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THE SCALING RELATIONS OF GALAXY CLUSTERS AND THEIR DARK MATTER HALOS

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ABSTRACT

Like early-type galaxies, nearby galaxy clusters also define fundamental plane, luminosity-radius, and luminosity–velocity dispersion relations, whose physical origins are still unclear. By means of high-resolution N-body simulations of massive dark matter halos in a ΛCDM (Λ cold dark matter) cosmology, we find that scaling relations similar to those observed for galaxy clusters are already defined by their dark matter hosts. The slopes, however, are not the same, and among the various possibilities in principle able to bring the simulated and the observed scaling relations into mutual agreement, we show that the preferred solution is a luminosity-dependent mass-to-light ratio ($M/L \propto L^{\sim 0.3}$) that corresponds well to what is inferred observationally. We then show that at galactic scales there is a conflict between the cosmological predictions of structure formation, the observed trend of the mass-to-light ratio in elliptical galaxies, and the slope of their luminosity–velocity dispersion relation (which significantly differs from the analogous one followed by clusters). The conclusion is that the scaling laws of elliptical galaxies might be the combined result of the cosmological collapse of density fluctuations at the epoch when galactic scales became nonlinear plus important modifications afterward due to early-time dissipative merging. Finally, we briefly discuss the possible evolution of the cluster scaling relations with redshift.

Subject headings: dark matter — galaxies: clusters: general

1. INTRODUCTION

Early-type galaxies are known to follow well-defined scaling relations involving their main observational properties, i.e., the luminosity L, effective radius R_e, and velocity dispersion σ; in particular, we recall here the Faber-Jackson (FJ; Faber & Jackson 1976), Kormendy (Kormendy 1977), and fundamental plane (FP; Djorgovski & Davis 1987; Dressler et al. 1987) relations. Within the limitations of poorer statistics, analogous relations have also been found to hold for galaxy clusters (Schaeffer et al. 1993, hereafter S93; Adami et al. 1998a, hereafter A98; for three-parameter scaling relations involving X-ray observables, see Annis 1994; Fujita & Takahara 1999a; Fritsch & Burchert 1999; Miller, Melott, & Gorman 1999). Besides their potential importance as distance indicators, these scaling relations are also useful to get insights on the structure and, possibly, the formation and evolutionary processes of galaxies and galaxy clusters.

Well-defined scaling relations that recall the observed ones are indeed expected on the basis of the simplest model for the formation of structures in an expanding universe, namely, the gravitational collapse of density fluctuations in an otherwise homogeneous distribution of collisionless dark matter (DM). In fact, the spherical top-hat model (Gunn & Gott 1972) predicts that, at any given epoch, all the existing DM halos have just collapsed and virialized; i.e., $M = r_{\rm vir}\sigma_{\rm vir}^2/G$ (where $r_{\rm vir} \equiv -U/GM^2$ and $\sigma_{\rm vir}^2 \equiv 2T/M$, U and T being the potential and kinetic energies of a halo of mass M, respectively). In addition, all the halos are characterized by a constant mean density ρ_Δ, given by the critical density of the universe at that redshift times a factor Δ depending on z and on the given cosmology (see, e.g., Peebles 1980; Eke, Cole, & Frenk 1996). For simplicity, we call r_Δ the radius of the sphere containing such a mean density, so that $M \propto r_\Delta^3$. In general, $r_{\rm vir} \neq r_\Delta$, but if for a family of density distributions $r_{\rm vir}/r_\Delta \simeq$ const, then the virial theorem can be rewritten as $M \propto r_\Delta \sigma_{\rm vir}^2$, and together with $M \propto r_\Delta^3$, it leads to $M \propto \sigma_{\rm vir}^3$, thus providing three relations that closely resemble the observed ones. Note that these expectations involve the *global three-dimensional* properties of DM halos, while the quantities entering the observed scaling relations are *projected* on the plane of the sky. However, if DM halos are structurally homologous[5] systems, as found in cosmological simulations (Navarro, Frenk, & White 1997, hereafter NFW), and are characterized by similar velocity dispersion profiles (e.g., Cole & Lacey 1996), their projected properties are also expected to follow well-defined scaling relations (with some scatter due to departures from perfect homology and sphericity).

Of course, the simple considerations above are not sufficient to account for the *observed* scaling relations of galaxy clusters, for at least two reasons. The first is that a given potential well (such as the one associated with the cluster DM distribution) can be filled, in principle, by very different distributions of "tracers" (such as the galaxies in the clusters, from which the scaling relations are derived). This means that the very existence of the cluster FP relation implies a remarkable regularity in their formation processes: galaxies must have formed or "fallen" in all clusters in a similar way. The second reason is that any trend of the cluster mass-to-light ratio (necessary to transform the masses involved in the theoretical relations into the luminosities[6] entering the observed ones) must be taken into

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[5] Strictly speaking, DM halos are only *weakly* homologous systems, since the low-mass halos are systematically more concentrated than the more massive ones (for a definition of weak homology, see Bertin, Ciotti, & Del Principe 2002).

[6] We recall here that the luminosity of a cluster refers to the sum of the luminosities of all its constituent galaxies, i.e., a sum over the cluster luminosity function.

account for a proper interpretation of the observed scaling relations.

A distinct but strongly related question about the origin and the meaning of the scaling laws naturally arises when also applying the predictions of the cosmology at galactic scales: in fact, while scale-invariant relations are predicted, different slopes of the FJ relation are observed for galaxies and galaxy clusters (see § 2 and Girardi et al. 2000). This suggests that different processes have been at work in setting or modifying the correlations at the two mass scales. The theoretical implications of the scaling laws for elliptical galaxies (Es) have been intensively explored (see, e.g., Bertin et al. 2002 and references therein), and several works have been devoted to their study within the framework of the dissipationless merging scenario in Newtonian dynamics (e.g., Capelato, de Carvalho, & Carlberg 1995; Nipoti, Londrillo, & Ciotti 2003a, 2003b, hereafter NLC03a, NLC03b; Evstigneeva, Reshetnikov, & Sotnikova 2002; Dantas et al. 2003; González-García & van Albada 2003). In particular, if the initial total energy of the system is nonnegative, the merger products are found to follow the observed edge-on FP relation (with the important exception of mergers dominated by accretion of small galaxies), but they badly fail at reproducing the FJ and Kormendy relations, in accordance with elementary predictions based on energy conservation and on the virial theorem (NLC03a, NLC03b). It has also been shown (Ciotti & van Albada 2001) that gas-free mergers cannot account at the same time for the FP and $M_{\rm BH}$-σ relations (which link the black hole mass and the stellar velocity dispersion of the host spheroid; Ferrarese & Merritt 2000; Gebhardt et al. 2000); the importance of gas dissipation in the formation of Es is also apparent from the observed color-magnitude and $\rm Mg_2$-σ relations (e.g., Saglia et al. 2000; Bernardi et al. 2003c). Much less effort has been devoted to the theoretical study of the FP relation of galaxy clusters: besides a work on the effects of dissipationless merging (Pentericci, Ciotti, & Renzini 1996), the few other theoretical studies mainly focus on the possible relation between the FP properties and the cluster age, the number of substructures, and the underlying cosmology (Fujita & Takahara 1999b; Beisbart, Valdarnini, & Buchert 2001). From the comparison between the FP relations of galaxies and galaxy clusters in the k-space, Burstein et al. (1997) derived support for the idea that, at variance with groups and clusters, gas dissipation must have had an important role on the formation and evolution of galaxies.

