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THE M_{BH}-σ_c RELATION AS A CONSTRAINT ON THE FORMATION OF ELLIPTICAL GALAXIES

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ABSTRACT

Most elliptical galaxies contain central black holes, whose masses scale proportionally to the observed central velocity dispersions of the host galaxies according to the so-called M_{BH}-σ_c relation. Here we discuss some consequences that can be derived by combining the M_{BH}-σ_c relation with the scaling relation describing the fundamental plane of elliptical galaxies. In particular, the possibility of substantial dissipationless merging in the formation and evolution of elliptical galaxies is discussed. We enforce the merger end products to satisfy the two scaling relations mentioned above, and a major role of dissipation in galaxy formation is strongly suggested by our analysis. Moreover, we show that existing observational data may shed some light on the complex process of black hole merging.

Subject headings: black hole physics — galaxies: elliptical and lenticular, cD — galaxies: formation — galaxies: kinematics and dynamics

1. INTRODUCTION

In recent years, the presence of massive black holes (BHs) in the centers of elliptical galaxies has been firmly established (for a review see, e.g., de Zeeuw 2001). The black hole masses (M_{BH}) lie in the range 10^6–10^9 $M_\odot$ and correlate surprisingly well with the stellar velocity dispersions in the central regions of the host galaxies:

$$M_{BH} \propto \sigma_c^{\alpha}. \tag{1}$$

The precise value of the slope α of this relation is still somewhat uncertain and appears to depend on the choice of galaxies and on the fitting procedure. For example, Gebhardt et al. (2000, hereafter G00) find $\alpha = 3.75 \pm 0.3$, whereas Ferrarese & Merritt (2000, hereafter FM00) find $\alpha = 4.8 \pm 0.5$; a discussion of the possible origin of this difference can be found in Merritt & Ferrarese (2001).

The existence of a relation between BH mass and stellar velocity dispersion may well provide valuable information on the process of galaxy formation, and some of its implications have already been discussed by Haehnelt & Kauffmann (2000, hereafter HK00). In fact, two substantially different scenarios for the formation of elliptical galaxies have been proposed. In the monolithic collapse picture, elliptical galaxies are formed at early times by dissipative processes (see, e.g., Eggen, Lynden-Bell, & Sandage 1962; Larson 1975), while in cold dark matter cosmologies elliptical galaxies result from a process of hierarchical merging. In the latter scenario, the assembly of massive elliptical galaxies requires several mergers, the last major merger taking place in relatively recent times, i.e., at $z < 1$ (see, e.g., White & Rees 1978; Kauffmann 1996). This latter view is supported by evidence of substantial galaxy merging in the cluster MS 1054−03 (van Dokkum et al. 1999). The data for this cluster show that the galaxies involved were typically E/S0 or early type S in the premerger stage.

Here we explore the consequences of combining equation (1) with other scaling relations followed by elliptical galaxies, such as the Faber-Jackson relation (Faber & Jackson 1976, hereafter FJ) and the fundamental plane (FP) relation between R_e, L, and σ_c (Dressler et al. 1987; Djorgovski & Davis 1987), where R_e is the effective radius, L is the total luminosity (for example, in the Johnson B band), and σ_c is the central velocity dispersion. Taking the point of view that early-type galaxies form mainly through merging and that the M_{BH}-σ_c and FP relations are preserved (or produced) by merging, we argue that either (1) the appropriate rule for adding the mass of merging BHs is substantially different from what is commonly assumed or (2) the merging process involves a significant dissipative phase. The requirement for significant dissipation has been recognized already on other grounds (see, e.g., Kauffmann & Haehnelt 2000; HK00; Burkert & Silk 2000).

2. SCALING RELATIONS AND THE DISSIPATIONLESS MERGING HYPOTHESIS

For simplicity, let us assume that the primordial building blocks of elliptical galaxies were stellar systems characterized by (L_0, $R_{e,0}$, $M_{BH,0}$, $\sigma_{c,0}$) and that during dissipationless merging the galaxy luminosities are added linearly. Then, by assembling n_1 primordial galaxies, we obtain $L_1 = n_1 L_0$, and equation (1) predicts that the central velocity dispersion $\sigma_{c,1}$ of galaxy L_1 correlates with $M_{BH,1}$, the mass of the BH resulting from the merger.

