ATTI DEL CONVEGNO

LO SVILUPPO DELLE RICERCHE IN MECCANICA ED IN ASTRONOMIA NELL'OTTOCENTO E NEL NOVECENTO
&
ASTRONOMIA ANTICA E ARCHEOASTRONOMIA

10th Annual Meeting on the History of Astronomy
Accademia delle Belle Arti di Reggio Calabria
Reggio Calabria, September 25 - 26, 1998

a cura di

Edoardo Proverbio e Pasquale Tucci

UNIVERSITÀ DEGLI STUDI DI MILANO
ISTITUTO DI FISICA GENERALE APPLICATA
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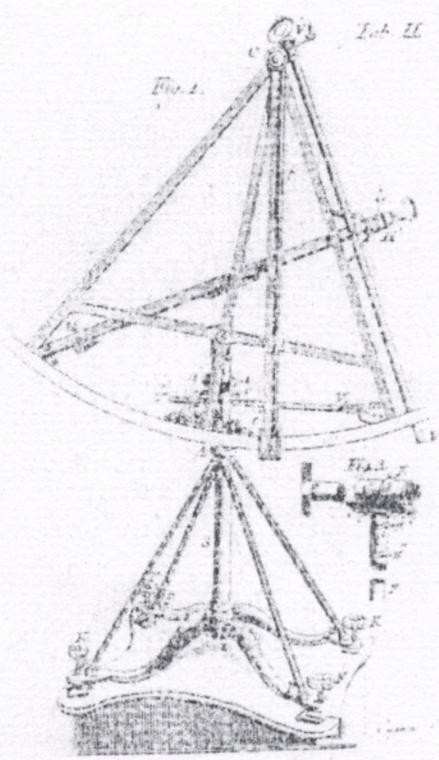

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PREMESSA

Gli Atti che qui si pubblicano sono quelli del decimo Convegno annuale di Storia dell'astronomia promosso dal Settore di Storia dell'astronomia della Società Astronomica Italiana, tenuto a Reggio Calabria il 25 e 26 settembre 1998.

Il Convegno, come i precedenti, è stato caratterizzato da un tema centrale, che in questa occasione si è ritenuto di dover caratterizzare dalle ricerche in meccanica ed in astronomia sulla fine dell'Ottocento e la prima metà del Novecento, per commemorare in particolare l'attività del grande astronomo James Jeans (1877-1946) del quale da poco si era ricordato il cinquantenario della morte, e al quale sono riferiti i contributi di Franco Selleri, Vittorio Banfi e Luca Ciotti.

Le altre comunicazioni presentate costituiscono un ulteriore prezioso contributo alla conoscenza della storia dell'astronomia e degli astronomi italiani.

Non sono infine mancati e pubblicati in questi Atti, interessanti contributi riguardanti temi di archeastronomia e di astronomia culturale che, in Italia, hanno ormai trovato nella organizzazione di questi Convegni, ancora in mancanza di pubblicazioni specialistiche, un degno luogo di presentazione, discussione e pubblicazione.

Il Convegno si è tenuto a Reggio Calabria presso la prestigiosa Accademia di Belle Arti. Sento il dovere di ringraziare oltre l'Accademia che ci ha ospitati, gli organizzatori locali, ed in particolare la professoressa Angela Misiano, che con grande entusiasmo, hanno saputo affrontare tanti problemi e difficoltà offrendo ai partecipanti una adeguata cornice al Convegno stesso, compresa una stupenda gita a Locri e Gerace.

Gli Atti che qui si presentano, a differenza dei precedenti, non appaiono nelle <u>*Memorie*</u> *della Società Astronomica Italiana a causa di difficoltà finanziarie.*

Sono quindi particolarmente grato al collega ed amico professor Pasquale Tucci, coeditore di questi stessi Atti, per aver accettato di accollarsi l'onere della loro pubblicazione nell'ambito delle attività e realizzazione di valorizzazione del patrimonio storico-scientifico perseguite da oltre un decennio dalla Sezione di Storia della Fisica dell'Università degli Studi di Milano.