In order to get a more complete view of the problems depicted above, we use high-resolution N-body simulations to study the scaling relations of very massive DM halos, which are thought to host the present-day galaxy clusters. The aim is to verify whether these relations are similar or not to the observed ones and to determine the assumptions required to bring them into mutual agreement. The results thus obtained and the empirical evidence of different slopes of the scaling relations at galactic and cluster scales are then discussed. The paper is organized as follows: In § 2 we present the scaling relations observed for nearby galaxy clusters and those followed by early-type galaxies. In § 3 we describe the high-resolution resimulation technique employed to build the sample of massive DM halos used for our analysis, and we derive their scaling relations. In §§ 4 and 5 we discuss under which hypothesis these scaling relations can be translated into the observed ones, focusing in particular on the astrophysical implications of the differences at galactic and cluster scales. The main results are summarized and discussed in § 6.

2. SCALING RELATIONS OF CLUSTERS AND EARLY-TYPE GALAXIES

From the observational point of view, only two works (namely, S93 and A98) report on the FP relation of galaxy clusters, i.e., the relation among their optical luminosity, scale radius, and velocity dispersion. The two groups agree about the existence of a tight and well-defined FP relation, even if quantitative differences in the numerical values of its coefficients are found that can be traced back to different choices of the radial variable. In fact, A98 use four density profiles (King, Hubble, NFW, and de Vaucouleurs) to describe the distribution of cluster galaxies, concluding that the best fit is provided by the first two models: consistently, they adopt the clusters' core radii to obtain the FP relation. S93 instead use in their analysis the cluster half-light projected radii (the standard choice in the vast majority of FP studies). For this reason, we choose to compare our results to those presented by S93: we note, however, that the FP coefficients derived by A98 when using the projected half-light radii derived from the de Vaucouleurs model agree within the errors with those reported by S93.

For their sample of 16 galaxy clusters at $z \leq 0.2$, S93 used the photometric parameters L and R_e in the V band (quoted by West, Oemler, & Dekel, 1989) and the velocity dispersion σ (given by Struble & Rood 1991), and they derived not only the FP relation but also FJ-like and Kormendy-like relations. However, instead of reporting the S93 scaling laws, which have been obtained by means of least-squares fits to the data, here we rederive them by minimizing the distance of the residuals perpendicular to a straight line (for the FJ and the Kormendy relations) or to a plane (for the FP relation). Results are anyway consistent with those of S93. The FJ and Kormendy relations we obtain are

$$L \propto \sigma^{2.18\pm0.52}, \tag{1}$$

in good agreement also with the results of Girardi et al. (2000), and

$$L \propto R_e^{1.55\pm0.19}, \tag{2}$$

where L is given in units of $10^{12}\ h^{-2}\ L_\odot$, σ in units of 1000 km s^{-1}, R_e in units of h^{-1} Mpc ($H_0 = 100\ h$ km s^{-1} Mpc^{-1}, and $h = 1$), and the errors on the exponents also take into account the observational uncertainties. As can be seen from Figure 1, where equations (1) and (2) are plotted together with the S93 data, the two relations above describe real scalings among the cluster properties, even if their scatter is quite large; without taking into account the observational errors, the rms dispersion of the data around the best-fit lines is 0.19 in both cases. Similarly to what happens for galaxies, a considerable improvement is achieved when combining all three observables together in an FP relation, which we have derived by performing a principal component analysis (PCA; see e.g., Murtagh & Heck 1987) on the data sample, thus obtaining the new orthogonal variables p_i, defined by

$$p_i \equiv \alpha_i \log R_e + \beta_i \log L + \gamma_i \log \sigma, \quad i = 1, 2, 3. \tag{3}$$

The numerical values of the coefficients α_i, β_i, and γ_i are listed in Table 1, while the resulting distribution of the observed clusters in the (p_1, p_3) and (p_1, p_2) spaces are shown in Figure 2, the former providing an exact edge-on view of the FP and making apparent its small thickness. A more intui-

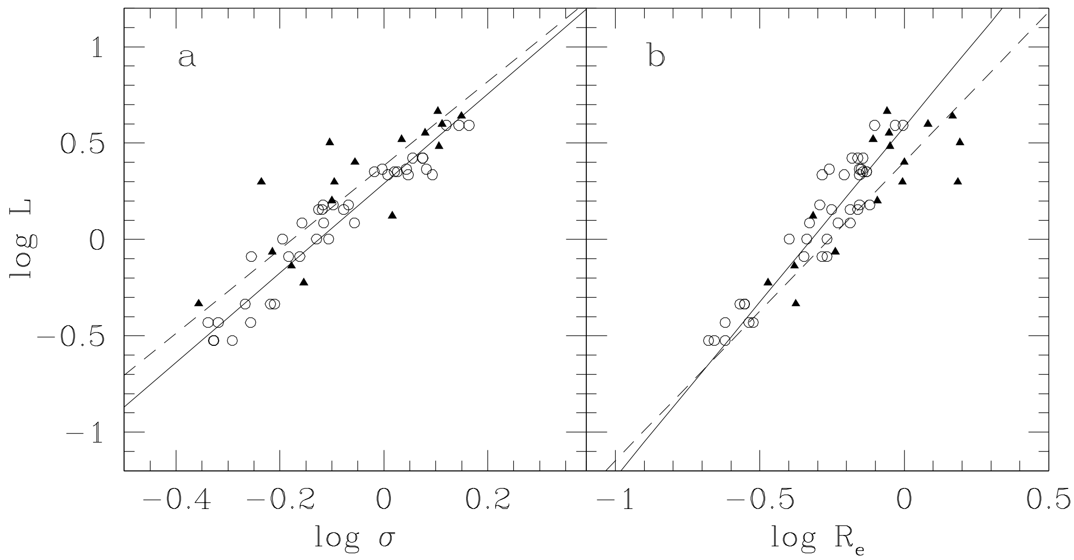

FIG. 1.—(*a*) FJ relation for the observed clusters (*triangles*) and the DM halos when eq. (11) is used for the mass-to-light ratio (*circles*). The corresponding best-fit relations are $L \propto \sigma^{2.18}$ (*dashed line*) and $L \propto \sigma^{2.32}$ (*solid line*), respectively. (*b*) Kormendy relation for the same data as in *a*. The best-fit relations are $L \propto R_e^{1.55}$ for the data (*dashed line*) and $L \propto R_e^{1.82}$ for the DM halos (*solid line*). Each DM halo is represented by three circles, corresponding to the three line-of-sight directions. Luminosities are normalized to $10^{12}\ h^{-1}\ L_\odot$ and velocity dispersions are in 1000 km s^{-1} and radii in h^{-1} Mpc.

tive representation of a (nearly) edge-on view of the FP can be obtained by solving equation (3) for $i = 3$ and using $p_3 \simeq$ const:

$$L \propto R_e^{0.9\pm0.15}\sigma^{1.31\pm0.22}. \quad (4)$$

This relation is shown in Figure 3 and is characterized by an rms dispersion of $\sim$0.07.