Unfortunately, how BH masses combine when they merge is not well known; energy (mass) can be radiated during merging, as gravitational waves and the total amount of energy radiated in a merger of two black holes is highly uncertain, because detailed calculations of these mergers are not yet available (see, e.g., Flanagan & Hughes 1998, hereafter FH98; Centrella 2000). However, using the entropy-area relation for BHs, it can be shown that in the maximally efficient (ME) radiative merging, the square of the final mass equals the sum of the squares of the initial masses. This is because $S_{BH} \propto A_{BH} \propto M_{BH}^2$ (where S_{BH} and A_{BH} are the BH entropy and surface area, respectively), and in the ME case the final area is equal to the sum of the initial BH areas (Hawking 1976; see also Peacock

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1999). To proceed we assume

$$M^{\mu}_{BH,1} = n_1 M^{\mu}_{BH,0}, \quad (2)$$

where $0 < \mu < 2$ is a free parameter. Thus, the ME and classical cases (i.e., when no energy is emitted as gravitational waves) are recovered, respectively, for $\mu = 2$ and $\mu = 1$. With $\mu \leq 1$ we can mimic in a simple way the case in which a significant amount of orbital kinetic energy (compared to $n_1 M_{BH,0} c^2$) is available just before the BH merging process and is converted into mass, resulting in a final BH with a mass greater than the linear sum of the BH masses involved in the merger.[5] Note, however, that work done so far on equal mass BH mergers indicates (in our notation) $\mu \gtrsim 1.4$ (FH98 and references therein) and that $\mu < 1$ seems physically unlikely.

From equations (1) and (2) it follows that $\sigma_{c,1}/\sigma_{c,0} = n_1^{1/\mu\alpha}$; therefore $L_1/L_0 = (\sigma_{c,1}/\sigma_{c,0})^{\mu\alpha}$. Let us now assume that another galaxy is assembled by the merging of n_2 primordial objects and that galaxies 1 and 2 also merge together in a second generation of mergers, and so on. It is trivial to show that if $L_3 = L_1 + L_2$, then $L_3/L_1 = (\sigma_{c,3}/\sigma_{c,1})^{\mu\alpha}$ and $L_3/L_2 = (\sigma_{c,3}/\sigma_{c,2})^{\mu\alpha}$. The same argument can be repeated for all successive merging events. Thus, *if the total luminosity is conserved during a merger, if μ is independent of the BH masses, and if the M_{BH}-σ_c relation is preserved, then the end products satisfy the relations*

$$M_{BH} \propto L^{1/\mu}, \quad (3)$$

$$L \propto \sigma_c^{\mu\alpha}. \quad (4)$$

Note that the treatment described is unaffected by the presence of dark matter halos. The first scaling relation above is just the Magorrian et al. (1998) M_{BH}-L relation if one adopts $\mu \simeq 1/1.2 \simeq 0.8$. The second relation is the FJ relation provided $\mu\alpha \simeq 4$. By using $\mu = 0.8$, one obtains $\alpha = 5$, a value compatible with that derived observationally by FM00. If instead of $\alpha = 4.8$ one uses $\alpha = 3.75$ (G00), then the FJ relation can be reproduced by assuming $\mu \simeq 1.1$, but then there is a conflict with the observed M_{BH}-L relation. Note, however, that the M_{BH}-L relation is characterized by a substantial scatter of at least 1 order of magnitude, and so its constraining power is not as strong as that of the FJ relation. Thus, according to the above analysis, *dissipationless merging as the main way to assemble massive elliptical galaxies could be a viable solution* (*marginally*) *consistent with the FJ, M_{BH}-L, and M_{BH}-σ_c scaling relations, provided one is willing to exclude ME radiative* (*or even classical*) *merging of BHs and α is high.*

Of course, in the idealized scenario here explored we cannot reproduce the scatter of the relations above, because the seed galaxies are assumed to be identical in *all* their properties. Possible sources of scatter in the two scaling relations given in equations (3) and (4) could be differences in the properties of the objects belonging to the "zeroth generation" and variations of the μ-value depending on the details of the merging process.

Fortunately, we can carry our analysis further by using a significantly tighter correlation, i.e., the FP relation. In the following, we require the end products of the mergers to lie on the FP, for which we choose the relation obtained by Jørgensen, Franx, & Kjærgaard (1996) in the B band for a large sample of galaxies:

$$R_e \sigma_c^{1.82} L^{-1.26} = \text{constant}, \quad (5)$$

where σ_c is the central velocity dispersion corrected to a circular aperture with radius $R_e/8$. The exponents in the equation above depend (slightly) on the wave band used for the observations, but as will be clear from the following discussion, these differences do not affect our conclusions. Although the FP and the FJ relations both describe a property of the distribution of elliptical galaxies in (R_e, L, σ_c)-space, they are essentially independent. The FP relation defines a plane in (R_e, L, σ_c)-space containing the large majority of elliptical galaxies, whereas the FJ relation describes a projection of the distribution of galaxies within that plane.

