ESO ASTROPHYSICS SYMPOSIA

ESO

European
Southern
Observatory

L. N. da Costa · A. Renzini
Editors

GALAXY SCALING RELATIONS

Springer

da Costa · Renzini (Eds.) ■ Galaxy Scaling Relations

This book is part of the ESO conference series meant to survey the current status of different areas of astronomical research and explore science opportunities with the European Very Large Telescope (VLT). The research presented focuses on the origins, evolution and applications of galaxy scaling relations. Reviews of the properties and scaling relations of local galaxies and several reports of recent results from observations of high redshift galaxies using the Hubble Space Telescope and the Keck Telescope are also included. Current theoretical efforts to explain the origin of the observed scaling relations and the constraints imposed by the new data, as well as the use of galaxy–galaxy lensing to probe the scaling properties of dark matter halos are investigated. The book includes contributions that consider the use of the scaling relations to measure distance of galaxies and map the peculiar velocity field of galaxies in the nearby universe, as well as constraints imposed on cosmological models from the analysis of cosmic flows.

ISBN 3-540-63822-9

9 783540 638223

http://www.springer.de

Volume Editors

Luiz Nicolaci da Costa
Alvio Renzini
European Southern Observatory
Karl-Schwarzschild-Strasse 2
D-85748 Garching, Germany
email: ldacosta@eso.org
arenzini@eso.org

Series Editor

Philippe Crane
European Southern Observatory
Karl-Schwarzschild-Strasse 2
D-85748 Garching, Germany

Cataloging-in-Publication data applied for
Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Galaxy scaling relations : origins, evolution and applications ; proceedings of the ESO workshop, held at Garching, Germany, 18 - 20 November 1996 / L. N. da Costa ; A. Renzini (ed.). - Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ; Santa Clara ; Singapore ; Tokyo : Springer, 1997
(ESO astrophysics symposia)
ISBN 3-540-63822-9

ISBN 3-540-63822-9 Springer-Verlag Berlin Heidelberg New York

Printed in Germany

Typesetting: Camera ready by authors/editors
Cover design: *design & production* GmbH, Heidelberg
SPIN: 10552376 55/3142-543210 - Printed on acid-free paper

The Physical Origin of the Fundamental Plane (of Elliptical Galaxies)

Luca Ciotti

Osservatorio Astronomico di Bologna,
via Zamboni 33, 40126 Bologna, Italy

Abstract. I review the basic problems posed by the existence of the Fundamental Plane, and discuss its relations with the Virial Theorem (VT). Various possibilities are presented that can produce the observed uniform departure from *homology* (structural, dynamical) and/or from a constant *stellar* mass-to-light ratio. The role of orbital anisotropy and its relation with the FP thickness are also discussed. None of the explored solutions – albeit formally correct – are easily acceptable from a physical point of view, due to the ever-present problem of the required fine-tuning.

1 Observational Facts

Three are the main global observables of elliptical galaxies (Es): the central projected velocity dispersion σ_0, the effective radius R_e, and the mean effective surface brightness within R_e, $\langle I\rangle_e = L_B/2\pi R_e^2$. It is well known that Es do not populate uniformly this three dimensional parameter space; they are rather confined to a narrow logarithmic plane (Dressler et al. 1987; Djorgovski and Davis 1987), thus called the *Fundamental Plane* (FP). For example, for Virgo ellipticals

$$R_e \propto \sigma_0^{1.4} \langle I\rangle_e^{-0.85} \ . \tag{1}$$

Bender, Burstein, and Faber (1992, hereafter BBF) have introduced the k coordinate system, in which the new variables are a linear combination of the observables:

$$k_1 \equiv (2\log\sigma_0 + \log R_e)/\sqrt{2} \ , \quad k_2 \equiv (2\log\sigma_0 + 2\log\langle I\rangle_e - \log R_e)/\sqrt{6} \ , \tag{2}$$

$$k_3 \equiv (2\log\sigma_0 - \log\langle I\rangle_e - \log R_e)/\sqrt{3} \ . \tag{3}$$

In the new space, the k_1-k_3 plane provides an almost edge-on view of the FP, and $k_3 = 0.15k_1 + \text{const.}$ (Fig. 1 of BBF). Here I consider some of the implications of the two main properties of the FP of Virgo and Coma cluster ellipticals: 1) the FP is remarkably thin, with a 1-σ dispersion $\sigma(k_3) \simeq \pm 0.05$ (Bender, private communication); and 2) this thickness is nearly constant along the FP. First, I will try to clarify some frequently misunderstood points about the relations between the FP and the VT, and the meaning of structural/dynamical non-homology.