How do cluster scaling relations compare with the analogous relations followed by early-type galaxies? For the FP relation, the agreement with that of galaxies is remarkable. For example, the FP relation of Es in the *B* band is given by $L \propto R_e^{\sim 0.8}\sigma^{\sim 1.3}$ (e.g., Dressler et al. 1987; Jørgensen, Franx, & Kjaergaard 1996; Scodeggio et al. 1998; Bernardi et al. 2003b). As is well known, the exponents of the galaxy FP relation depend on the adopted photometric band (see, e.g., Scodeggio et al. 1998; Treu 2001), but the differences from the values reported above become significant only when using the *K* band (Pahre, Djorgovski, & de Carvalho 1998). Thus, the agreement between the cluster and galaxy FP relations can be regarded as robust. The situation is similar for the Kormendy relation: in fact, $L \propto R_e^{\sim 1.7\pm0.07}$ has been reported for Es in the *B* band (Davies et al. 1983; see also Schade, Barrientos, & López-Cruz 1997; Bernardi et al. 2003a). However, this agreement should be regarded as less robust than that for the FP relation, because largely different values of the Kormendy slope for various selections of the data sample have been reported in the case of galaxies (Ziegler et al. 1999). In any case, the FJ relation of galaxies is different from that of the clusters: in fact, $L \propto \sigma^4$ for (luminous) Es (Faber & Jackson 1976; Forbes & Ponman 1999; Bernardi et al. 2003a), while for clusters the reported slope is around 2 (a value of $\sim$1.58 is also found by A98). The additional facts that the slope of the galaxy FJ relation seems to drop below 3 for Es with $\sigma < 170$ km s^{-1} (see, e.g., Davies et al. 1983) and that the exponent of the $M_{\rm BH}$-σ relation (Ferrarese & Merritt 2000; Gebhardt et al. 2000) is remarkably similar to that of the FJ relation are discussed in § 5.

3. DM HALOS SCALING RELATIONS

3.1. *The Simulations*

To investigate whether the DM hosts of galaxy clusters, as obtained by numerical simulations, do define scaling relations similar to the observed ones, a large enough sample of very massive DM halos is needed. Therefore, we employed dissipationless simulation with 512^3 particles of $6.86 \times 10^{10}\ h^{-1}\ M_\odot$ mass each, where the volume of the universe is sufficiently large for this purpose: the box side is 479 h^{-1} Mpc comoving, with $h = 0.7$ (see Yoshida, Sheth, & Diaferio 2001). The adopted cosmological model is a ΛCDM (Λ cold dark matter) universe with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ (e.g., Ostriker & Steinhardt 1995), spectral shape $\Gamma = 0.21$, and normalization to the local cluster abundance $\sigma_8 = 0.9$. From this simulation, we have randomly selected a subsample of 13 halos at $z = 0$, with masses between $10^{14}\ h^{-1}\ M_\odot$ and $2.3 \times 10^{15}\ h^{-1}\ M_\odot$. They span a variety of shapes, from nearly round to more elongated. The richness of their environment also changes from case to case, with the less isolated halos usually surrounded by pronounced

TABLE 1

PCA PARAMETERS FOR THE OBSERVED CLUSTERS

Parameter	α_i	β_i	γ_i
p_1	−0.67	−0.51	−0.94
p_2	0.88	−0.005	−1.22
p_3	0.55	−0.61	0.80

NOTE.—The parameters p_i are defined in eq. (3).

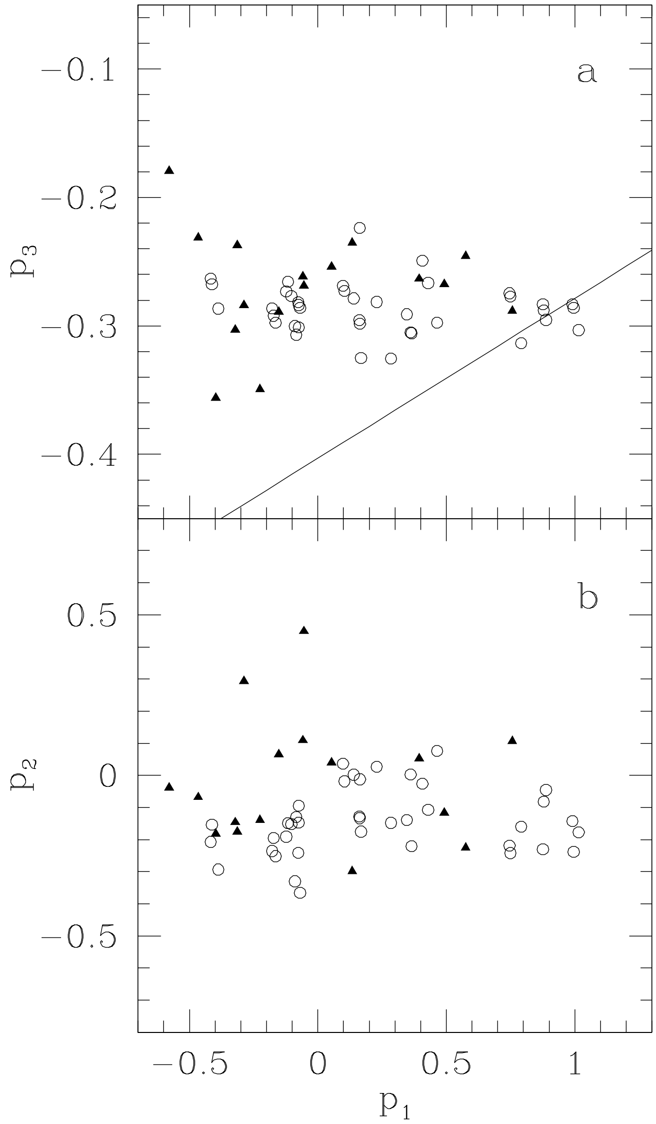

FIG. 2.—(*a*) Edge-on and (*b*) face-on views of the FP for the observed clusters (*triangles*) and the DM halos when eq. (11) is used for the mass-to-light ratio (*circles*). The solid line represents the best fit of the DM halos when assuming a constant mass-to-light ratio $\Upsilon = 280\ h\ (M/L)_\odot$. Note that mass increases for decreasing p_1.

filamentary structures containing massive neighbors (up to 20% of the selected halo in mass).

