Astron. Astrophys. 363, 647–656 (2000)

ASTRONOMY
AND
ASTROPHYSICS

Revision of the initial-to-final mass relation

V. Weidemann

Universität Kiel, Institut für Theoretische Physik und Astrophysik, 24098 Kiel, Germany (weidemann@astrophysik.uni-kiel.de)

Received 6 June 2000 / Accepted 14 September 2000

Abstract. The initial-to final mass relation is revised in view of new theoretical and observational data. Recent stellar evolution models predict core masses at the beginning of the thermally pulsing asymptotic giant branch which practically coincide with final white dwarf masses derived for NGC 3532 and the binary white dwarf PG 0922+162. The Hyades white dwarfs must be relocated at higher initial masses since Hipparcos data lead to a reduced cluster age, and move also closer to the first TP relation. Comparison with theoretically predicted initial to final mass relations support the new evolutionary models with exponential diffusive overshoot which undergo core mass reduction by a strong third dredge-up. The upper mass limit for C/O white dwarf production is smaller than hitherto assumed, around 6.5 $M_{\odot}$ with white dwarf masses below 1 $M_{\odot}$. More massive white dwarfs must have Ne/O cores after off-center carbon burning or are due to mergers or rotational lifting. Differential mass loss seems to be small thus justifying the use of an universal $M_{\rm i}/M_{\rm f}$ relation for calculations of stellar and galactic evolution. It differs from the widely used 1987 relation and is steeper especially in the mass range from 3 to 4 $M_{\odot}$

Key words: stars: evolution – stars: AGB and post-AGB – stars: mass-loss – stars: white dwarfs

1. Introduction

Ever since it became clear that stars with initial masses far above the Chandrasekhar mass produce white dwarfs the question of the existence of a relation between initial main sequence and final white dwarfs mass has been discussed. The classical case of the Hyades white dwarfs was investigated by Weidemann (1977) in his paper on "Mass loss towards the white dwarf stage". Koester & Weidemann (1980) demonstrated how empirically established data on white dwarfs – especially the narrow mass distribution and the ratio of white dwarf to supernova birth rates – constrain models of galactic evolution. We concluded that the data favor relatively strong mass loss during the stellar lifetime and that the average upper limiting initial mass for white dwarf production is probably higher than 5 solar masses. Koester & Reimers (1981) embarked on a program of systematic search and observations of white dwarfs in young open clusters for which the progenitor masses had to be larger than the turn-off masses. A striking success was the finding of three massive white dwarfs in the cluster NGC 2516 (Reimers & Koester 1982). Considering the luminosity function of the cluster they concluded that the limiting mass for white dwarfs should be around 8 solar masses. A more detailed method to obtain progenitor masses for individual white dwarfs was also applied: determination of the cooling age from spectroscopically derived surface gravities and effective temperatures by comparison with theoretical white dwarf cooling tracks (Koester 1972), followed by subtraction of the cooling age from the cluster age to obtain the total pre-white dwarf evolutionary age which is then fitted to theoretical stellar evolution model calculations of total ages as a function of initial mass (and composition). The method and the results were then discussed and semi-empirical initial- final mass relations presented by Weidemann & Koester (1983). An upper mass limit around 8 solar masses for white dwarf production was confirmed, it agreed in those days with the theoretical upper mass limit for the development of a degenerate carbon-oxygen core, 8-9 solar masses (Iben & Renzini 1983). It thus was predicted that all intermediate mass stars undergo heavy mass loss in order to reach the relatively small final white dwarf masses – a fact that since then has been confirmed by observations on and around AGB stars which produce circumstellar shells and planetary nebulae.