2 The FP and its Relation with the VT

The characteristic dynamical time of Es (e.g., within R_e) is

$$T_{dyn} \simeq (G < \rho >_e)^{-1/2} \approx 10^8 \text{yrs} \ , \tag{4}$$

and their collisionless relaxation time is of the same order (Lynden-Bell 1967), i.e., both are short with respect to the age of Es. As a consequence, only highly perturbed galaxies are presumably caught in a non-stationary phase. Stationarity is a *sufficient* condition for the validity of the VT, and so for Es the Virial Theorem holds. For a galaxy of total stellar mass M_* embedded in a dark matter halo of total mass M_h, the scalar VT can be written as

$$< V_*^2 >= (G\Upsilon_* L_B/R_e) \times (|\tilde{U}_{**}| + \mathcal{R}|\tilde{W}_{*h}|) \ , \tag{5}$$

where $\mathcal{R} \equiv M_h/M_*$ and $\Upsilon_* = M_*/L_B$ is the *stellar* mass-to-light ratio, here defined using the galaxy total *blue* luminosity. The dimensionless functions $\tilde{U}_{**}$ and $\tilde{W}_{*h}$ are the stellar gravitational self-energy and the interaction energy between the stars and the dark matter halo. They depend only on the stellar density profile and on the relative distribution of the stellar matter with respect to the gradient of the dark matter potential, i.e., the dimensionless function on the r.h.s. of equation (5) *depends only on the galaxy structure* (see, e.g., Ciotti, Lanzoni, and Renzini 1996, CLR). Moreover, $\tilde{U}_{**}$ and $\tilde{W}_{*h}$ are known to be *weakly dependent* on the particular density profiles (see, e.g., Spitzer 1969, Ciotti 1991, Dehnen 1993). Obviously the same comments apply also to the stellar velocity virial velocity dispersion $< V_*^2 >$, that necessarily results to be *independent* of the particular internal dynamics of the galaxy (e.g., the amount of orbital anisotropy).

$< V_*^2 >$ is related to σ_0^2 through a dimensionless function that depends on the galaxy structure, its specific internal dynamics and on projection effects:

$$< V_*^2 >= C_K(\textit{structure}, \textit{anisotropy}, \textit{projection}) \times \sigma_0^2 \ . \tag{6}$$

It is important to note that – also in absence of orbital anisotropy – C_K *is very sensitive to galaxy-to-galaxy structural differences*, much more than $\tilde{U}_{**}$ and $\tilde{W}_{*h}$, because it relates a weakly structure dependent quantity ($< V_*^2 >$) to a *local* one (σ_0^2). So, in practice structural non-homology always implies a *strong* variation in the σ^2 profile. Some authors call this phenomenon *dynamical* non-homology, but I (strongly!) suggest to call this effect *kinematical* non-homology, at least for two reasons. First, it is not very useful to call with two different terms (structural vs. dynamical non-homology) the *same* phenomenon: you cannot break structural homology without breaking also kinematical homology. Second, *dynamical* homology should be used to describe galaxies (for example) with the same relative amount of ordered rotation or orbital anisotropy. So, according to this nomenclature, kinematical non-homology can be induced by structural non-homology and/or dynamical non-homology. Moreover, globally isotropic galaxies with different density profiles are structurally non-homologous but dynamically homologous, their $R_e < V_*^2 > /G\Upsilon_* L_B$ are similar, but their C_K's can be significantly different. As a final remark, it is important to note that the strong dependence of C_K on galaxy structure is essentially due to its definition: in fact, using larger and larger aperture velocity dispersions instead of the *central* velocity dispersion σ_0^2, the projected Virial Theorem is better and better approximated, and for a spherical system without dark matter $C_K \to 3$ independently of the galaxy orbital structure (e.g., Ciotti 1994; cfr. also the results of the numerical simulations of mergers of Capelato et al. 1995).