Given the mass resolution of the simulation, fewer than 1500 particles compose a halo of $10^{14}\ h^{-1}\ M_\odot$, and as a result of discreteness effects, its properties defining the FP relation cannot be accurately determined. We have therefore *resimulated*, at higher resolution, the halos in our sample by means of the technique introduced in Tormen, Bouchet, & White (1997); here we recall only its relevant aspects (for more details see Lanzoni, Cappi, & Ciotti 2003). The first step is to select in a given cosmological simulation the halo one wants to "zoom in" on. Then, the region defined by all the particles composing the selected halo and its immediate surroundings is detected in the initial conditions of the parent simulation, and the number of particles within it is increased by the factor needed to attain the suited mass resolution. Such a region is therefore called "the high-resolution region" (HRR). Since the mean interparticle separation within the HRR is smaller than in the parent simulation, the corresponding high-frequency modes of the primordial fluctuation spectrum are added to those on the larger scales originally used in the parent simulation, and the overall displacement field is also modified consequently. At the same time, the distribution of surrounding particles is smoothed by means of a spherical grid, whose spacing increases with the distance from the center: in such a way, the original surrounding particles are replaced by a smaller number of *macroparticles*, whose mass grows with distance from the HRR. Thanks to this method, even if the number of particles in the HRR is increased, the total number of particles to be evolved in the simulation remains small enough to require reasonable computational costs, while the tidal field that the overall particle distribution exerts on the HRR remains very close to the original one. For the new initial configuration thus produced, vacuum boundary conditions are adopted; i.e., we assume a vanishing density fluctuation field outside the spherical distribution of particles, with diameter equal to the original box size L. A new N-body simulation is then run, starting from these new initial conditions, and allowing us to reobtain the selected halo at the required resolution.

We have applied this technique to the 13 selected massive DM halos. For eight of them the resolution has been increased by a factor of $\sim$33, by means of high-resolution particles of mass $\sim 2.07 \times 10^9\ h^{-1}\ M_\odot$ each, while a further increase of a factor of 2 has been adopted for the five intermediate-mass halos (the particle mass is $10^9\ h^{-1}\ M_\odot$ in this case). The gravitational softening used for the HRR is $\epsilon = 5\ h^{-1}$ kpc (roughly Plummer equivalent), corresponding to about 0.2% and 0.5% of the virial radius of the most and least massive halos, respectively. This scale length represents the spatial resolution of the resimulations, to be compared with that of 30 h^{-1} kpc of the original one. To run the resimulations, the parallel dissipationless tree code GADGET (Springel, Yoshida, & White 2001) has been used. At $z = 0$, the DM halos have

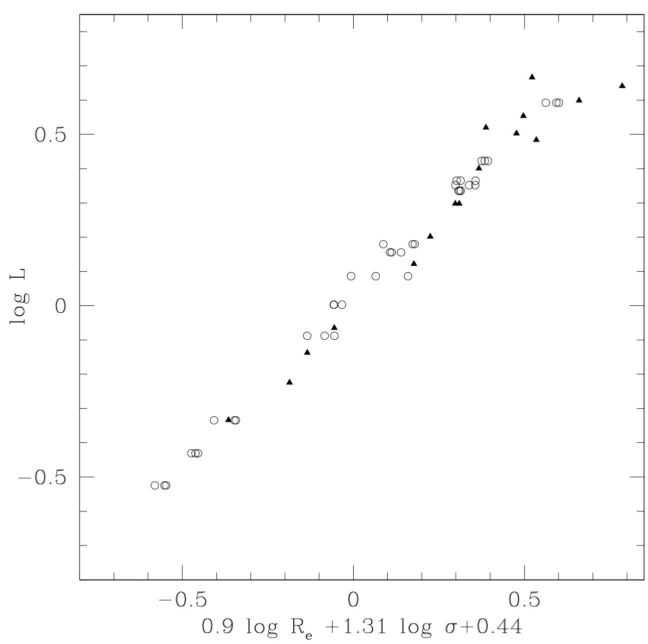

FIG. 3.—FP relation of observed clusters (*triangles*) and DM halos when eq. (11) is used for the mass-to-light ratio (*circles*).

TABLE 2

Properties of the DM Halos at $z = 0$

Name	M ($10^{14}\ h^{-1}\ M_\odot$)	r_Δ (h^{-1} Mpc)	$r_{\rm vir}/r_\Delta$	$\sigma_{\rm vir}$ (km s^{-1})	$R_{h,x}$ (h^{-1} Mpc)	$R_{h,y}$ (h^{-1} Mpc)	$R_{h,z}$ (h^{-1} Mpc)	$\sigma_{h,x}$ (km s^{-1})	$\sigma_{h,y}$ (km s^{-1})	$\sigma_{h,z}$ (km s^{-1})
1542	0.82	0.99	0.77	760	0.22	0.24	0.21	471	470	511
3344	1.09	0.99	0.83	815	0.30	0.24	0.29	459	554	480
914	1.45	1.09	0.75	967	0.27	0.28	0.28	616	605	541
4478	2.92	1.37	1.02	1075	0.54	0.52	0.45	556	656	689
1777	3.83	1.50	0.94	1177	0.40	0.54	0.46	783	638	742
564	4.91	1.63	1.01	1271	0.65	0.59	0.47	696	878	766
689	6.08	1.75	1.01	1354	0.56	0.65	0.69	837	749	761
245	6.50	1.79	1.03	1332	0.51	0.70	0.76	855	799	764
51	10.78	2.12	0.84	1794	0.62	0.70	0.52	1114	1018	1241
696	11.37	2.16	0.98	1669	0.74	0.72	0.74	1062	1048	959
72	11.77	2.18	0.90	1777	0.70	0.71	0.55	1105	993	1207
1	13.99	2.31	0.88	1870	0.66	0.72	0.69	1187	1136	1189
8	23.42	2.75	0.92	2262	0.79	0.93	0.99	1459	1395	1319

Notes.—M is the DM halo mass, r_Δ and $r_{\rm vir}$ are the truncation and virial radii, respectively, and $\sigma_{\rm vir}$ is the virial velocity dispersion. The projected half-mass radii and the corresponding line-of-sight velocity dispersions are given for three orthogonal directions. Note that R_h/r_Δ is in the range $\sim$0.2–0.4.

been selected by means of a spherical overdensity criterion (Lacey & Cole 1994; Tormen et al. 1997); i.e., they are defined as spheres centered on maximum-density peaks in the particle distribution and with mean density equal to the virial density ρ_Δ predicted by the spherical top-hat model for the adopted ΛCDM cosmology ($\rho_\Delta \simeq 97\rho_{\rm crit}$ at $z = 0$). The corresponding masses, linear scales, and velocity dispersions are listed in Table 2.

3.2. *Scaling Relations for the DM Halos*

According to the selection method described in the previous section, the DM halos are all characterized by the same ρ_Δ and should also be nearly virialized systems. The first condition (and thus the relation $M \propto r_\Delta^3$) is satisfied by construction, while the second can be easily verified. In fact, we find that all the selected halos follow the relations $M \propto r_\Delta \sigma_{\rm vir}^2$, with an rms scatter of only 0.03, and $M \propto \sigma_{\rm vir}^{3.1}$, with rms $\simeq 0.05$. In Table 2 we list the ratio $r_{\rm vir}/r_\Delta$ for each halo.