Since classical stellar evolution calculations predicted at that time a strict core mass – luminosity relation, AGB tip luminosities and luminosity functions in clusters – discernible practically only in the Magellanic Cloud clusters with numerous stars – could also be taken to derive final masses – an approach which was included in a thorough discussion of the initial-final mass relation in the galactic disk and the Magellanic Clouds (Weidemann 1987). Especially the absence of brighter AGB stars in the famous young LMC cluster NGC 1866 was evaluated to predict a final mass of 0.8 solar masses for an initial mass around 4.5 solar masses – the exact values depending on the LMC distance modulus and the degree of overshooting included in the stellar evolution models. The strong dependence of the semi-empirically derived initial masses on overshooting was demonstrated also for the white dwarfs in galactic clusters

Send offprint requests to: V. Weidemann in that paper, especially it has been shown that evolution models with no overshoot predicted initial masses far above the limit for carbon ignition. The above mentioned agreement of that theoretical limit, $M_{\rm up}$, with the predicted limit from the extrapolated $M_{\rm i}/M_{\rm f}$ relation of Weidemann & Koester (1983) could only be upheld for models with medium or strong overshoot. Since medium overshoot at that time seemed to be favoured, I tabulated in the 1987 paper a best estimate of the $M_{\rm i}/M_{\rm f}$ relation and the mass fraction returned to the interstellar medium which in the following years has been widely used in galactic evolution calculations.

However in the meantime several changes occurred which necessitate a revision of the $M_{\rm i}/M_{\rm f}$ relation. These changes are based partly on the progress of white dwarf observations – better spectra for faint objects with CCDs and larger telescopes – and partly on more sophisticated stellar evolution models due to an enormous increase of computational possibilities during the last decade. The present investigation thus will be based on the latest papers on AGB evolution which incorporate hydrodynamically predicted exponentially declining overshoot at the base of convective instable layers and predict a breakdown of the classical core mass – luminosity relation. The next section will be devoted to these changes which are important for the prediction of theoretical $M_{\rm i}/M_{\rm f}$ relations. In the third section we will discuss changes in the semi-empirically derived $M_{\rm i}/M_{\rm f}$ relations based on mostly new observations of white dwarfs and cluster data. In Sect. 4 we discuss the implications of these changes by a comparison of the new theoretical and semi-empirical $M_{\rm i}/M_{\rm f}$ relations and point to some open questions especially concerning the end of the AGB evolution. We suggest a new $M_{\rm i}/M_{\rm f}$ relation. The following section 5 is devoted to the question of the upper mass limit for white dwarf production. In Sect. 6 we consider the importance of differential mass loss. Finally we summarize our results.

2. Theoretical evolution models and predicted $M_{\rm i}/M_{\rm f}$ relations

Numerous new sets of stellar evolution models have been calculated during the last years, to mention only the Padua group (Marigo et al. 1999, Girardi et al. 2000), the Frascati group (D'Antona & Mazzitelli 1996, Straniero et al. 1997, Ventura et al. 1998, Dominguez et al. 1999) and the Potsdam group (Blöcker 1995, Herwig et al. 1997, 1998, 1999a, Blöcker et al. 2000, Herwig 2000). They differ in their assumptions about convective overshoot as well as mass and composition range. Whereas the evolution through main sequence, first giant branch, central helium burning and early asymptotic giant branch (E-AGB) phase towards the beginning of the thermally pulsing asymptotic giant branch (TP-AGB) is calculated by standard procedures, the evolution on the TP-AGB itself is dealt with very differently. Since the exact calculation through thermal pulses is extremely time consuming, the evolution has been often followed only through a few or exemplary pulses and is then replaced by synthetic models (Groenewegen & de Jong 1993, 1994, see also Wagenhuber & Groenewegen 1998) towards the end of the AGB which is determined by the assumed mass loss law. The final white dwarf mass thus depends on how far the carbon/oxygen core can grow during the TP-AGB. Since the mass loss mechanism is still not understood from first physical principles, different laws have been assumed – e.g. the Reimers (1975) formula with constant or variable η parameter (Boothroyd & Sackmann 1988, Bryan et al. 1990, Forestini & Charbonnel 1997), the Bowen 1988 shocked wind law with modifications (Blöcker 1995), or the Mira pulsation period calibrated law (Vassiliadis & Wood 1993). Accordingly different final masses are predicted (see Weidemann 1993a for a discussion of total mass loss on the AGB).