Edoardo Proverbio

THE CONTRIBUTIONS OF SIR J.H. JEANS TO STELLAR DYNAMICS

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ABSTRACT. I present a short review of the main contributions of Sir James Jeans to the field of Stellar Dynamics. The paper is organized in three parts. In the first and second part I give a qualitative presentation of three key–concepts of Stellar Dynamics, i.e., the concept of "collisional" vs. "collisionless" stellar system, the "Jeans Theorem", and the "Jeans Equations". In the third and last part of this review I briefly summarize the contents of the four classic Jeans papers on Stellar Dynamics, where the previous concepts are discussed and/or introduced.

1. Introduction

The discipline known as *Stellar Dynamics* (SD) deals with the study of the motion of stars inside *stellar systems*, from Open Clusters through Globular Clusters, Galaxies, and Clusters of Galaxies. Albeit an autonomous research field, many other old and honoured disciplines as Analytical, Celestial, and Statistical Mechanics, Fluidodynamics and Plasma Physics share techniques and results with SD. For this reason it is very difficult, and sometimes impossible, to give a definitive priority attribution for many important results of SD. With a very simplicistic approach, we can say that from a mathematical point of view the central problem of SD is the study of the *behavior* and *structure* of gravitational N–body systems, over temporal and spatial scales of astrophysical interest. From this point of view, we can say (arbitrarily) that for $N < 10$ we are in the domain of Celestial Mechanics, while for $N > 10$ we are properly discussing a SD problem. Moreover, the *structure* of the phase–flow induced in the phase–space by the differential equations of the N–body problem (one of the central problems of Analytical Mechanics)

$$\frac{d^2\mathbf{x}_i}{dt^2} = -G \sum_{j\neq i, j=1}^{N} m_j \frac{\mathbf{x}_i - \mathbf{x}_j}{||\mathbf{x}_i - \mathbf{x}_j||^3} \tag{1}$$

(m_i= particle masses, $i = 1, ..., N$) present many formal similarities with the motions investigated by Fluidodynamics. Finally, a particular (but extremely interesting) class of systems discussed in SD, the so-called *collisionless* systems, needs for its treatment many mathematical tools developed by Plasma Physics and Statistical Mechanics.

In the next Section I briefly introduce three main results of SD, namely, the concept of "collisional" and "collisionless" stellar systems, the "Jeans Theorem", and the "Jeans Equations". In fact, at least a qualitative understanding of these three points is necessary in order to better appreciate the contributions of Sir James Jeans to the development of SD.

2. Three basic concepts of SD

2.1. Collisional vs. Collisionless stellar systems

One of the basic concepts of SD is that of *collisional* and *collisionless* stellar system. The "collisions" involved in these definitions are not *geometrical* collisions, i.e., we are not considering physical collisions in which the distance of two stars (ignoring their tidal distortions) is less than the sum of the two radii, but *gravitational deflections* from the hypothetical orbit that each star would trace if the true mass distribution of the system is replaced with a *smooth* one. Obviously, the two trajectories differ more and more as time increases. The characteristic time in which the two orbits become significantly different is called *two-body relaxation time* (t_{2b}), and correspondingly a system is called *collisional* when its life is greater than its relaxation time. In the opposite case, the system is said to be *collisionless*. The quantitative determination of t_{2b} as a function of the system characteristics (i.e., the evaluation of the cumulative effects of the "granularity" of the system true density distribution with respect to the orbit described in the smooth potential associated to the smoothed density) is a very complex problem. Here it is sufficient to remember that the gravitational interaction is a long–range, always attractive force, and its slow decrease (only as r^{-2}) is responsible for the fact that even the most distant parts of any system contribute in general to the force at any point. For a general discussion of this problem the interested reader can consult the classic books of Chandrasekhar (1960), Spitzer (1987), Binney & Tremaine (1987, BT87). It can be shown that for a gravitationally bound system, made of N particles of the same mass,

$$t_{2b} \sim \frac{N}{8 \ln N} \times t_{cross}, \tag{2}$$

where t_{cross} is an estimate of the *crossing time*, i.e., the characteristic time scale required to a representative particle to cross the system. For Open Clusters t_{2b} is of few Gyrs, for Globular Clusters is of the same order of the age of the Universe, and for galaxies is million of times the age of the Universe. For this reason galaxies are a classical example of collisionless system, while Open and Globular Clusters are collisional systems.