However, *projected* quantities are involved in the observations, and the first step of our analysis is the determination of which (if any) scaling relations are satisfied by the DM halos when projected. We have therefore constructed the projected radial profiles of the selected halos by counting the DM particles within concentric shells around the center of mass for three arbitrary orthogonal directions (x, y, and z), and we defined R_h as the projected radius of the circle containing half of the total number of particles. Then, the velocity dispersion σ_h was computed from the line-of-sight (barycentric) velocity of all the particles within R_h. Since the DM halos, as well as real clusters, are not spherical, such a procedure gives different values of R_h and σ_h for the three lines of sight (the maximum variations, however, never exceed 33% and 21% for the two quantities, respectively; see Table 2), and we decided to build our data sample by considering all three projections for each halo. With the projected properties R_h and σ_h now available, we have determined the best-fit relations between M and R_h and between M and σ_h by minimizing the distance of the residuals perpendicular to a straight line and thus obtaining the DM analogs of the observed FJ and Kormendy relations:

$$M \propto \sigma_h^{3.02\pm0.15} \tag{5}$$

and

$$M \propto R_h^{2.36\pm0.14}, \tag{6}$$

with rms $\simeq$ 0.12 and 0.15, respectively. With a PCA of these data we determined the relation analogous to equation (4):

$$M \propto R_h^{1.1\pm0.05}\sigma_h^{1.73\pm0.04}, \tag{7}$$

with rms = 0.04. Compared to those among the virial properties, these relations have larger scatters, as expected. The FJ and FP relations have slopes similar to those obtained for the virial quantities, while the M-R_h relation appears to be significantly flatter, as a consequence of the different density concentration of low- and high-mass halos. Note that the NFW concentration parameter varies only weakly in the mass range covered by the halos in our sample (e.g., Eke, Navarro, & Steinmetz 2001; Bullock et al. 2001). This is consistent with our results; in fact, from equation (6) and $M \propto r_\Delta^3$, we find that $R_h/r_\Delta \propto M^{\sim 0.09}$.

We stress that while scaling relations between M, r_Δ (or $r_{\rm vir}$), and $\sigma_{\rm vir}$ were expected, a tight correlation between projected properties is a less trivial result; in fact, structural and dynamical nonhomology can, in principle, produce significantly different effective radii and projected velocity dispersion profiles for systems characterized by identical M, $r_{\rm vir}$, and $\sigma_{\rm vir}$. It is also known that weak homology, coupled with the virial theorem, does indeed produce well-defined scaling laws (see, e.g., Bertin et al. 2002). Therefore, the scaling relations presented in equations (5)–(7) are a first interesting result of our study. The difference between the values of the exponents appearing in equations (5)–(7) and those in the virial relations is the direct evidence of weak homology of the halos; in fact, strong homology would result in exactly the same exponents, while a strong nonhomology would disrupt any correlation. We checked this further, and we found that the DM halos in our sample still show well-defined scaling relations (although with different exponents) when the projected radius encircling any fixed fraction of the total mass and the corresponding projected velocity dispersion within it (see Table 3) are considered. Note that this finding is in agreement with the results already pointed

TABLE 3
Exponents and rms Scatter of the Scaling Relations

	$M \propto \sigma_f^a$		$M \propto R_f^b$		$M \propto R_f^c \sigma_f^d$		
Fraction	a	rms	b	rms	c	d	rms
0.8..........	3.01	0.12	2.67	0.10	1.47	1.43	0.04
0.5..........	3.02	0.12	2.36	0.15	1.10	1.73	0.04
0.3..........	3.10	0.13	2.29	0.17	0.97	1.95	0.04
0.1..........	3.12	0.13	2.15	0.19	0.88	2.02	0.03

Note.—M is halo total mass, R_f is the projected radius containing the fraction f of the total mass M, and σ_f is the mean projected velocity dispersion within R_f: $R_{0.5} \equiv R_h$.

out by several groups, namely, the fact that DM halos obtained with numerical N-body simulations in standard cosmologies are characterized by significant structural and dynamical weak homology (e.g., Cole & Lacey 1996; NFW; Subramanian, Cen, & Ostriker 2000 and references therein).

4. FROM THE SIMULATED TO THE OBSERVED SCALING RELATIONS

Comparison of equations (5)–(7) with equations (1), (2), and (4) reveals that the FJ, Kormendy, and FP relations of simulated DM halos are characterized by slopes different from those derived observationally. What kind of regular and systematic trends with cluster mass of the galaxy properties and distribution are implied by these differences? In order to answer this question, we define the dimensionless quantities $\Upsilon \equiv M/L$, $\mathcal{R} \equiv R_h/R_e$, and $\mathcal{S} \equiv \sigma_h/\sigma$. Focusing first on the edge-on FP relation, from equations (4) and (7) we obtain

$$\frac{\Upsilon}{\mathcal{R}^{1.1}\,\mathcal{S}^{1.73}} \propto R_e^{0.2}\sigma^{0.42}. \tag{8}$$

Thus, in order to transform the DM halos' FP relation into the observed FP relation, the product $\Upsilon\mathcal{R}^{-1.1}\mathcal{S}^{-1.73}$ must systematically increase as $R_e^{0.2}\sigma^{0.42}$, which in turn, again from equation (4), is approximately proportional to $L^{0.3}$. In principle, Υ, $\mathcal{R}$, and $\mathcal{S}$ could all vary in a *combined* and *regular* way from cluster to cluster, so that equation (8) is satisfied. Of course, given the small scatter around the best-fit relation (eq. [4]), this kind of solution requires a remarkable fine tuning of the variations of the three parameters. Alternatively, it is possible that only one of the three parameters varies significantly, while the other two are approximately constant. This situation is analogous to that faced in the studies of the physical origin of the FP tilt of Es, where the so-called orthogonal exploration of the parameter space is often adopted (see, e.g., Renzini & Ciotti 1993; Ciotti 1997). In this approach, all but one of the available model parameters are fixed to constant values, and the goal is to determine what kind of variation of the "free" parameter is necessary to reproduce the observed FP tilt. Several quantitative results have been derived in this framework (see, e.g., Ciotti, Lanzoni, & Renzini 1996; Ciotti & Lanzoni 1997 and references therein), even though, by construction, it cannot provide the most general solution to the problem, and the choice of the specific parameter responsible for the tilt is somewhat arbitrary (see Bertin et al. 2002; Lanzoni & Ciotti 2003). In the present context some of this arbitrariness can be removed; in fact, here we assume that (1) the DM distribution in real clusters is described by the simulated DM halos and that galaxies are merely dynamical *tracers* of the total potential well, (2) in addition to the edge-on FP relation, we also consider the constraints imposed by the FJ and the Kormendy relations. These two points will *allow us to use the orthogonal exploration approach for determining what is the most plausible origin of the tilt between the simulated and the observed cluster FP relations*.