From equations (2)-(3) and (5)-(6), defining

$$\Theta \equiv [2\pi G\,(|\tilde{U}_{**}| + \mathcal{R}|\tilde{W}_{*h}|)]/C_K \ , \tag{7}$$

one finally obtains

$$k_1 = \log(\Theta \times \Upsilon_* \times L_B/2\pi)/\sqrt{2} \ ; \quad k_3 = \log(\Theta \times \Upsilon_*)/\sqrt{3} \ . \tag{8}$$

Note that the VT does not imply any FP, in fact for fixed L_B different galaxies, all satisfying the VT, can in principle have very different Θ and Υ_*, and so be scattered everywhere in the k-space. So, the statement that *the FP deviates from the VT* – often stated because of the difference between the exponents in eq. (1) and those in the VT [eq. (5)] – is wrong: the FP deviates from *homology* (in a broad sense).

Summarizing, three ingredients are necessary for a class of hot dynamical systems to flatten about a FP: 1) to be virialized, 2) to have similar structures and internal dynamics, 3) to exhibit a small dispersion of Υ_* for any given L_B. Observations tell us that $\Theta \times \Upsilon_*$ is a very well defined function of the galaxy parameters with an intrinsic nearly constant scatter less than 12%. In particular,

$$\delta(\Theta \times \Upsilon_*)/(\Theta \times \Upsilon_*) < 0.12 \ . \tag{9}$$

As galaxies in the BBF sample span a factor ~ 200 in L_B, the tilt corresponds to a factor ~ 3 increase of $\Theta \times \Upsilon_*$ along the FP, from faint to bright galaxies. If Υ_* and Θ are not finely anticorrelated, this implies a very small dispersion, *separately* for both quantities, at any location on the FP. This sets a very severe restriction on $\Theta \times \Upsilon_*$, which translates into strong constraints on the range that each parameter can span at any location on the FP. It is evident that fine tuning is required to produce the tilt, and yet preserve the tightness of the FP. Note also how, from eqs. (8), galaxies with fixed L_B and various internal dynamics, structure, Υ_*, *move along straight lines* in the k-space, with $k_3 = \sqrt{2/3}k_1 +$ const.. The inclination of this line with respect to the FP given by BBF is equal to $\arctan(\sqrt{2/3}) - \arctan(0.15) \simeq 30\,$deg: this is the reason why in numerical simulations the end-products of the merging of systems initially placed on the FP are found near the FP itself[1]. In conclusion, what is important is not the attempt to understand, perhaps by the finding of a "good" set of observational quantities, why the FP is "distant" from the VT, but, on the contrary, why galaxies are so similar in structure and dynamics, with such a small scatter.

---

[1] Moreover, this relation between k_3 and k_1 helps reduce slightly the problem of the FP thickness.

3 Exploring Various Possibilities

For simplicity the origin of the FP tilt can be sought in two *orthogonal* directions: either due to a *stellar population* effect, in which case $\Upsilon_* \propto L_B^{0.2}$ and Θ=const, or due to *structural/dynamical* effects, i.e., $\Theta \propto L_B^{0.2}$ and Υ_*=const.

3.1 A Stellar Origin: Changing the IMF

A systematic change of the stellar initial mass function (IMF) is explored in Renzini and Ciotti (1993, hereafter RC). Υ_* is obtained by convolving the present mass of the stars M with the IMF, where $M = M_i$ for the initial mass $M_i < M_{TO}$ (the turnoff mass), and $M = M_R$ (the remnant mass) for $M_i \geq M_{TO}$. For the IMF we adopt $\psi(M_i) = AL_B M_i^{-(1+x)}$, where L_B is the present day blue luminosity of the population, or a multi-slope Scalo IMF with a variable slope for $M < 0.3M_\odot$. For details see RC.

Changing M_{inf}. In this case we assume a *decrease* of M_{inf}, the lowest stellar mass, for increasing L_B. We found that – in the case of a single-slope IMF – for no value of x small values of Υ_* (characteristic of the FP faint-end) are obtained, unless M_{inf} is unrealistically high. Reducing the slope does not help: for x below ~ 0.65 Υ_* increases again, since then the mass in remnants increases more than how much the mass in the lower main sequence stars is reduced. Only with the multi-slope Scalo IMF a low Υ_* can be realized (Fig. 1 in RC).