It is important to stress that 1) the subdivision of stellar systems between collisional and collisionless is not intrinsic, but on the contrary depends on the time scale over which the systems are observed; 2) the dynamical evolution of collisional stellar systems is extremely complicate, presenting various interesting peculiar phenomena, as evaporation, gravothermal catastrophe, core–collapse, etc.. A quantitative and global picture of the dynamical evolution of collisional systems is still lacking, albeit the enormous amount of work done on this subject.

2.2. The "Jeans Theorem"

For a generic stellar system is useful the introduction of the concept of *Distribution Function* (DF), i.e., a nowhere negative function $f = f(\mathbf{x}, \mathbf{v}; t)$ defined on the extended one–particle phase–space $(\Re^6 \times \Re)$ by the property that for every phase–space volume $\Delta\mathbf{x}\Delta\mathbf{v}$

$$\int_{\Delta\mathbf{x}\Delta\mathbf{v}} f(\mathbf{x}, \mathbf{v}; t) d^3\mathbf{x} d^3\mathbf{v} = \Delta M, \tag{3}$$

where ΔM is the total mass of the stars found in $\Delta\mathbf{x}\Delta\mathbf{v}$ at time t. From the previous definition, it follows that the mass density of any stellar system at $\mathbf{x}$ can be expressed as

$$\int_{\Re^3} f(\mathbf{x}, \mathbf{v}; t) d^3\mathbf{v} = \rho(\mathbf{x}; t). \tag{4}$$

The importance of the DF cannot be underestimated: in fact, when known, the DF can be used to obtain all the required *macroscopic* (i.e., in principle observable) informations for a given system. More explicitly, let us assume that a *microscopic* function $F = F(\mathbf{x}, \mathbf{v}; t)$ is assigned: the associated macroscopic counterpart $\overline{F} = \overline{F}(\mathbf{x}; t)$ is given by

$$\overline{F}(\mathbf{x}; t) = \frac{1}{\rho(\mathbf{x}, t)} \int_{\Re^3} F(\mathbf{x}, \mathbf{v}; t) f(\mathbf{x}, \mathbf{v}; t) d^3\mathbf{v}. \tag{5}$$

For example, when $F = 1$ then $\overline{F} = \rho(\mathbf{x}; t)$; when $F = v_i$ then $\overline{F} = \bar{v}_i(\mathbf{x}; t)$, the i-th component of the *streaming velocity field*; when $F = (v_i - \bar{v}_i)(v_j - \bar{v}_j)$ then $\overline{F} = \sigma^2_{ij}(\mathbf{x}; t)$, the so–called *velocity dispersion tensor*.

In this framework, the fundamental problem of SD is the determination of the temporal evolution of the DF for assigned initial conditions. Obviously, here it is not possible even to recall the many issues raised by this approach (the interested reader can consult as an introductory text BT87). It is sufficient to remember that a rigorous treatment can be obtained in the collisionless regime, starting from the *Liouville Equation* in the $6N$–dimensional phase–space. Applying the so–called *BBGKY–hierarchy* the evolutionary equation for the DF can be reduced to an equation in $\Re^6 \times \Re$ with sources. When the source term is developed in power series using the *locality hypothesis*, one obtains the well–known *Fokker–Planck* equation, a diffusive equation (in phase–space), used in the study of (weakly) collisional stellar systems. The more extreme case of collisionless systems is obtained showing that the source term is zero under the (unproven!) *non–correlation hypothesis*.