From equations (4) and (5), the following scaling relation should be satisfied by galaxies formed through dissipationless merging:

$$R_e \propto \sigma_c^{1.26\mu\alpha - 1.82}. \quad (6)$$

Note that the quantity σ_c appearing in equations (1) and (5) is defined in the same way only in the case of assuming the FM00 slope (in their analysis, FM00 use the observed velocity dispersion inside a circular aperture of radius $R_e/8$, while G00 adopt the average velocity dispersion inside the effective radius, σ_e). However, in the following we discuss also the case of the G00 slope, mainly for two reasons: the first is to investigate the degree of robustness of our results with respect to possible changes in the value of α, and the second is because Merritt & Ferrarese (2001) showed that the mean value of the ratio σ_e/σ_c (for their sample of 27 galaxies and with the exception of the Milky Way) is 1.01.

To compare the prediction in equation (6) with observations, we considered three galaxy samples, obtained by combining the data (in the Johnson B band) from Bender, Burstein, & Faber (1992; 87 objects in their Table 1, excluding dwarf spheroidal galaxies) and from Jørgensen, Franx, & Kjærgaard (1995a, 1995b; 83 galaxies from the clusters Abell 194, Abell 3574, S753, and DC 2345−28). We used galaxies with $\sigma_c \geq 100$ km s^{-1} only, in order to restrict our analysis to galaxies (except two cases) for which the central BHs have secure mass estimates (see Table 1 in FM00 and Table 1 in Merritt & Ferrarese 2001). In particular, sample 1, containing 170 galaxies, is the combination of the samples of Bender et al. and Jørgensen et al. described above. In sample 2 (151 galaxies), 19 bulges from Bender et al. in sample 1 have been excluded. Sample 3 (83 galaxies) consists of the galaxies from Jørgensen et al.

With the standard least-squares method (see, e.g., Bevington 1969), we fitted the two parameters of the linear relation

$$\log R_e = a + b \log \sigma_c, \quad (7)$$

where R_e is given in units of kiloparsecs and σ_c in kilometers per second. The results for the three samples are given in Table 1; they agree within the errors.

In agreement with the preceding discussion based on the M_{BH}-L and FJ relations, the main result obtained by using the FP is that the assumption of ME (or classical) BH merging cannot be reconciled with the observations. The above conclusion may be illustrated by estimating upper and lower limits for the predicted slope $b = 1.26\mu\alpha - 1.82$ (see eq. [6]) from the observed value of α and the cases of ME and classical BH

[5] In fact, given $a_i \geq 0$ ($i = 1, \ldots, n$) and two numbers $q \geq p \geq 0$, then $(\sum a_i^p)^{1/p} \geq (\sum a_i^q)^{1/q}$.

TABLE 1
GALAXY SAMPLES

Sample	N	$a \pm \sigma_a$	$b \pm \sigma_b$	χ^2	$\mu\alpha$
1	170	-1.86 ± 0.53	1.14 ± 0.23	0.15	2.35
2	151	-1.34 ± 0.51	0.94 ± 0.22	0.13	2.19
3	83	-1.49 ± 0.70	1.06 ± 0.31	0.12	2.29

NOTE.—The parameters a and b of eq. (5), with their errors σ_a and σ_b, and the reduced χ^2 of the fit for the three data samples. N is the number of galaxies in each sample. In the last column we give the product $\mu\alpha$ required in eq. (6) to reproduce observations.

merging. This yields, respectively, $b \simeq 10.3$ ($\mu = 2$ and $\alpha = 4.8$) and $b \simeq 2.9$ ($\mu = 1$ and $\alpha = 3.75$), to be compared with the observed value $b \simeq 1$ (Table 1). Another way to present this inconsistency is to determine the expected μ-value from the last column in Table 1; adopting $\alpha = 3.75$, we obtain $\mu \simeq 0.6$, while for $\alpha = 4.8$, $\mu \simeq 0.5$.

As is well known, the dependence of the FP coefficients on the photometric band is weak, and therefore, the argument above is not affected by the choice of the wave band; for example, the *minimum* b-value obtained (when $\mu = 1$ and $\alpha = 3.75$) using the FP coefficients given in Table 5 of Jørgensen et al. (1996) is 2.7.

In Figure 1 we present the ($\log R_e$, $\log \sigma_c$)-plane for sample 1. The data show a large dispersion around the best fit (*thick solid line*). The lines representing the *expected* R_e-σ_c correlations in the case of ME and classical BH merging for the two adopted values of α are well above the data points, however; in other words, the expected effective radii of massive galaxies are far too large.