Changing the IMF Slope. The previous requirement of a low Υ_* at the FP faint-end forces the choice of a low x, but then Υ_* is quite insensitive to variations of M_{inf}; only for a steep IMF Υ_* is sensitive to M_{inf}. As a consequence a mere variation of M_{inf} with a constant IMF slope cannot account for the observed trend. Hence, a variation of slope is required, by an amount Δx which depends on the adopted M_{inf}. We conclude that *a major change of the IMF slope in the lower main sequence is necessary to account for the FP tilt.* There remains to consider the thickness of the FP. In RC it is shown that in order to preserve the $\sim 12\%$ upper limit on $\sigma(\Upsilon_*)$, the galaxy-to-galaxy dispersion in M_{inf} and x should be extremely small, $< \pm 10\%$ and $< \pm 0.15$, respectively. Such very small galaxy-to-galaxy dispersion, coupled to a large systematic variation of x, is a rather demanding constraint, and we conclude that fine tuning is required to produce the observed tilt of the FP, while preserving its constant thickness: the IMF should be virtually universal for a given galaxy mass, and yet exhibit a large trend with galaxy mass. A more accurate analysis of this scenario, using stellar population synthesis models, is given by Maraston (1996).

3.2 A Structural/Dynamical Origin

In this case, assuming Υ_* =const., we explore under which conditions structural/dynamical effects may cause the tilt in k_3 via a systematic increase of Θ

[RC; CLR; Ciotti and Lanzoni 1996, (CL); Lanzoni and Ciotti 1996 (LC)]. In all these investigations spherical, non rotating, two–component galaxy models are constructed, where the light profiles resemble the $R^{1/4}$ law when projected. The internal dynamics is varied using the Osipkov-Merritt formula. Here for shortness reasons only the main results are summarized.

Dark Matter Content and Distribution. In this case we assume *global isotropy*, i.e., all models are *dynamically homologous*, and we ascribe all the FP tilt to systematic variations of $\mathcal{R} = M_h/M_*$ or $\beta = r_h/r_*$ (r_h is a characteristic radius of the dark matter distribution). Obviously, the larger β, the larger the variations of $\mathcal{R}$ that are required to produce the tilt. For Hernquist+Hernquist models and Hernquist+Plummer models, for $\beta \simeq 5$ exceedingly large values of $\mathcal{R}$ are required to produce the FP tilt ($\mathcal{R} \simeq 30 - 175$). An increasing $\mathcal{R}$ may be at the origin of the observed tilt, provided that $\beta < 5$. The same problem affects all the Jaffe+Quasi-isothermal models that we have considered, for every values of β and r_t. For Jaffe+Jaffe models, $\mathcal{R}$ never becomes larger than 10 thus every value of this parameter is acceptable, for every explored value of β.

Structural Non-Homology. Systematic deviations of the Es light distribution from the standard $R^{1/4}$ profile may also possibly cause the FP tilt (Djorgovski 1995; Hjorth and Madsen 1995). We investigate this *morphological* option using isotropic $R^{1/m}$ models without dark matter, thus ascribing to a systematic variation of m the origin of the tilt. Assuming the faintest galaxies to be $R^{1/4}$ systems, in order to produce the tilt m has to increase from 4 up to ~ 10 along k_1 (CLR). If one assumes $m = 2$ for the faintest galaxies, the required variation is even larger, about a factor of 4, up to ~ 8, and its permitted variation at each FP location remains very small: a scatter of $m < 10\%$ at any location on the FP should be associated to a large variation of it with galaxy luminosities. A systematic trend of m with galaxy luminosity has been reported (Caon, Capaccioli and D'Onofrio 1993), with m increasing from ~ 1 up to ~ 15, thus spanning a much wider range than required to produce the tilt. We conclude that further observational studies are required in order to determine whether a progression of light-profile shapes along the FP really exists among cluster ellipticals.