Table 1. Core masses at the beginning of the TP-AGB

$M_{\rm c}$	0.53	0.56	0.50	0.62	0.80	0.86	0.92	0.99
$M_{\rm i}$	1.0	1.5	2.0	3.0	4.0	5.0	6.0	7.0

Herwig/Blöcker (priv. comm.). Cb: off-center carbon burning

In order to check the influence of the new model calculations it is thus advisable to compare first their predictions of the core mass at the beginning of the TP-AGB (the first TP, or the end of the E-AGB). Such a comparison has been made by Wagenhuber & Groenewegen 1998, Fig. 7, for three different compositions. The general trend is as follows: for low mass stars, from 1 to about 2.5 $M_\odot$ (the limiting helium flash mass) the first-TP core masses are nearly constant, around 0.52 $M_\odot$ then strongly increase for the intermediate mass stars up to 4 $M_\odot$, followed by a slower growth due to the second dredge-up (which reduces the core mass) until off-center carbon ignition occurs around 6 to 7 $M_\odot$ which terminates the TP phase at core masses around 1 $M_\odot$. For the more recent model calculations the results are plotted in Fig. 1. It can be seen that models with overshoot (as those of Girardi et al. 2000) predict larger core masses than those without overshoot (Dominguez et al. 1999). Girardi et al. 2000 assume intermediate core overshoot ($\Lambda = 0.5$) 0.5) and somewhat stronger envelope undershoot ($\Lambda = 0.7$) similar to the earlier Padua calculations of Bressan et al. 1993 and following.

The most recent 1.TP relation calculated with the exponential diffusive overshoot and undershoot prescriptions of Herwig et al. 1997, 1998, 1999b by Herwig and Blöcker 2000 (private communication) is given in Table 1 and plotted in Fig. 1.

For a 3 $M_\odot$ star with solar composition the core masses are 0.58, 0.60 and 0.62 $M_\odot$ for the cases of no, intermediate and exponential diffusive overshoot respectively. For larger initial masses the trend is inverse: at 5 $M_\odot$ the core masses are equal, 0.86 $M_\odot$, for 6 $M_\odot$ Dominguez et al. find 0.94, Herwig and Blöcker 0.92 $M_\odot$ whereas Girardi obtains off-center carbon burning already at 0.97 $M_\odot$. $M_{\rm up}$, the limiting mass for C/O white dwarf production is dependent on metallicity (see Umeda et al. 1999 for a discussion, especially their Fig. 7). Present day models which use OPAL opacities and overshooting lower $M_{\rm up}$ compared to the classical models (Becker & Iben 1979) from 9 $M_\odot$ to 8 $M_\odot$ or 7 $M_\odot$ (Umeda or Dominguez, without over-

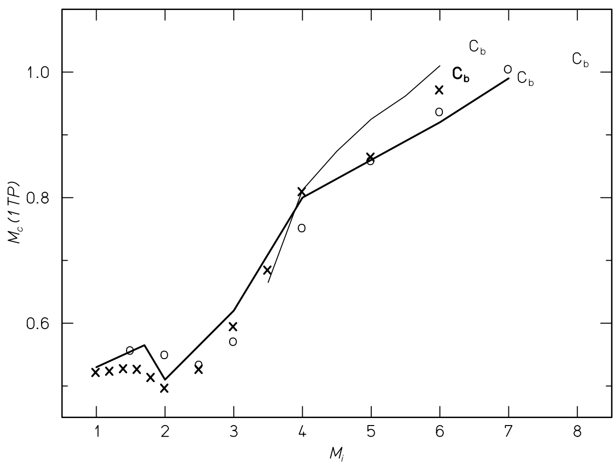

Fig. 1. Core masses at the beginning of the TP-AGB. Crosses: Girardi et al. 2000, circles: Dominguez et al. 1999, thick line: Herwig& Blöcker (priv. comm.), thin line: Mazzitelli et al. 1999. Cb indicates off-center carbon ignition.

shooting) to 6 $M_\odot$ (Girardi, with overshooting) for solar composition, $Z = 0.02$, and further down for lower metallicity (5 $M_\odot$ at $Z = 0.008$, Girardi). We shall come back to the question of $M_{\rm up}$ in Sect. 5. After the thermal pulsing phase is reached, further growth of the core towards the final white dwarf mass depends on the details of the AGB evolution.

