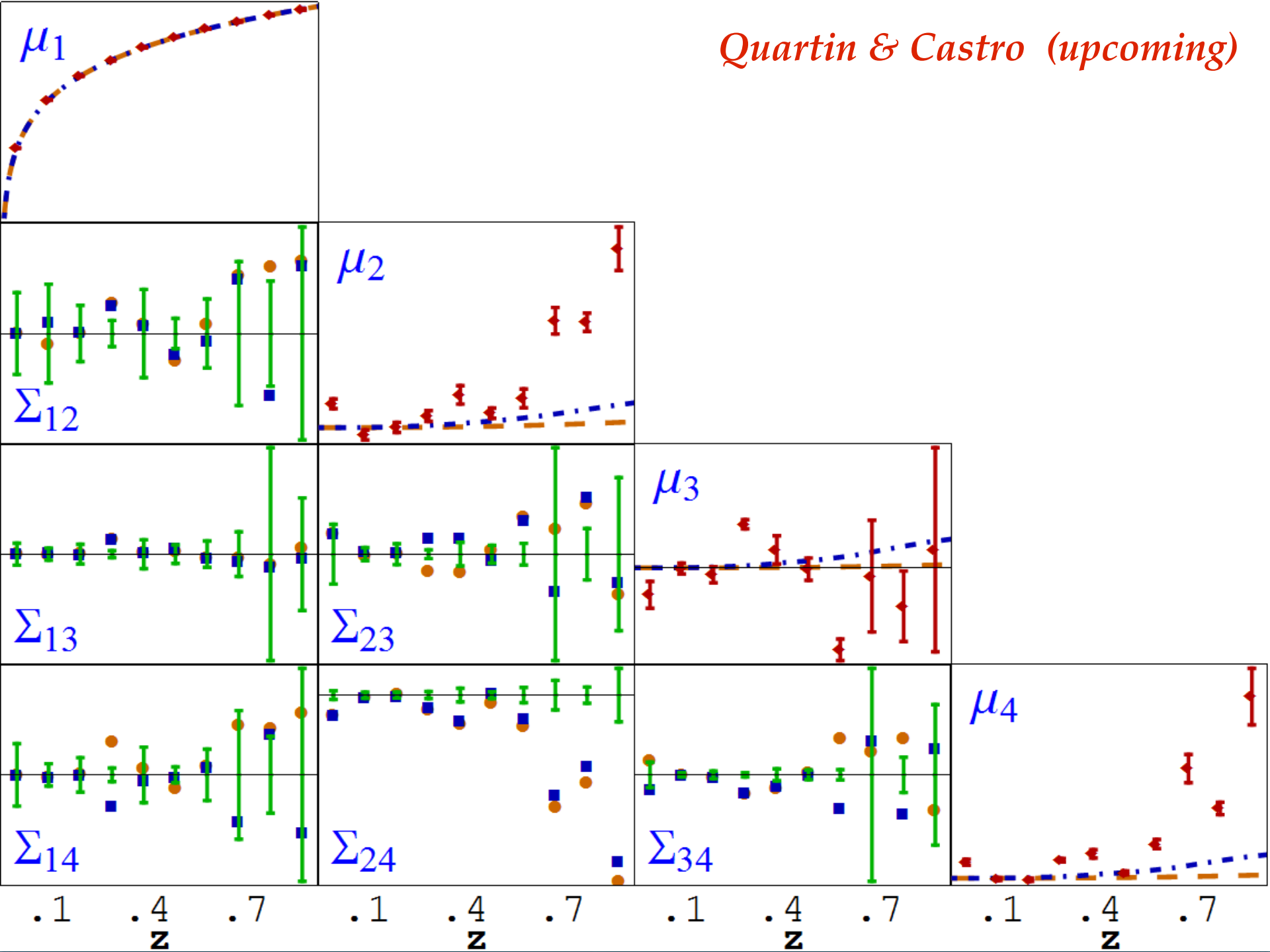


What IS a standard candle?

- Supernovae are assumed to be a standard candle
 - Intrinsic magnitude $M \rightarrow \text{const. in } z + \text{ gaussian scatter}$
 - A fine-tuned $M(z) \rightarrow \text{no acceleration} \rightarrow \text{no Nobel prize}$
- So why a Nobel prize?
 - It agrees with CMB & BAO (baryon acoustic oscillations)
 - Occam's Razor \rightarrow acceleration is the simplest model!
- Apply same reasoning for intrinsic non-gaussianity
 - Add nuisance parameters for intrinsic central moments
 - We tested this idea with the SNLS 3-year catalog

Proof-of-concept: SNLS 3-year data





Conclusions

- SNe Lensing has **already** been detected at $\sim 3\sigma$ (1307.2566)
 - But **not** detected from SNe data **alone!**
- Detailed lensing modeling important to avoid biases
- Lensing degradation smaller than previous estimate
- Supernova can constrain also **perturbation** parameters!
 - σ_8 to percent level with LSST.
 - SNLS3 (at face value) $\rightarrow \sigma_8 < 1.6 @ 2\sigma$
- Can also constrain halo profiles and dark matter clustering *Fedeli & Moscardini (1401.0011)*

Grazie!

Extra slides