The Role of Anisotropy. In this case we ascribed the entire tilt of the FP to a trend with L_B in the anisotropy degree of the galaxies (described in the Osipkov-Merritt formulation by the anisotropy radius r_a), assuming no dark matter. For Hernquist, Jaffe, and $R^{1/m}$ models constrained to the FP we found that, above a certain luminosity, the phase–space distribution function runs into negative values. So we conclude that anisotropy alone cannot be at the origin of the tilt, because the extreme values of r_a that would be required correspond to dynamically inconsistent models (CLR, CL, LC). A special problem with anisotropy is raised by the FP thinness: for galaxies with a positive distribution function a strong fine tuning between L_B and r_a could appear to be required. However in CL and LC we showed, using a semi–quantitative global instability indicator, that in $R^{1/m}$ models the excursion in anisotropy permitted by *stability* is much less than that given by the simple dynamical consistency, and the induced scatter on the FP is inside the observed spread in k_3. We are well aware that this result is very qualitative (where is it placed in the k–space the end-product of a radially unstable galaxy model initially on the FP?), and so we feel that this result is worth to be further studied, using N-body simulations.

4 Conclusions

Our exploration indicates that all structural/dynamical/stellar population solutions to the FP tilt are rather unappealing, though some are more so than others. This comes from the strong *fine tuning* that is required, no matter whether the driving parameter is the amount of dark matter ($\mathcal{R}$), its distribution relative to the bright matter (β), the shape of the surface brightness distribution (m), or finally the properties of the IMF. In addition to this, we have excluded a trend in the anisotropy as possible cause of the tilt, because it leads to physically inconsistent models. Finally, we showed in a semi-quantitative way that probably a fine tuning of anisotropy with the galaxy luminosity is not required for stability arguments, but deeper investigations, both analytical and numerical, are required.

References

Caon, N., Capaccioli, M., D'Onofrio, M., (1993): MNRAS, **265**, 1013
Capelato, H.V., de Carvalho, R.R., Carlberg, R.G., (1995): ApJ, **451**, 525
Bender, R., Burstein, D., Faber, S.M., (1992): ApJ, **399**, 380, (BBF)
Ciotti, L., (1991): A&A, **249**, 99
Ciotti, L., (1994): Celestial Mechanics, **60**, 401
Ciotti, L., Lanzoni, B., (1996): A&A, *in press*, (astro-ph/9610251) (CL)
Ciotti, L., Lanzoni, B., Renzini, A., (1996): MNRAS, **282**, 1 (CLR)
Dehnen, W., (1993): MNRAS, **265**, 250
Djorgovski, M., Davis, S., (1987): ApJ, **313**, 59
Dressler, A., Lynden-Bell, D., Burstein, D., Davies, R.L., Faber, S.M., Terlevich, R.J., Wegner, G., (1987): ApJ, **313**, 42
Hjorth, J., Madsen, J., (1995): ApJ, **445**, 55
Lanzoni, B., and Ciotti, L., (1996): *this workshop* (LC)
Lynden-Bell, D., (1967): MNRAS, **136**, 101
Maraston, C., (1996): *this workshop*
Renzini, A., Ciotti, L. (1993): ApJL, **416**, L49 (RC)
Spitzer, L. (1969): ApJL, **158**, L139

Anisotropic $R^{1/m}$ Models. Velocity Profiles, and the FP of Elliptical Galaxies

Barbara Lanzoni[1]* and Luca Ciotti[2]

[1] Dipartimento di Astronomia di Bologna, Via Zamboni 33, Bologna, Italy
[2] Osservatorio Astronomico di Bologna, Via Zamboni 33, Bologna, Italy

Abstract. We study the dynamical properties of spherical galaxies with surface luminosity profile described by the $R^{1/m}$-law, in which a variable degree of orbital anisotropy is allowed. The stability of the models against radial–orbit instability is studied, and the limits on the maximum anisotropy allowed for each model are determined. The consequent constraints on the the projected velocity dispersion imply that no fine-tuning for anisotropy is required along the fundamental plane (FP) in order to maintain its small thickness. Finally, the velocity profiles are constructed, and their deviations from a gaussian discussed.

1 The Models

The models surface brightness profile is described by the $R^{1/m}$ law (Sersic 1968):

$$I(R) = I_0 \exp\left[-b(m)\,(R/R_e)^{1/m}\right], \tag{1}$$

which seems to give a better fit to the spheroidal galaxies surface brightness profiles (e.g., Caon et al. 1993; Graham et al. 1996, and references therein) than the "standard" $R^{1/4}$ law [de Vaucouleurs 1948; Eq.(1) for $m = 4$].