A third dredge-up (3DUP) is necessary to explain the existence of carbon stars. It has been a longstanding "carbon star mystery" (Iben 1981) that classical calculations predicted 3DUP only for stars above 5 $M_\odot$ whereas LMC observations demonstrated their occurrence at much smaller initial masses. Since computational efforts to obtain 3DUP at smaller masses were unsuccessful several authors tried to reproduce the carbon star luminosity function by parametrization, i.e. an adjustment of a dredge-up parameter lambda which measures the ratio of dredged-up matter to burned matter between consecutive pulses and an assumption about the minumum core mass at which 3DUP begins (Groenewegen & de Jong 1993, Forestini & Charbonnel 1997) or the minimum temperature at the base of the convective envelope (Marigo et al. 1999). However the recent concept of exponentially declining diffusive overshoot from all convective layers, especially also below the helium flash convection zone during thermal pulses (Herwig et al. 1997, 1998, 1999a, Herwig 2000) causes 3DUP already at about the third pulse for a 3 $M_\odot$ initial mass star at a core mass of 0.64 $M_\odot$ (Herwig & Blöcker 1999). Since lambda becomes even larger than one, a core mass reduction occurs, although the luminosity increases further (see Herwig et al. 1998), even stronger for $M_{\rm i} = 4\,M_\odot$. Above 4 $M_\odot$ hot bottom burning operates and destroys carbon thereby explaining the observed absence of brighter C-stars. The very efficient 3DUP and the simultaneous treatment of mixing and burning also explains the existence of lithium-rich AGB stars (Blöcker et al. 2000). The dredge-up from the C/O core below the helium flash convection tongue (called 4thDUP by Iben 1999) leads to an enrichment of C and O in the intershell region which is in agreement with the unusual C and O abundances found for hydrogen-deficient post-AGB stars (PG 1159 stars: Dreizler & Heber 1998, Werner 2000). Herwig (2000) uses the fact that these abundances depend on the strength of the dregde-up from the C/O core to constrain his overshoot efficiency parameter f (which originally had been calibrated from the width of the observed main sequence to be $f = 0.016$ and was used with that same value at all evolutionary stages) to be between 0.01 and 0.03. Since these longtime unexplained phenomena can thus be understood with the concept of hydrodynamically caused overshooting and its description by an exponentially declining diffusion, I shall prefer the results of the Potsdam group in the further discussion.