An alternative approach to the derivation of the evolutionary equation for the DF of collisionless stellar systems relies on Fluidodynamical techniques. In fact, this approach uses the *Reynolds Transport Theorem*. This states that for a generic vector field $\mathbf{w}(\mathbf{x}; t)$ in $\Re^n$ the *Lagrangian derivative*

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + w_i \frac{\partial}{\partial x_i} \tag{6}$$

applied to the (phase–space) volume integral

$$H(t) = \int_{V(t)} h(\mathbf{x}; t) d^n\mathbf{x} \tag{7}$$

equals

$$\frac{dH}{dt} = \int_{V(t)} \left[\frac{Dh}{Dt} + h\frac{\partial w_i}{\partial x_i} \right] d^n\mathbf{x}. \tag{8}$$

In eqs. (7-8) $V(t)$ is a generic phase–space volume obtained applying the phase–flow (induced by the field w) to the initial condition volume $V(0)$. If the quantity H is conserved during the motion of any control volume $V(t)$ (i.e., $dH/dt = 0$), then the evolutionary equation for h is

$$\frac{Dh}{Dt} + h\frac{\partial w_i}{\partial x_i} = \frac{\partial h}{\partial t} + \frac{\partial h w_i}{\partial x_i} = 0. \tag{9}$$

Moreover, if the phase–flow induced by the vector field w is *solenoidal* (i.e., $\partial w_i/\partial x_i = 0$, as in the case of incompressible fluids or hamiltonian flows) then from the previous equation

$$\frac{Dh}{Dt} = \frac{\partial h}{\partial t} + w_i\frac{\partial h}{\partial x_i} = 0. \tag{10}$$

In the specific case of a collisionless stellar system, for $N \to \infty$ the relaxation time t_{2b} becomes infinite, as can be seen from eq. (2). As a consequence, in this situation it is plausible that the (true) stellar orbits are more and more similar to those described in the potential $\phi = \phi(\mathbf{x}; t)$ associated to the smooth density ρ [see eq. (13)], and so the evolutionary equation for the phase–space DF is well described by

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + v_i\frac{\partial f}{\partial x_i} - \frac{\partial \phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0, \tag{11}$$

where the initial condition is given by

$$f(\mathbf{x}, \mathbf{v}; 0) = f_0(\mathbf{x}, \mathbf{v}). \tag{12}$$

Note that the potential ϕ is related to the DF by the *Poisson Equation*

$$\triangle\phi(\mathbf{x}; t) = 4\pi G \int_{\Re^3} f(\mathbf{x}, \mathbf{v}; t) dtxv, \tag{13}$$

where $\triangle$ is the *Laplace* operator. Equation (11) is the *Collisionless Boltzmann Equation* (CBE). From a strictly mathematical point of view, the problem of the integration of the CBE is that of the integration of a *quasi–linear* partial differential equation. The formal solution of such a problem is obtained as a function of the *characteristic* curves generated by the coefficients of the equation itself. In our specific case, the problem is reduced to the integration of the equations of motion for a particle in the three–dimensional space, under the action of a (potential) force field, generally non–linear and time–dependent. As well–known, this problem is beyond the present mathematical possibilities.

In any case, it is possible to obtains many informations on the DF in the particular (but very important) case of *stationary* stellar systems, i.e., stellar systems in which the term $\partial f/\partial t$ in eq. (11) vanishes identically. We have the

Jeans Theorem: The DF of any stationary stellar system depends on the phase–space coordinates $(\mathbf{x}, \mathbf{v})$ *only* through the *regular first integrals of motion* $I_i = I_i(\mathbf{x}, \mathbf{v})$ associated to the potential ϕ:

$$f = f(I_1, I_2, ...). \tag{14}$$

The Jeans Theorem is one of the most powerful tools available to SD. Obviously, even in the stationary cases not all the difficulties are eliminated by the Jeans Theorem, because at present general methods to determine *all* the regular integrals of motion permitted by a given potential are not known. So, as a rule, we are today able to construct *exact* DFs for collisionless stellar systems only in special cases, when the regular integrals of motion are known for some basic reason. The commonest case is represented by the use of the *classical* (regular) integrals of motion, as *energy* and *angular momentum*, that depend on the symmetry of the system under study.

In conclusion, I also recall to the reader that an entire field of research originates from the Jeans Theorem, i.e., that of the solution of the *inverse problems* of SD.