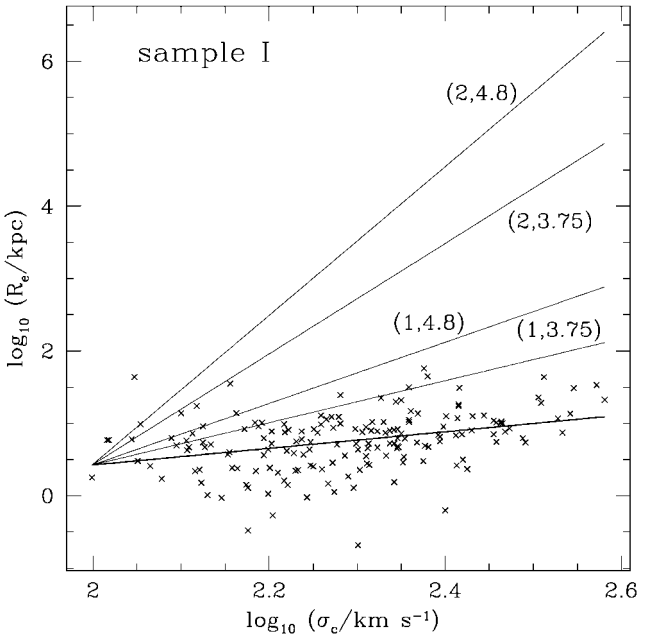

FIG. 1.—The (logarithmic) (R_e, σ_c)-plane for our sample 1 (170 galaxies). The thick line is the best fit ($b = 1.14$), and the other lines are eq. (5) evaluated with the (μ, α) pairs given in the figure.

3. DISCUSSION AND CONCLUSIONS

The inconsistency between the $M_{\rm BH}$-σ_c relation and dissipationless hierarchical merging with $\mu \geq 1$ on the one hand and the FP on the other hand can be explained in physical terms as follows. The number of mergers required to increase σ_c by a factor r is $n = r^{\mu\alpha}$. Thus, an increase of a factor r in the central velocity dispersion corresponds to an increase of $r^{\mu\alpha}$ in the luminosity. If the resulting galaxies are required to lie on the FP relation, then R_e must increase considerably because of the increase in luminosity (see eq. [5]). In other words, *the basic reason of the large increase in the effective radius is the large number of merging events required to increase the central velocity dispersion*.[6] HK00 suggest that "frequent merging of galaxies in hierarchical cosmogonies moves galaxies along the correlation in the $M_{\rm BH}$-σ_c plane, even when galaxies are gas-poor and their BHs grow mainly by the merging of preexisting BHs"; it would be of great interest to test if in this case the elliptical galaxies resulting from the mergers actually lie on the observed FP and at the appropriate location.

Following the general discussion above, three possibilities can be suggested that may solve this inconsistency:

1. In the case of purely dissipationless merging, the addition of BH masses in a merger is the result of neither ME nor classical merging; in other words, a (fine-tuned) significant fraction of the orbital kinetic energy of the merging BHs (just before the merging event) is converted into mass. In this case, μ is decreased below unity, leading to agreement with the results of § 2. However, in the commonly accepted scenario in which the BHs merge, dissipating their orbital kinetic energy and dynamically heating the stellar background of the newly formed galaxy (see, e.g., Faber et al. 1997), it is not clear whether the amount of extra kinetic energy required is available.

2. ME or classical merging of black holes may be a viable possibility in case gas is present in the merging galaxies. In fact, a gaseous component could affect the merging in two ways. First, the central BH in the merger remnant may accrete gas. Equation (2) is then no longer valid; a source term must be added on the right-hand side. This situation can be mimicked with a decreased *effective* μ in equation (2), possibly to the small values ($\mu \simeq 0.5$) that are apparently required by the data, in combination with ME or classical merging of the BHs in the progenitor galaxies. Second, the formation of stars from a centrally condensed gas component in the merger remnant might lead to a decrease of the—overall—effective radius. Of course, in both cases fine-tuning between the amount of gaseous accretion/dissipation and the masses of the merging BHs and the parent galaxies is needed because the $M_{\rm BH}$-σ_c and FP relations must be maintained. In addition to this difficulty, it has been pointed out that BH mergers that radiate too much energy anisotropically may also radiate a substantial amount of linear momentum (see, e.g., FH98), and the consequent recoil of the final BH could correspond to a kick velocity that is a moderate fraction of the speed of light, sufficient to expel the resulting BH from the galaxy. If this were confirmed, ME merging can perhaps not be reconciled with the merging scenario for elliptical galaxies, even in the presence of a substantial gaseous dissipation.

3. Finally, the more likely possibility is that the bulk of elliptical galaxies are not formed by merging at late times but as a consequence of strongly dissipative processes at high redshift. In this case, the $M_{\rm BH}$-σ_c correlation (as well as the metallicity–velocity dispersion relation) is the product of the complicated feedback processes associated with galaxy formation (see, e.g., Ostriker 1980; Burkert & Silk 2000).

[6] Note that from the virial theorem and energy conservation the *virial* velocity dispersion in dissipationless galaxy merging cannot increase. In this scenario, the increase of σ_c with $M_{\rm BH}$ must be due to nonhomology effects. But observational evidence points to dynamical and structural homology in elliptical galaxies (see, e.g., Gerhard et al. 2000) and thus provides a strong argument against dissipationless merging.

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