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Phase–Space Constraints on Visible and Dark Matter Distributions in Elliptical Galaxies

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Abstract. There are observational and theoretical indications that both the visible (stars) and the dark matter density distributions in elliptical galaxies increase significantly up to the galactic center. I present here some analytical results obtained with the aid of self–consistent, spherically symmetric two component galaxy models. These results suggest the possibility that this similar behavior could be a direct consequence of the structural and dynamical constraints imposed by the request of positivity of the phase–space distribution function (DF) of each density component.

1. Introduction

High resolution observations of elliptical galaxies and bulges of spirals show that their (spatial) central stellar densities are well described by a power–law profile,

$$\rho_*(r) \propto r^{-\gamma}, \quad r \sim 0, \tag{1}$$

where $0 \leq \gamma < 2.4$ (e.g., Gebhardt et al. 1996; Carollo & Stiavelli 1998). Moreover, it is now commonly accepted that a non negligible fraction of the total mass in elliptical galaxies is made of a dark component, whose density distribution at large radii differs significantly from that of the visible one. The radial distribution of the dark matter in the galactic central regions is less well known. In the past a commonly used, centrally flat model for the dark matter distribution was the so called quasi isothermal (QI) density distribution ($\beta = 2$ in eq. [6]), but, more recently, high resolution numerical simulations of dark matter halo formation showed that also for the dark matter density distribution

$$\rho_{\rm DM}(r) \propto r^{-\beta} \tag{2}$$

in the central regions, where $\beta \geq 1$ (e.g., Dubinsky & Carlberg 1991; Navarro, Frenk, & White 1997). Thus, *there are indications that both the visible and the dark matter density distributions increase significantly up to the center of galaxies.* A natural question arises: is the origin of this qualitatively similar behavior independent for the two density distributions, or it is related for some reason?

From a theoretical point of view it is well known that in any physically acceptable multicomponent galaxy model the phase–space DF of *each* density component must be non negative. A model satisfying this minimal requirement is called a *consistent* model. Thus, the question above can be reformulated as follows: *can the request of positivity of the DF of each density component in multicomponent galaxy models tell us something about the relative distribution of dark and visible matter in real galaxies?* In order to answer this question I studied the problem of the construction and investigation of the DFs of two component galaxy models with variable amount and distribution of dark matter, and variable orbital anisotropy.

2. Technique

For a multi component spherical system, where the (radial) orbital anisotropy of each component is modeled according to the OM parameterization (Osipkov 1979; Merritt 1985), the DF of the density component ρ_k is given by:

$$f_k(Q_k) = \frac{1}{\sqrt{8}\pi^2}\frac{d}{dQ_k}\int_0^{Q_k}\frac{d\varrho_k}{d\Psi_T}\frac{d\Psi_T}{\sqrt{Q_k-\Psi_T}}, \quad \varrho_k(r) = \left(1+\frac{r^2}{r_{ak}^2}\right)\rho_k(r), \tag{3}$$

where $\Psi_T(r) = \sum_k \Psi_k(r)$ is the total relative potential, and $Q_k = \mathcal{E} - L^2/2r_{ak}^2$. $\mathcal{E}$ and L are respectively the relative energy and the angular momentum modulus per unit mass, r_{ak} is the *anisotropy radius*, and $f_k(Q_k) = 0$ for $Q_k \leq 0$ (e.g., Binney & Tremaine 1987).

Preliminary information on the model consistency can be easily obtained using the following results (Ciotti & Pellegrini 1992, CP; Ciotti 1996, C96; Ciotti 1999, C99). A *necessary* condition for the non negativity of each f_k is:

$$\frac{d\varrho_k(r)}{dr} \leq 0, \quad 0 \leq r \leq \infty. \tag{4}$$

If the NC is satisfied, *strong* and *weak sufficient* conditions for the non negativity of each f_k are:

$$\frac{d}{dr}\left[\frac{d\varrho_k(r)}{dr}\frac{r^2\sqrt{\Psi_T(r)}}{M_T(r)}\right] \geq 0, \quad \frac{d}{dr}\left[\frac{d\varrho_k(r)}{dr}\frac{r^2}{M_T(r)}\right] \geq 0, \quad 0 \leq r \leq \infty. \tag{5}$$

The explored two component galaxy models are made of various combinations of spherically symmetric density distributions, as the *flat-core* mass distributions

$$\rho(r) = \frac{\rho_0 r_c^\beta}{(r_c^2 + r^2)^{\beta/2}}, \tag{6}$$

the *centrally-peaked* spatial density associated to the deprojection of the Sersic (1968) profile

$$I(R) = I(0)\exp[-b(m)(R/R_e)^{1/m}], \quad \rho(r) \sim r^{(1-m)/m}, \quad r \sim 0, \tag{7}$$