The evolution after the first TP towards the final mass depends again on the mass loss law but now under different circumstances: Whereas for 3 and 4 $M_\odot$ stars core mass reduction inhibits the further core growth (see Fig. 1 in Herwig et al. 1998), the luminosity increases above the classical core mass-luminosity relation. One may thus expect the final mass to be nearly equal to the core mass at the beginning of the TP phase. In the case of 5 $M_\odot$ and above hot bottom burning occurs but 3DUP remains strongly efficient. In the 6 $M_\odot$ case considered especially by Mazzitelli et al. (1999) which also adopt the concept of exponential diffusive overshoot (however with the Canuto Full Spectrum Turbulence instead of mixing length description for the convective unstable layers) the 3DUP even removes the helium buffer zone completely such as to suppress thermal pulses and to produce C/O white dwarfs around 1 $M_\odot$. However Blöcker et al. (2000) found no quenching of TPs for their 6 $M_\odot$ model. The different behavior may come from the fact that the core mass at the first TP is smaller for Herwig & Blöcker 2000 (private communication), 0.92 $M_\odot$, than for Mazzitelli et al. 1999, 1.01 $M_\odot$. In summary: for intermediate mass stars we expect the theoretical $M_{\rm i}/M_{\rm f}$ relation to follow closely the first-TP relation for $M_{\rm i}$ larger than 3 $M_\odot$. However for smaller initial masses a prediction is not that easy. Herwig et al. (1999b) investigated the mass range from 1 to 3 $M_\odot$ as well for solar composition as for reduced metallicity $(Z = 0.02, 0.01, 0.001)$ in order to check the occurence of 3DUP and transformation to carbon stars. Whereas the 1 $M_\odot$ star left the AGB already after the second TP (no 3DUP), the 1.5 $M_\odot$ $(Z = 0.02)$ star showed continuous 3DUP after the 6.TP (at $M_{\rm c}$ = $0.584\,M_\odot$), the 1.7 $M_\odot$ star showed stronger DUPs beginning at the 5. pulse and became a carbon star after the 9.TP (at $M_{\rm c}$ = $0.603\,M_\odot$), the 2 $M_\odot (Z = 0.02)$ star needed 22 TPs to become a carbon star at $M_{\rm c}$ = $0.601\,M_\odot$, and the 3 $M_\odot$ $(Z = 0.02)$ star 7 TPs at $M_{\rm c}$ = $0.628\,M_\odot$. Since Herwig et al. (1999b) were not interested to reach the end of the AGB they assumed a rather small Reimers mass loss law with $\eta = 0.5$ for initial masses below the limiting helium flash mass between 1.7 and 2 $M_\odot$ and $\eta = 1.0$ above, thus allowing many TPs. In reality the mass loss law will be much stronger and thus determine the final masses. Forestini & Charbonnel 1997 e.g. assume for $M_{\rm i} = 3\,M_\odot$ η to increase from 2.5 to 5 in the final AGB stage. Their final mass of 0.60 $M_\odot$ is reached after 19 TPs from a core mass of 0.54 $M_\odot$ at the first TP – whereas Herwig et al. (1999b) start the TP-AGB at a core mass of 0.62 $M_\odot$ (much higher since they used overhooting models up to the AGB stage whereas Forestini and Charbonnel use models with no overshoot) and find the above mentioned core mass reduction very soon at $M_c = 0.63\,M_\odot$. Dominguez et al. 1999, with no-overshoot models, start the TP-AGB at 0.58 $M_\odot$ and reach the end after 26 TPs at $M_f = 0.71 M_\odot$ – calculated by Straniero et al. 1997 with a mass loss prescription according to Groenewegen & de Jong 1994 ($\eta = 5$), whereas Blöcker (1995) starting at 0.53 $M_\odot$ needed 20 TPs to reach a final mass of 0.63 $M_\odot$ with his fairly strong Bowen mass loss law. These examples demonstrate that it is not yet possible to determine theoretical final masses convincingly, but that they depend strongly on mass loss laws and overshooting prescriptions. For smaller initial masses one should expect rather numerous TPs to occur since below the transition region from degenerate to nondegenerate helium ignition (around 2 $M_\odot$) the first-TP core masses are small (between 0.51 and 0.56 $M_\odot$) and the empirically established final masses (Hyades masses around 0.7 $M_\odot$ at M_i around 2.7 $M_\odot$) are larger (see next section). Also white dwarfs in the maximum of the observed mass distribution with M_f around 0.60 $M_\odot$ must be produced by low mass stars in the initial mass interval between 1 and 2.5 $M_\odot$. The calculations of the Potsdam group (Herwig et al. 1999b) do not show core mass reduction due to 3DUP in this range of initial masses (depending on metallicity however: core mass reduction occurs even for a 2 $M_\odot$ initial mass star with $Z = 0.001$).

3. New data on the semiempirical initial-final mass relation

The semiempirical M_i/M_f relation has been determined predominantely by the open cluster method described in the introduction. The three most important clusters are the Hyades, NGC 3532 and NGC 2516 which shall be discussed in more detail first.