2.3. The "Jeans Equations"

As briefly discussed in the previous section, from the knowledge of the DF it is possible to derive *all* the properties of a stellar system. Unfortunately, such complete knowledge can be obtained only for the restrict class of stationary systems for which some regular integral of motions are available. This is not an happy situation, especially when trying to model *real* stellar systems. An alternative method to the solution of the CBE is given by the *moments method*. In this approach, one starts from the CBE, and consider its *mean values* over the velocity space, after multiplication for some microscopic function $F = F(\mathbf{x}, \mathbf{v}; t)$. The obtained equations are called *Jeans Equations* (JEs). Focusing on a more formal aspect of the problem, the basic relation satisfied by the JE for the F is

$$\int_{\Re^3} F \frac{Df}{Dt} d^3\mathbf{v} = 0. \tag{15}$$

After some manipulation, the previous identity gives:

$$\frac{\partial \rho \overline{F}}{\partial t} + \frac{\partial \rho \overline{F v_i}}{\partial x_i} = -\rho \frac{\partial \phi}{\partial x_i} \overline{\frac{\partial F}{\partial v_i}} + \rho \overline{\frac{\partial F}{\partial t}} + \rho \overline{v_i \frac{\partial F}{\partial x_i}}. \tag{16}$$

The JEs used in SD are obtained for $F = 1$:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \overline{v_i}}{\partial x_i} = 0, \tag{17}$$

and $F = v_i$ $(i = 1, 2, 3)$:

$$\frac{\partial \rho \overline{v_i}}{\partial t} + \frac{\partial \rho \overline{v_i v_j}}{\partial x_i} = -\rho \frac{\partial \phi}{\partial x_i}. \tag{18}$$

Using the identity

$$\overline{v_i v_j} = \bar{v}_i \bar{v}_j + \sigma^2_{ij}, \tag{19}$$

the three equations (18) can be rewritten as

$$\frac{\partial \rho \overline{v_i}}{\partial t} + \frac{\partial \rho \bar{v}_i \bar{v}_j}{\partial x_i} + \frac{\partial \rho \sigma_{ij}^2}{\partial x_i} = -\rho \frac{\partial \phi}{\partial x_i}. \tag{20}$$

A stellar system in which at any point

$$\sigma_{ij}^2(\mathbf{x};t) = \sigma^2(\mathbf{x};t)\delta_{ij} \tag{21}$$

is called an *isotropic* stellar system. The reader can note the strict analogy between the JEs (17-18) and the equations of Fluidodynamics, where the velocity dispersion tensor is replaced by the *thermodynamical pressure*. This analogy is not casual: in fact, the equations of Fluidodynamics can be derived in a similar way using the Reynolds Transport Theorem with the velocity field of the fluid as differential field. But the analogy is not complete: the JEs describes a *stellar fluid* much more complicate than usual fluids! In fact, at variance with standard Fluidodynamics, two main new aspects are present: the first is that (due to the underlying non–collisionality assumption), the "pressure" can be *anisotropic*; and the second is that, due to the lack of thermodynamical relations between density and velocity dispersion (or even higher order velocity moments), no *natural closure* is in general possible for the JEs. In principle, the JEs are an *infinite set* of equations, obtained for $F = v_i v_j v_k$, $F = v_i v_j v_k v_l$, and so on. As a consequence, in common applications the JEs are closed with more or less arbitrary assumptions on the nature of the velocity dispersion tensor.

3. The contributions of Sir J.H. Jeans to SD

The main contributions of Sir J.H. Jeans to SD concerns basically the three points previously described. Particularly, I will focus on a discussion of his four papers appeared on the *Monthly Notices of Royal Astronomical Society* (MNRAS).

- *On the "Kinetic Theory" of Star-clusters*, J.H. Jeans, MNRAS, **74**, 109, 1913.
- *On the theory of Star-Streaming and the Structure of the Universe*, J.H. Jeans, MNRAS, **76**, 71, 1915.
- *On the theory of Star-Streaming and the Structure of the Universe*, J.H. Jeans, MNRAS, **76**, 552, 1916.
- *On the Law of Distribution in Star-Clusters*, J.H. Jeans, MNRAS, **76**, 567, 1916.