(where the explicit expression for $b(m)$ can be found in Ciotti & Bertin 1999), and finally the γ models (Dehnen 1993)

$$\rho(r) = \frac{3-\gamma}{4\pi}\frac{M r_c}{r^\gamma (r_c + r)^{4-\gamma}}, \quad 0 \leq \gamma < 3. \tag{8}$$

The mass, scale–length, and anisotropy radius of each component are free parameters. For example, in C96 and C99 the family of two component self–consistent galaxy models, where one density distribution follows a γ_1 profile, and the other a γ_2 profile [(γ_1, γ_2) models], is presented.

3. Results

The main results can be summarized as follows.

In CP, by *numerical* investigation of the inequalities given in eqs. (4)-(5), it was shown that it is not possible to add an isotropic QI or Hubble modified halo ($\beta = 3$ in eq. [6]) to a $R^{1/4}$ galaxy ($m = 4$ in eq. [7], de Vaucouleurs 1948), because their DFs run into negative values near the model center. On the contrary, the isotropic $R^{1/4}$ galaxy was found to be consistent for any value of the superimposed halo mass and scale–length (i.e., over all the parameter space). At variance with the previous case, the isotropic two component models where both density distributions are characterized by a centrally flat profile (the Hubble modified + QI models), or by a centrally peaked profile (the $R^{1/4}$+$R^{1/4}$ models), were found to be consistent over all the parameter space.

In C96 and C99 the analytical DF for both components of OM anisotropic (1,1) and (1,0) models is found in terms of elliptic functions. Moreover, the method described by eqs. (4)-(5) is applied *analytically* to the more general family of (γ_1, γ_2) models. It is proved that for $1 \leq \gamma_1 < 3$ and $\gamma_2 \leq \gamma_1$ the DF of the γ_1 component in isotropic (γ_1, γ_2) models is nowhere negative, independent of the mass and concentration of the γ_2 component. As a special application of this result, it follows that a black hole (BH) of any mass can be consistently added at the center of any isotropic member of the family of γ models when $1 \leq \gamma < 3$, and that both components of isotropic (γ, γ) models (with $\gamma \geq 1$) are consistent over all the parameter space. As a consequence, the isotropic $\gamma = 1$ component in (1,1) and (1,0) models is consistent. In the anisotropic case, it is shown that an analytic estimate of a minimum value of r_a for one component γ models with a massive BH at their center can be explicitly found. As expected, this minimum value decreases for increasing γ. The region of the parameter space in which (1,0) models are consistent is successively explored using the derived DFs: it is shown that, unlike the $\gamma = 1$ component, the $\gamma = 0$ component becomes inconsistent when the $\gamma = 1$ component is sufficiently concentrated, even in the isotropic case. The combined effect of the mass concentration and orbital anisotropy is also investigated, and an interesting behavior of the DF of the anisotropic $\gamma = 0$ component is found and explained: it is analytically shown that there exists a small region in the parameter space where a sufficient amount of anisotropy can compensate for the inconsistency produced by the $\gamma = 1$ component concentration on the structurally analogous but isotropic case.

4. Conclusions

Some of the general trends that emerge when comparing different one and two component models with OM anisotropy, as those investigated in CP, Ciotti, Lanzoni, & Renzini (1996), Ciotti & Lanzoni (1997), C96 and C99, can be summarized as follows.

1) In the isotropic case, when the two density components (i.e., stars and dark matter) are similarly distributed in the galactic center, the corresponding models are consistent for any choice of the mass ratio and scale–length ratio.

2) On the contrary, when the two density components are significantly different in the galactic central regions, the mass component with the steeper density is "robust" against inconsistency. The mass component with the flatter density in the central regions is the most "delicate" and can easily become inconsistent.

3) For sufficiently small values of the anisotropy radius, OM anisotropy may produce a negative DF outside the galaxy center, while the halo concentration affects mainly the DF at high (relative) energies (i.e., near the galaxy center).

4) The trend of the minimum value for the anisotropy radius as a function of the halo concentration shows that a more diffuse halo allows for a larger amount of anisotropy: in other words, the possibility of sustaining a strong degree of anisotropy is weakened by the presence of a very concentrated halo.

As a consequence of previous points 1 and 2, one could speculate that in the presence of a centrally peaked dark matter halo, centrally flat elliptical galaxies should be relatively rare, or that a galaxy with a central power law density profile cannot have a dark halo too flat in the center. Thus, *the qualitatively similar behavior of visible and dark matter density profiles in the central regions of elliptical galaxies could be a consequence of phase–space constraints.*

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