3.1. The Hyades

The Hyades white dwarfs have been studied extensively in the past (Weidemann 1977, Weidemann et al. 1992, Weidemann 1993a, 1993b, 1993c, 1997). Reid (1996) obtained new high resolution spectra and was able to determine improved redshifts by observation of the sharp NLTE-cores of Balmer lines. He used the data together with physical parameters for the stars obtained by the Bergeron group (Bergeron et al. 1995) to derive masses for six Hyades white dwarfs. Initial masses were then calculated from cooling and cluster ages by comparison with evolutionary models of the Geneva group (Schaller et al. 1992, Schaerer et al. 1993). The necessary individual distances were determined with the moving cluster method as outlined by Weidemann et al. (1992) however with slightly different proper motions obtained by Reid (1992). The resulting initial masses were very similar (Table 2). An independent study by Herwig (1995) using effective temperatures and surface gravities by Bergeron et al. (1995) and Wood (1995) evolutionary tracks (with thick hydrogen surface layers) to determine masses and cooling ages for five Hyades white dwarfs reached practically the same results for the final and initial masses (see Table 2).

Table 2. M_f and M_i for Hyades White Dwarfs

Des.	M_f				M_i			
Author	1	2	3	4	1	2	3	5
HZ4	.62	.74	.71	.71	3.0	2.90	3.00	3.51
LB227	–	.80	–	.79	–	2.85	–	3.50
VR7	.62	.70	.68	.675	2.6	2.52	2.67	2.95
VR16	.65	.69	.70	.70	2.5	2.42	2.58	2.85
HZ7	.63	.67	.65	.645	2.6	2.45	2.62	2.90
HZ14	.62	.66	.68	.68	2.5	2.39	2.55	2.80

Explanations. Des.: McCook Sion Catalogue (1987) Authors: 1) Weidemann, Jordan, Iben, Casertano (1992) 2) Reid (1996) M_f based on redshift 3) Herwig (1995) M_f based on spectroscopic gravity and effective temperature, with Wood thick layer models 4) Weidemann (1997) average of $M(g)$,$M(R)$ and $M(red)$ 5) this paper, M_i with lower Hyades age (see text)

In an extensive comparative investigation (1997) I demonstrated how the average final white dwarf masses had been increased from 0.59 $M_\odot$ (Koester et al. 1979) to 0.62 $M_\odot$ by using thin-layer cooling models (Wood 1992) instead of zero-temperature configurations for the mass-radius relation (Bergeron et al. 1992, Weidemann et al. 1992) and then to 0.66 $M_\odot$ by an increase of the spectrosopically determined surface gravities (Bergeron et al. 1995) and finally – by using thick-layer cooling models instead – to 0.68 $M_\odot$ (Herwig 1995). Practically the same value was reached by Reid (1996) with his redshift data, reduced also with thick-layer cooling models. Both Herwig and Reid pointed out that the higher value for M_f would imply a change of the initial-final mass relation compared to Weidemann 1987 in suggesting a steeper increase in the initial mass interval from 1 to 2.5 $M_\odot$ the average initial mass for the Hyades white dwarfs. However the evaluation of the HIPPARCOS observations for the Hyades by Perryman et al. (1998) presented new and more reliable cluster parameters which I have checked for changes in our derived data. First the average cluster distance is now determined to be $(m - M) = 3.33 \pm 0.01$, (we had used 3.35 in 1992). Second the individual distances derived by the moving cluster method are changed by a revised cluster velocity V to $45.93 \pm .23$ km/s (we and Reid used 46.6) and by a change of the converging point CP from $\alpha = 97.65°$ and $\delta = +5.98°$ to $97.91°$ and $6.66°$. The formula for the distance determination gives $d \propto V sin\lambda/\mu$, where λ measures the angular distance of the star from the CP and μ the proper motion. Since the recalculated angular distances changes are minor the distances change according to $\Delta log d = \Delta log V = --0.0063$, i.e. the stars are somewhat closer and the radii (for a given surface flux) correspondingly smaller. A straightforward parallax determination for the Hyades white dwarfs could not been made by HIPPARCOS since the Hyades white dwarfs are too faint, around $V = 14$, for direct observation. Anyway the use of individual proper motion determined distances for the Hyades white dwarfs results in a much smaller scatter around the theoretical mass-radius relation than the use of an average cluster distance (cf. Weidemann 1997 vs. Schmidt 1996). Although Perryman et al. 1998 conclude that the concept of parallel motion is justified for the derivation of the cluster parameters, small deviations are present which of course also influence the derived individual white dwarf distances. The somewhat smaller radii cause a change of about $+0.01\,M_\odot$ for the white dwarfs masses derived from the mass-radius relation, $M(R)$, compared to the tabulated data in Weidemann 1997. For the determination of the gravitational redshift one needs to correct the observed redshift for the radial velocity which also is derived by the moving cluster method, $V cos\lambda$. Again we predict a change of the order of $\Delta logv(red) = 0.007$. The velocity increase is minor and the derived masses $M(red)$ increase by $0.007\,M_\odot$ in the average, compared to Reid 1996 well within the listed uncertainties.