The velocity dispersion tensor of our models is described by the Osipkov–Merritt parameterization (Osipkov 1979; Merritt 1985), and characterized by the *anisotropy radius* r_a: in the limit $r_a \to \infty$ the velocity dispersion tensor is globally isotropic, while in general the radial anisotropy increases with radius.

2 Stability

The stability of the anisotropic models is investigated in a semi-quantitative way using the radial-orbit instability indicator ξ (e.g., Fridman and Polyachenko 1984). A model is likely to be unstable if

$$\xi \equiv \frac{2K_r}{K_t} \gtrsim 1.5 \div 2, \tag{2}$$

where K_r and K_t are the radial and the tangential kinetic energies, respectively. This parameter is quite independent of the assumed density distribution profile,

---

[*] Present address: Osservatorio Astronomico di Capodimonte, Via Moiariello 16, Napoli, Italy.

and for any globally isotropic system $\xi = 1$, while in presence of radial anisotropy $2K_{\rm r} > K_{\rm t}$, and so $\xi > 1$.

For the investigated models, $\xi = \xi(r_{\rm a}, m)$, and for a fixed m it decreases towards unity for increasing $r_{\rm a}$ (see Fig.1), according to the previous discussion. So, assuming a fiducial critical value of ξ for stability (e.g., $\xi = 1.7$), a minimum value for the anisotropy radius $(r_{\rm a})_\xi$ is obtained, i.e., all models with $r_{\rm a} < (r_{\rm a})_\xi$ are unstable.