The reader interested in a short but instructive historical discussion of CBE (and Jeans contributions to this subject) should consult the following paper: *Vlasov Equation?*, M. Hénon, Astronomy & Astrophysics, **114**, 211, 1982.

3.1. Paper 1: On the "Kinetic Theory" of Star–clusters

In this paper Jeans focuses on the problem of the determination of the relaxation time for stellar systems. The development is carried out in a simplified manner with respect to the more detailed analysis developed in the '40s by Chandrasekhar (1960) and Spitzer (1987): notwithstanding this, the order of magnitude (and the consequent astrophysical implications as well) is correct. The conclusion is that the geometrical impacts between stars are extremely improbable in stellar systems under normal conditions. Moreover, the characteristic relaxation time (a concept whose origin Jeans attributes to Maxwell) is enormously long for galaxies (million times the age of the universe!). Finally, Jeans describes the concept of *evaporation* for collisional stellar systems (as Globular Clusters), stating (correctly) that this phenomenon proceeds by weak "gradual scattering".

3.2. Paper 2: On the theory of Star-Streaming and the Structure of the Universe

In my opinion this paper is the most important contribution of Jeans to SD.

Firstly, he derives (in a not very elegant way, it must be admitted) the evolutionary equation for the DF of collisionless stellar systems, and successively Jeans recognizes the formal identity of his equation with that derived by Boltzmann for gases, when the effect of molecular collisions is neglected. For this reason the equation derived by Jeans in this paper is called Collisionless Boltzmann Equation.

Secondly, Jeans explicitly states the content of the so-called Jeans Theorem, and presents two simple cases of his application, namely the case of a globally isotropic stellar system (the DF depending only on energy), and an anisotropic case in which the DF depends on energy and on the angular momentum component along the simmetry axis of the system. Jeans correctly identify the properties of the velocity dispersion tensor in these cases.

In a third remarkable point, Jeans states without proof that a self-gravitating system whose DF depends only on energy is necessarily spherically symmetric: this is in fact true, but the formal proof of this result was obtained only in the '70s using sophisticated methods developed in order to prove the existence of solutions for non-linear elliptic partial differential equations.

Finally, from an historical point of view it can be of interest the fact that Jeans in this paper made a serious (but understandable) conceptual error. In particular, he believed that the observational evidence of a stellar velocity field considerably different from that predicted by its DFs can be conclusively considered a proof of non-stationarity of our galaxy (and so of the impossibility to apply the Jeans Theorem). The mistake in this argument is the belief that the *only* regular integrals of motions are the *classical* ones: in the '60s it was discovered numerically, and successively analytically, that this is not the case. Obviously a DF depending on these non-classical integrals of motion admits more complicate stellar velocity fields, so invalidating the argument of Jeans.

3.3. Paper 3: On the theory of Star-Streaming and the Structure of the Universe

At variance with Paper 2, this paper does not contain any remarkably new idea of SD, and so it is here included only for completeness. The developed subject is the problem of the origin of star-streaming.

3.4. Paper 4: On the Law of Distribution in Star–Clusters

This last paper is a very important one for SD. The starting point is the derivation of the Poisson equation for (spherically symmetric) collisionless stellar systems, whose DF is formally similar to the relation between density and potential for *gaseous polytropes*. As a consequence, the derived Poisson Equation is the well–known *Lane–Emden* equation. For a particular value of the politropic exponent, Jeans derives from a dynamical point of view the solution found by Schuster in 1883 for gaseous polytropes and successively applied to Globular Clusters by Plummer. The interest of this work is clearly not in the results form themselves (they were already known at that time), but in the *method*, i.e., in a coherent and rigorous derivation "ab–initio" of an important result in the field of SD. Quite surprisingly, in the MNRAS immediately after the Jeans paper, a paper appeared by Sir A. Eddington on the physical meaning of the "Plummer law", this time using concepts of gas dynamics only. The two Authors completely ignored each other.

Acknowledgements

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