The largest change however occurs for the derived initial masses: it is caused by the new determination of a cluster age by Perryman et al. 1998. Whereas Weidemann et al. (1992) and Herwig (1995) used a cluster age of 700 Myr, and Reid (1996) 800-900 Myr, Perryman et al. now derive 625 ± 50 Myr by comparison of a much clearer defined main sequence and isochrones from newly calculated stellar models with Hyades abundances, $Y = 0.26, Z = 0.024$ (corresponding to $[\mathrm{Fe/H}] = +0.14$). The smaller cluster age for a given cooling age also reduces the progenitor ages and thereby increase the derived initial masses, in the average by 0.28 $M_\odot$ (Table 2). The progenitor ages were reduced to initial masses with evolutionary ages up to the first TP by models of Girardi et al. (2000). The new data are plotted in the $M_\mathrm{i}/M_\mathrm{f}$ plane (Fig. 2). The steepness of the $M_\mathrm{i}/M_\mathrm{f}$ relation above 1 $M_\odot$ is thus again somewhat reduced and more close to the former 1987 relation (see Herwig 1995). Moreover it is striking that the new points are located very closely above the 1.TP relations of Fig. 1 at $M_\mathrm{i} \approx 2.9\,M_\odot$, and for HZ4, (EG 26) even at the 1.TP relation for $M_\mathrm{i} \approx 3.5\,M_\odot$. It is interesting to note that this interval of progenitor masses, 3 to 4 $M_\odot$ is just the one in which Herwig et al. 1998 first showed the core mass reduction by the third dredge-up to occur, thus our new results for the Hyades white dwarfs give empirical support for that concept (see also the presentation in Herwig 2000).

3.2. NGC 3532

White dwarfs in NGC 3532 have been first detected and spectroscopically observed by Reimers & Koester 1989 and subsequently by Koester & Reimers 1993. At a turn-off mass of about 4 $M_\odot$ and initial masses between 3.5 and 5 $M_\odot$ the final masses showed a large scatter between 0.65 and 1 $M_\odot$, possibly an indication of differential mass loss (see Fig. 1 of Herwig 1995). Since the higher mass objects, 3532-1, -5 and -6 had been observed with lower quality in 1988, they have been reobserved by Koester and are now found at normal masses of 0.72, 0.75 and 0.77 $M_\odot$ (see Fig. 4 in Finley & Koester 1997). Herwig 1995 has redetermined the initial masses for the other three white dwarfs found by Koester & Reimers (1993), 3532-8, -9 and -10, using Wood 1995 tracks with thick hydrogen layers for the cooling ages, and Schaller models for the reduction of progenitor ages. Plotted in Fig. 2 these results show initial masses between 3.5 and 4 $M_\odot$ fully coincident with the Herwig & Blöcker (priv. comm.) core mass relation for five of the six stars and a higher M_i of 5 $M_\odot$ for 3532-10, with $M_\mathrm{f} = 0.9\,M_\odot$ again close to the Herwig/Blöcker 1.TP relation. However in this case the Potsdam models predict hot bottom burning with only slight core mass reduction during the AGB (see Blöcker et al. 2000). The results for NGC 3532 thus strengthen the conclusion already obtained for the Hyades that the $M_\mathrm{i}/M_\mathrm{f}$ relation follows closely the first-TP relation for stellar models with exponential declining diffusive overshoot.