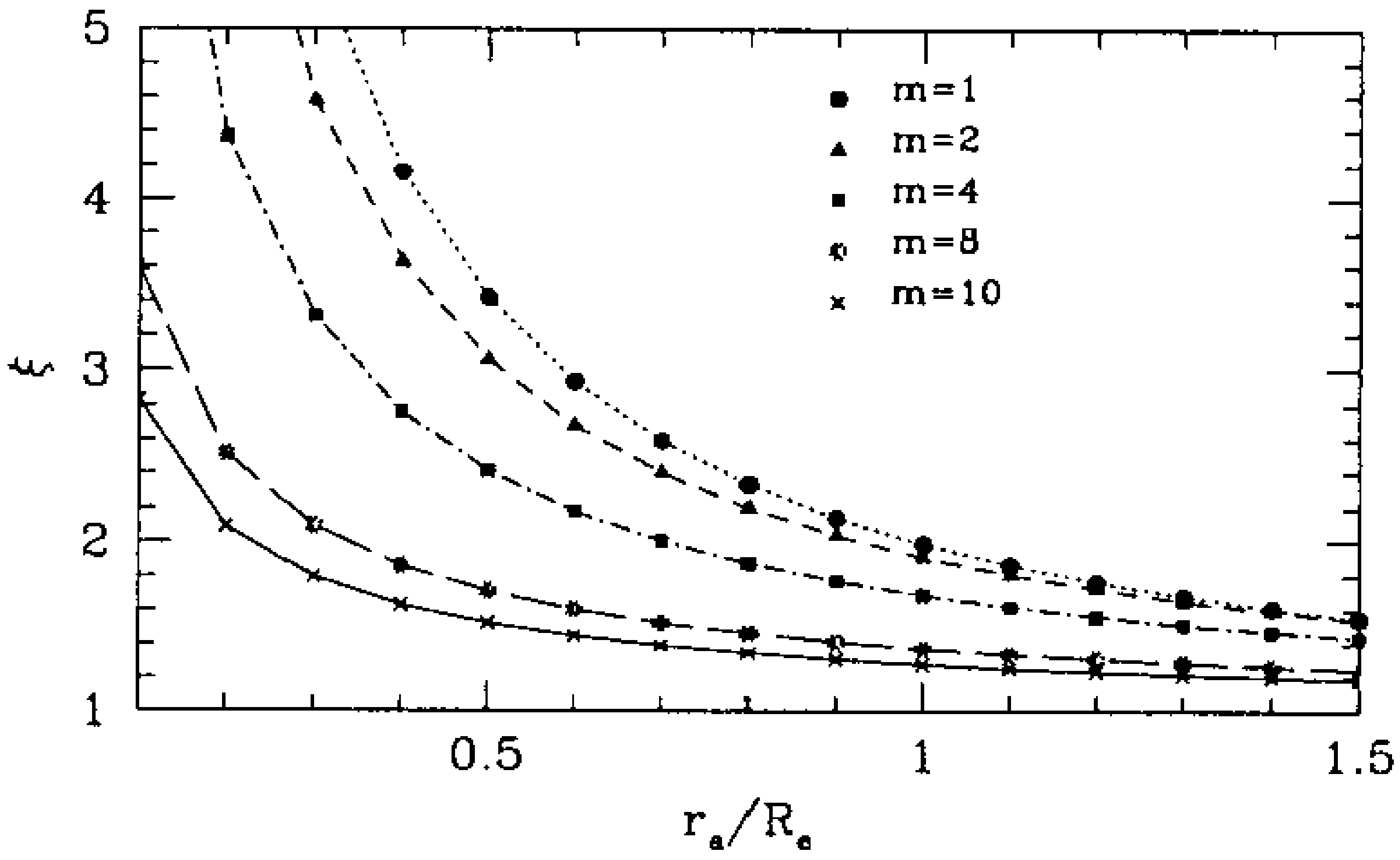

Fig. 1.

For stable models we compute the spatial velocity dispersion profile by integration of the Jeans equation. After the operations of projection and mean, we obtain the *aperture* velocity dispersion $\sigma_{\rm a}$, which mimics the observed central one (see Ciotti and Lanzoni 1997).

3 Implications for the FP

In principle, one of the possible origins of the FP tilt could be a systematic increase of radial anisotropy from faint to bright galaxies (Ciotti et al. 1996). If this is the case, a variation of factor 3 from isotropic to anisotropic squared central velocity dispersion $\sigma_{\rm a}^2$ is required, in the assumption of structural homology (i.e., the same m for all galaxies). On the contrary, for our models we find that, if the maximum degree of anisotropy allowed by the stability requirement is considered, the *radial anisotropy cannot produce the tilt* (a conclusion already reached for different galaxy models in Ciotti et al. 1996).

As concerns the problem of the very small thickness of the FP, the limits imposed by the stability requirement imply that the variations between the isotropic and the maximum anisotropic velocity dispersion are so small that the *anisotropy is not required to be fine-tuned with the galaxy luminosity in order to maintain the small observed FP scatter* (see Ciotti and Lanzoni 1997).

4 Velocity Profiles

The velocity profile (VP) at a certain projected distance from the galaxy center is the distribution of the stars line–of–sight velocities at that point. The analysis of the deviations of VPs from gaussianity may give important insights on the dynamical structure of a galaxy (e.g., van der Marel 1994).

We numerically recover the VPs of our models (as in Carollo et al. 1995), and expand them on the Gauss-Hermite basis (Gerhard 1993; van der Marel and Franx 1993), thus obtaining the values of the coefficient h_4 at various distances from the galaxy center. Generally, a negative h_4 indicates a flat-topped VP, while a positive one indicates a VP more centrally peaked than a Gaussian.

In Fig.2 the radial behaviour of h_4 for various m and for isotropic (left panels) and anisotropic ($\xi = 1.7$, right panels) models is shown.