In order to make the DM halos' FP relation reproduce the observed one within the framework of the orthogonal exploration approach, we have three different possibilities, corresponding to the choice of Υ, $\mathcal{R}$, or $\mathcal{S}$ as the key parameter, while keeping constant the remaining two in the left-hand side of equation (8). Note that the two choices based on variations of $\mathcal{R}$ or $\mathcal{S}$ should be interpreted from an astrophysical point of view as systematic differences in the way galaxies populate the cluster DM potential well as a function of the cluster mass. However, the orthogonal analysis of the FJ and the Kormendy relations strongly argues against these two solutions, since from equations (1), (2), (5), and (6) one obtains

$$\frac{\Upsilon}{\mathcal{S}^{3.02}} \propto \sigma^{0.84} \tag{9}$$

and

$$\frac{\Upsilon}{\mathcal{R}^{2.36}} \propto R_e^{0.81}. \tag{10}$$

Thus, it is apparent that any attempt to reproduce equation (8) by a variation of $\mathcal{R}$ (or $\mathcal{S}$) alone will fail at reproducing the FJ (or the Kormendy) relation; in fact, the only parameter common to equations (8), (9), and (10) is the mass-to-light ratio Υ.[7]

Therefore, while a *purely structural* ($\mathcal{R}$) and a *purely dynamical* ($\mathcal{S}$) origin of the tilt between the DM halos' FP and the clusters' FP both seem to be ruled out for the above reasons, a systematically varying mass-to-light ratio, for constant $\mathcal{R}$ and $\mathcal{S}$, could in principle account for all the three considered scaling relations. In particular, from equation (8), $\Upsilon \propto L^{\alpha}$, with $\alpha \sim 0.3$. Guided by this indication, we tried to superpose the points corresponding to the simulated DM halos

[7] Note that the constraints imposed by the FJ and the Kormendy relations should not be considered redundant with respect to those imposed by the edge-on FP relation; in fact, these two relations, albeit with a large scatter, describe how galaxies are distributed on the face-on FP relation.

on the sample of observed clusters by using $\Upsilon \propto M^{\beta}$, and we found that if

$$\Upsilon = 280\ h\left(\frac{M}{10^{14}\ h^{-1}\ M_{\odot}}\right)^{0.23}\left(\frac{M}{L}\right)_{\odot}, \tag{11}$$

the edge-on FP relation of DM halos is practically indistinguishable from that of real clusters (see Figs. 2*a* and 3); the value of α derived from this assumption is $\alpha = \beta/(1-\beta) \simeq 0.3$. Note that such a trend of Υ is in agreement not only with the expectations of equation (8) but also with what is inferred observationally in the *B* band (S93; Girardi et al. 2002; Bahcall & Comerford 2002; but see Bahcall, Lubin, & Dorman 1995; Bahcall et al. 2000; and Kochanek et al. 2003 for claims of a constant mass-to-light ratio at large scales) and what is inferred from the comparison between the observed *B*-band luminosity function (LF) of virialized systems and the halo mass function predicted in CDM cosmogonies (Marinoni & Hudson 2002). A remarkable agreement is also found with the results of van den Bosch, Yang, & Mo (2003), who, using a completely different approach, obtain $\Upsilon \propto M^{0.26}$ for their model A, which allows for a nonconstant Υ at the cluster scales (see their eq. [15] and Table 2). In addition, $\Upsilon \sim 280\ h\,(M/L)_{\odot}$ for $M \simeq 10^{14}\ h^{-1} M_{\odot}$ clusters and $\Upsilon \sim 475\ h\ (M/L)_{\odot}$ for $M \simeq 10^{15}\ h^{-1} M_{\odot}$ clusters are values well within the range of the various estimates for galaxy clusters (e.g., Adami et al. 1998b; Mellier 1999; Wilson, Kaiser, & Luppino 2001; Girardi et al. 2000, 2002). It is also remarkable (since it is not a necessary consequence) that by adopting equation (11) *the face-on FP and the FJ and Kormendy relations are also very well reproduced*, as apparent from Figs. 2*b*, 1*a*, and 1*b*, respectively.

What could be the physical interpretation of the required trend of the mass-to-light ratio with cluster mass? We note that equation (11) can be formally rewritten as $\Upsilon_{\rm gal}(M/M_{\rm gal}) \propto M^{\beta}$, where *M* is the DM mass of the clusters (~their total mass), $M_{\rm gal}$ is their total stellar content in galaxies, and

$$\Upsilon_{\rm gal} \equiv \frac{\int N_{\rm gal}(L)\Upsilon_{*}(L)L\,dL}{\int N_{\rm gal}(L)L\,dL}, \tag{12}$$

where $N_{\rm gal}(L)$ is the cluster LF and finally, $\Upsilon_{*}(L)$ is the mean stellar mass-to-light ratio of a galaxy of total luminosity *L*. Thus, the trend of Υ with *M* could be ascribed to $\Upsilon_{\rm gal} \propto M^{\beta}$ for $M/M_{\rm gal} = {\rm const}$ or to $M_{\rm gal} \propto M^{1-\beta}$ for $\Upsilon_{\rm gal} = {\rm const}$ (or, more generally, to a combined effect of these two quantities). Both possible solutions have interesting astrophysical implications. For example, a constant $\Upsilon_{\rm gal}$ from cluster to cluster is obtained only if all clusters have the same population of galaxies, i.e., if they are characterized by a universal LF and a similar morphological mix such that the distribution of the stellar mass-to-light ratios of their galaxies is also the same. In such a case, the required trend $M_{\rm gal} \propto M^{0.77}$ should be entirely explained by a systematic increase with *M* of the total number of galaxies, in remarkable agreement with several studies of the halo occupation numbers that find $N_{\rm gal} \propto M^{a}$, with *a* in the range 0.7–0.9 (Peacock & Smith 2000; Scranton 2002; Berlind & Weinberg 2002; Marinoni & Hudson 2002; van den Bosch et al. 2003). To study the ratio $M/M_{\rm gal}$, an estimate of the stellar mass in galaxies can also be obtained from the observed total luminosity in the near-infrared (since Es are the dominant component of the cluster population, and their luminosity in the *K* band gives a reasonable measure of their stellar mass, i.e., $M_{\rm gal} \propto L_K$). However, observational results are still uncertain and controversial: a constant or weakly decreasing M/L_K with *M* is reported by Kochanek et al. (2003) for the 2MASS (Two Micron All-Sky Survey) clusters, while an increasing M/L_K is claimed by Lin, Mohr, & Stanford (2003) for the data from the same survey.

In any case, the universality of the cluster LF is still under debate, since it appears to be appropriate in many cases, but variations in some individual clusters have also been reported (see, e.g., Yagi et al. 2002, De Propris et al. 2003, and Christlein & Zabludoff 2003 for detailed discussions and recent results). In particular, the LF of early-type (or quiescent) galaxies appears to vary significantly among clusters (but see Andreon 1998): if the richer clusters contain a proportionally larger fraction of Es (Balogh et al. 2002), which are characterized by a Υ_{*} higher than that of spirals, a cluster-dependent $\Upsilon_{\rm gal}$ should be expected even in presence of a universal LF.

5. CLUSTER VERSUS GALAXY SCALING RELATIONS

As discussed in § 2, the cosmological-collapse model predicts the same scaling relations at all mass scales, and remarkably, the edge-on FP relation and the (although more dispersed) Kormendy relation of clusters and Es are very similar. In particular, the interpretation of the FP tilt of Es as the combined result of the virial theorem and strong structural and dynamical homology implies $\Upsilon_{*} \propto L^{0.3}$ in the *B* band (e.g., Faber et al. 1987; van Albada, Bertin, & Stiavelli 1995; Bertin et al. 2002), in agreement with the result for clusters (eq. [11]). In § 4, we showed that equation (5), coupled with $\Upsilon \propto L^{0.3}$, does reproduce the observed FJ relation at cluster scales, but clearly this cannot work in the case of galaxies; in fact, the FJ relation of Es is characterized by a significantly higher exponent (~4) than that of clusters (~2). Indeed, Υ_{*} should actually *decrease* for increasing galaxy luminosity in order to reproduce the observed FJ relation, at variance with all the available indications (e.g., Faber et al. 1987; Jørgensen et al. 1996; Scodeggio et al. 1998).

One is then forced to assume that (1) the relation $M \propto \sigma_{\rm vir}^{3}$ does not apply in the case of Es, perhaps because of a failure of standard cosmology at small scales, or (2) evolutionary processes have modified the ratio $\sigma_{*}/\sigma_{\rm vir}$, where σ_{*} is the galaxy central velocity dispersion (from which the FJ relation is derived). As for point 1, this might be another problem encountered by the current cosmological paradigm at small scales, in addition to the well-known overprediction of the number of DM satellites (the so-called DM crisis; see Moore et al. 1999, but see also Stöhr et al. 2002 and the recent results on the "running spectral index" obtained by Peiris et al. 2003 from the *Wilkinson Microwave Anisotropy Probe* data) and the cusp-core problem for low surface brightness and dwarf galaxies (e.g., Swaters et al. 2003a, 2003b and references therein). However, this point is at present very speculative, and thus in the following we restrict our discussion to the standard CDM paradigm, which predicts $M \propto \sigma_{\rm vir}^{3}$, also at galactic mass scales. Two physical processes (namely, gas dissipation and early-time merging) certainly played a major role in galaxy evolution, thus supporting point 2. The effects of these two processes have been already discussed in the context of *k*-space by Burstein et al. (1997); here we focus on the implications that can be derived from the simpler FJ relation, taking into account the additional information from recent numerical simulations (NLC03a, NLC03b) and the constraints imposed on galaxy merging by the recently discovered $M_{\rm BH}$-σ relation (Ferrarese

& Merritt 2000; Gebhardt et al. 2000), according to which $M_{\rm BH} \propto \sigma_*^{\sim 4}$, with very small dispersion.

We then adopt the point of view that at the epoch of the detachment from the Hubble flow, the seed galaxies were also characterized by the "universal" scaling law $M \propto \sigma_{\rm vir}^3$, and we discuss the possibility that galaxy merging and gas dissipation originated the observed FJ relation of Es. We start by considering the effect of dissipationless merging alone. From this point of view, it is clear that merging played different roles in the evolutionary history of clusters and galaxies. In fact, while clusters can be thought of as formed from the *collapse* of density perturbations at scales that *very recently became nonlinear*, galaxies are presently in a highly nonlinear regime, and the *merging* they suffered since their separation from the Hubble expansion can no longer be interpreted in terms of the cosmological collapse of density fluctuations. The differences between these two dynamical processes have important consequences for the present discussion. In fact, in the *cosmological-collapse* case the initial conditions correspond to those of a "cold" system (i.e., $2T_i + U_i = V < 0$), and so virialization increases the virial velocity dispersion of the end products as $T_f = T_i - V$; the systematic increase of $\sigma_{\rm vir}$ with M in clusters is basically due to this process. At highly nonlinear scales (such as, for instance, in the case of galaxies in the outskirts of clusters or groups), the situation is considerably different. In fact, if merging occurs, it cannot be interpreted as the collapse of a cold system. On the contrary, the initial conditions of the merging pair are in general characterized by a null or a positive V (see, e.g., Binney & Tremaine 1987, chap. 7); under these conditions, the virial velocity dispersion of the remnant will not increase. In fact, numerical simulations of one- and two-component galaxy models show that successive dissipationless merging at galactic scales, while preserving the edge-on FP relation, does not reproduce the FJ relation, the end products being characterized by too low a σ_* (i.e., nearly constant) compared to the expectations of the FJ relation (NLC03a, NLC03b). In other words, dissipationless merging at galactic scales in general will produce a relation $M \propto \sigma_{\rm vir}^\alpha$, with $\alpha > 3$. Obviously, this could be an interesting property in our context, since we are exactly looking for a mechanism able to increase α from 3 to 4. We also note that faint Es do follow an FJ relation with a best-fit slope significantly lower than 4 (Davies et al. 1983 report a value of $\sim$2.4) and more similar to the cosmological predictions; a simple interpretation of this fact could be that small galaxies experienced fewer merging events than giant Es, so their scaling relations are more reminiscent of the cosmological origin. However, even if the exponent of the M-σ relation can be increased, several theoretical and empirical arguments clearly indicate that purely dissipationless merging cannot be at the origin of the spheroids. For example, why the exponent should increase to the observed value is unclear, even though it has been claimed that the FJ relation of Es is the result of cumulative effects of inelastic merging and passing encounters taking place in a cluster environment (Funato, Makino, & Ebisuzaki 1993; Funato & Makino 1999). These authors already discuss some weaknesses of the scenario they suggest (for instance, the difficulty of accounting for the analogous Tully-Fisher relation of spirals), and we add that it is not clear whether the other scaling laws are also reproduced by their simulations or how the FJ relation could also be followed by the field Es if resulting from dynamical interactions in cluster environment. Moreover, a further difficulty is added by the recently discovered $M_{\rm BH}$-σ relation: if the FJ is the cumulative result of random dynamical processes that changed the galaxy velocity dispersion, how can the $M_{\rm BH}$-σ relation be so tight? Finally, there is an even stronger empirical argument against substantial dissipationless merging for Es: in fact, if the merging end products are forced to obey both the edge-on FP and $M_{\rm BH}$-σ relations, then they are characterized by exceedingly large effective radii with respect to real galaxies (Ciotti & van Albada 2001).

Concerning the effect of gas dissipation on the predicted relations at galactic and cluster scales, we note that an important difference already resides in *what* is effectively observed in the two cases when deriving the scaling laws: the galaxy distribution, tracing the gravitational potential of the DM, in the case of clusters, and the stellar population within a fraction of the effective radius, where stars themselves are the major contributors to the total gravitational field, in the case of Es. Since the top-hat model properly describes the (dissipationless) collapse of the DM component, it is not surprising that it cannot be accurately applied to predicting the properties of the baryonic (dissipative) matter where it is dominant. In addition, gas dissipation is thought to be negligible for the formation and evolution of clusters, while it empirically appears to be increasingly important with mass for the galaxies, as mirrored by their Mg_2-σ and color-magnitude relations and by the observed metallicity gradients (e.g., Saglia et al. 2000; Bernardi et al. 1998, 2003c). As discussed by Ciotti & van Albada (2001), its effects also represent a possible solution to the exceedingly large effective radii found for (dissipationless) merger products forced to follow both the FP and $M_{\rm BH}$-σ relations. Thus, from the considerations above, gas dissipation is a good candidate to explain the difference between the cluster and galaxy FJ relations. However, we note that *baryonic dissipation alone* cannot solve the discrepancy; in fact, its effect is to increase σ_*, thus further decreasing the exponent in the galactic FJ relation. This can be seen in a more quantitative way as follows: given that galactic DM halos follow $M \propto \sigma_{\rm vir}^3$, then the FJ relation implies that $(M/M_*)\Upsilon_*(\sigma_*/\sigma_{\rm vir})^3 \propto \sigma_*^{-1}$; this means that at fixed M/M_* and Υ_*, dissipation should have been more important in low-mass galaxies, contrary to the existing observational evidence.