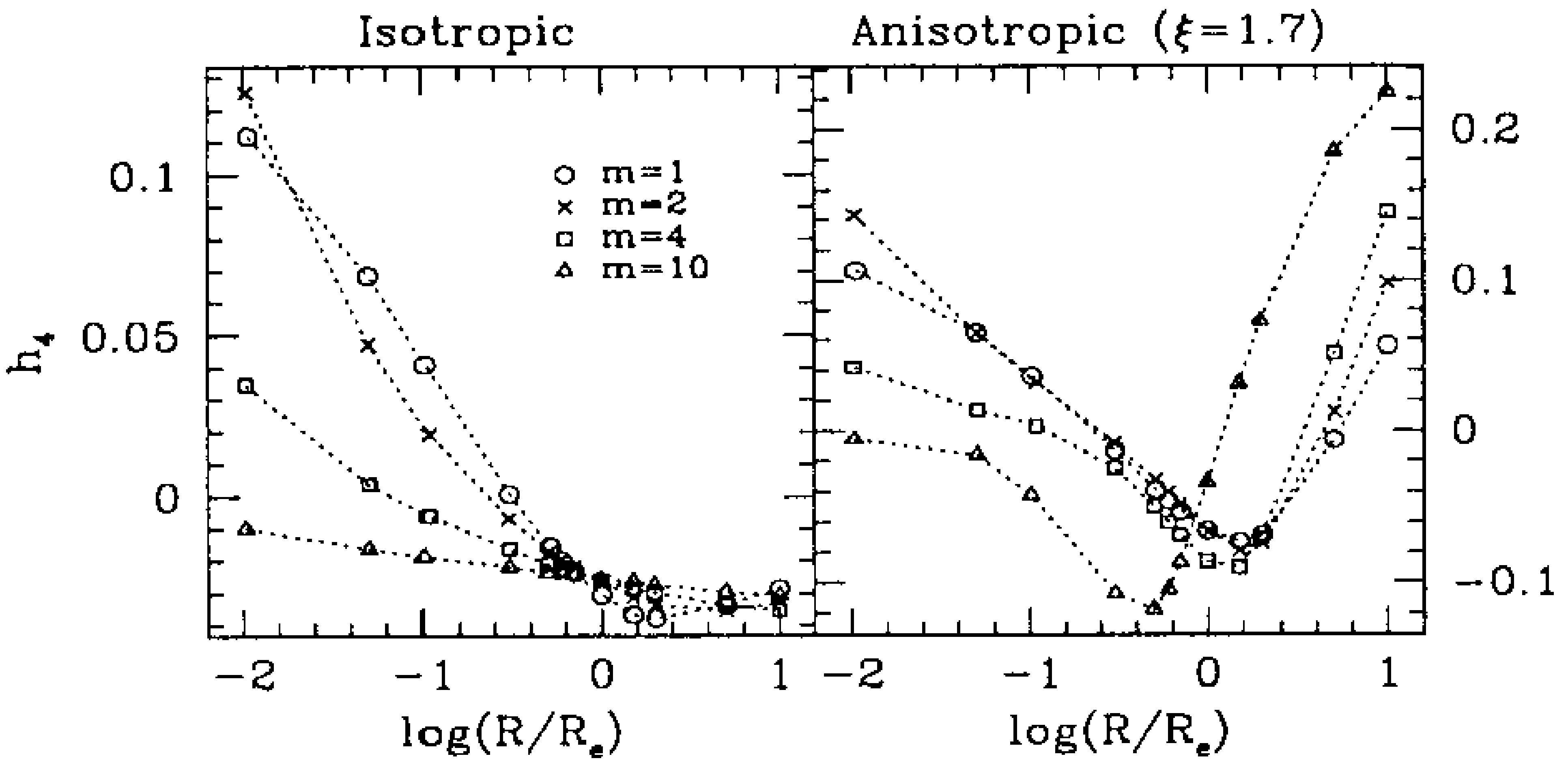

Fig. 2.

Because of the differences in the trend of h_4 between the isotropic and the anisotropic cases, and because of the growing evidence that $R^{1/m}$ law appropriately describe the surface brightness profiles of elliptical galaxies, we suggest that a detailed study of the VPs along the FP could be in principle a tool to study the effect of orbital anisotropy on its tilt and thickness.

References

Caon N., Capaccioli M., D'Onofrio M., (1993): MNRAS, **265**, 1013
Carollo C.M., de Zeeuw P.T., van der Marel R.P., (1995): MNRAS, **276**, 1131
Ciotti L., Lanzoni B., (1997): A& A, in press (astro-ph/9610251)
Ciotti L., Lanzoni B., Renzini, A., (1996): MNRAS, **282**, 1
de Vaucouleurs G., (1948): Ann. d'Astroph., **11**, 247
Fridman A.M., Polyachenko V.L., (1984): Physics of Gravitating Systems, 2 vols., New York: Springer
Gerhard O.E., (1993): MNRAS, **265**, 213
Graham A., Lauer T.R., Colless M., Postman M., (1996): ApJ, **465**, 534
Merritt D., (1985): AJ, **90**, 1027
Osipkov L.P., (1979): Pis'ma Astron.Zh., **5**, 77
Sersic J.L., 1968, Atlas de Galaxias Australes, Cordoba: Observatorio Astronomico
van der Marel R.P., (1994): Ph.D. Thesis
van der Marel R.P., Franx M., (1993): ApJ, **407**, 525