In summary, the arguments presented above strongly suggest that merging *and* gas dissipation played a fundamental role in the formation and evolution of galaxies. In particular, given the competitive effect of these two processes on the slope of the M-σ relation, they *both* appear to be necessary for modifying the $M \propto \sigma_{\rm vir}^3$ into the observed FJ relation. Coupled with the necessity of dissipative merging, the observational evidence that the bulk of stars in Es, even at high z, are old puts a strong constraint on the time when the mass of spheroids was assembled: say, $z \gtrsim 2$. As already recognized by several authors, this is in agreement with the available information about the star formation history of the universe and the redshift evolution of the quasar LF (see, e.g., Haehnelt & Kauffmann 2000; Ciotti & van Albada 2001; Burkert & Silk 2001; Yu & Tremaine 2002; Cavaliere & Vittorini 2002; Haiman, Ciotti, & Ostriker 2003). A tight connection between the star formation and the quasar activity in Es is further supported by the striking similarity of slopes between the FJ and the $M_{\rm BH}$-σ relations, thus suggesting a common origin for these two empirical laws.

Based on these considerations, it seems likely that the scaling relations of Es are the result of the cosmological collapse of the density fluctuations at the epoch when galactic scales became nonlinear, plus important modifications afterward due to early-time merging in gas-rich systems. On the contrary, since the present-day nonlinearity scale is that of galaxy clusters and since the role of gas dissipation is thought to be marginal in clusters life, the cosmological-collapse model and a mass-to-light ratio increasing with cluster mass seem to be sufficient to account for their scaling relations.

6. DISCUSSIONS AND CONCLUSIONS

In this paper we have addressed the question of whether high-mass DM halos (which are thought to host clusters of galaxies) do define the FP and the other scaling relations observed for nearby clusters. For that purpose, we have analyzed a sample of 13 massive DM halos, obtained with high-resolution N-body simulations in a ΛCDM cosmology. After verifying that the DM halos do follow the predictions of the spherical collapse model for virialized systems, we have found that their projected properties also define FJ, Kormendy, and FP-like relations. This latter result is not trivial, and it can be traced back to the weak (structural and dynamical) homology shown by the DM halos assembled via hierarchical cosmological merging. However, the slopes of the DM halo scaling laws do not coincide with the observed ones, and we have discussed what kind of systematic variations of one or more of the structural and dynamical properties of the galaxy distribution with cluster mass could account for such differences. We have shown that two of the three basic options can be discarded just by requiring the simultaneous reproduction of the (edge-on) FP, FJ, and Kormendy relations. A solution that instead works remarkably well is to assume a cluster mass-to-light ratio Υ increasing as a power law of the luminosity. The required normalization and slope agree well with those estimated observationally for real galaxy clusters. We have discussed two possible causes for such a trend of Υ, namely, a systematic increase of the *galaxy* mass-to-light ratio with cluster luminosity and a decrease of the baryonic mass fraction for increasing cluster total mass, concluding that both possibilities are consistent with the available observational data. In any case, it appears that *the FJ, Kormendy and FP relations of nearby clusters of galaxies can be explained as the result of the cosmological collapse of density fluctuations at the appropriate scales, plus a systematic trend of the total mass-to-light ratio with the cluster mass*. Note that this is by no means a trivial result: in fact, it is known that numerical simulations of galaxy dissipationless merging (where the mass-to-light ratio is kept fixed) do reproduce well the edge-on FP relation, but badly fail with the FJ and the Kormendy relations (NLC03b).

We next focused on the fact that Es follow an FJ relation with a significantly different slope ($\sim$4) compared to that of clusters ($\sim$2) but show a similar trend of the mass-to-light ratio with the system luminosity. We have discussed the implications of this observational evidence, under the assumption that at the epoch of the detachment from the Hubble flow, $M \propto \sigma_{\rm vir}^3$ at galactic scales also, as required by standard cosmology and taking into account several pieces of empirical and theoretical evidence suggesting that gas dissipation and merging must have played an important role in galaxy evolution. Since dissipationless merging and baryonic dissipation have competitive consequences on the system velocity dispersion, we argue that the *combined* effects of these two processes are required to account for the slope of the FJ relation of Es. We therefore conclude that *the scaling relations of Es might be the result of the cosmological collapse of density fluctuations at the epoch when galactic scales became nonlinear, plus successive modifications due to (early-time) dissipative merging*. This scenario also seems to be supported by the similarity between the slopes of the FJ and $M_{\rm BH}$-σ relations and between the peak of the star formation rate of the universe and that of the quasar luminosity function.

Before concluding, we point out that while no observational data are yet available for the scaling relations of galaxy clusters at high redshift, the results that we have obtained allow us to speculate on what could be expected. On the one hand, the predictions of the top-hat model are the same at any redshift, and also weak homology of DM halos in numerical simulations appears to hold at any epoch; thus, if the dependence of the cluster Υ on L changes according to passive evolution, scaling relations with the same slopes should also be found at high redshift. Even if with some uncertainties, observational evidence for passive evolution of these cluster properties already exists. In fact, the characteristic luminosity of the cluster LF increases with redshift consistently with pure, passive luminosity evolution models (De Propris et al. 1999), and the same is found for the stellar mass-to-light ratio Υ_* of Es, as derived from the studies of their FP relations up to redshift $z = 1.27$ (e.g., van Dokkum & Franx 1996; Jørgensen et al. 1999; Kelson et al. 2000; Treu et al. 2002; van Dokkum & Stanford 2003). On the other hand, a more complicated evolution of $\Upsilon = \Upsilon_{\rm gal} M/M_{\rm gal}$ with z might also be expected, and thus the redshift variation of the FP zero point and slope could give important information on cluster and galaxy evolution. Although the fraction of Es in clusters appears not to change with redshift, a substantial increase in the relative number of spiral galaxies with respect to that of S0 galaxies at higher z is reported (Dressler et al. 1997; Fasano et al. 2000). In addition, $M/M_{\rm gal}$ might also increase with z, if clusters form by continuous accretion of galaxies from the field, in a DM potential well that has already settled on shorter timescales. Of course, a detailed prediction of the evolution of the cluster scaling relations in this latter case is much more difficult.

As a result of the interest in the questions addressed above, it is apparent that with more available data (e.g., from the SDSS [Sloan Digital Sky Survey], RCS [Red-Sequence Cluster Survey], MUNICS [Munich Near-Infrared Cluster Survey], 2dF [Two-Degree Field], and 2MASS surveys), a strong effort should be devoted to better determine the cluster scaling relations while, on theoretical ground, a larger set of simulated clusters, spanning a wider mass range and taking into account also the presence of gas, should be performed and analyzed for different choices of the cosmological parameters.

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