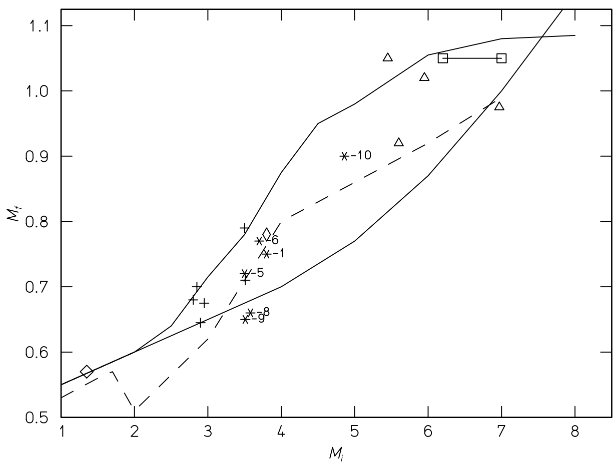

Fig. 2. Semi-empirical data for final masses, and $M_\mathrm{i}/M_\mathrm{f}$ relations $+$: Hyades, $*$: NGC3532, rhombus: PG 0922+162A, triangles: NGC2516, squares: Pleiad, diamond: M67. Long dashed line: 1.TP relation of Herwig & Blöcker (priv. comm), full lines: semi-empirical $M_\mathrm{i}/M_\mathrm{f}$ relations by Weidemann 1987 (below) and Herwig 1995 (above)

3.3. NGC 2516

Since this cluster has been most important for the test of the $M_\mathrm{i}/M_\mathrm{f}$ relation at the high mass end, the white dwarfs have also been reobserved by Koester & Reimers 1996. The resulting final masses, 0.91, 1.05, 0.975 and 1.02 $M_\odot$ for 2516-1, -2, -3 and -5 respectively and evaluation with Schaller et al. 1992 models and a cluster age of 140 Myr leads to initial masses between 5.5 and 7 $M_\odot$. Koester & Reimers 1996 discussed the uncertainties and pointed out that aside from observational errors the main sources were uncertain distance and cluster age. Herwig 1997 has thus redetermined the cluster age by observation of main sequence B- and A stars and use of theoretical isochrones by Meynet et al. 1993. He finds an indication of a slightly lower age of 120 Myr which would imply larger initial masses between 6.5 and 9.5 $M_\odot$ (the latter value for 2516-5). On the other hand Jeffries (1997) has recently claimed that the initial masses are about 1 to 1.7 $M_\odot$ smaller (from 4.5 to 5.3 $M_\odot$) since NGC 2516 appeared to be metal-poor by factor of two. He recalculated the progenitor ages using the same cooling ages as Koester & Reimers 1996 or Herwig 1997 but an increased cluster age of 185 Myr compared to 141 Myr for a given turn-off color (B-V) and Geneva group models with $Z = 0.008$ instead of 0.02. He demonstrates that the new locations in the $M_{\rm i}/M_{\rm f}$ plane are close to the theoretical first-TP relations for $Z = 0.02$ and $Z = 0.008$ for evolutionary models of the Padua group which predict an upper limit for white dwarf production at the onset of carbon ignition around 6 $M_\odot$ and thus smaller than the canonical 8 $M_\odot$ suggested and confirmed by Koester & Reimers 1996. Indeed Koester and Reimers as well as Herwig had used normal metallicity evolution models in their reduction, and thus the run of the $M_{\rm i}/M_{\rm f}$ relation at the upper mass end was open to question. However in a more recent paper Jeffries, James and Thurston (1998) redetermined the metallicity of NGC 2516 by observations of late type star color-magnitude diagrams and conclude that the best values obtained are between [Fe/H] +0.05 and -0.18, thus favoring again a normal metallicity and therefore the Koester/Reimers results. But even if we assume the metal deficiency to be real one must state two remarks: first, the new extensive models by Girardi et al. 2000 produce 1.TP relations which run to higher initial masses than those applied by Jeffries 1997 (his Fig. 1). If the data for NGC 2516 are reduced with $Z = 0.008$ Girardi models the stars move to an $M_{\rm i}$ range from 5.2 to 6.2 $M_\odot$ instead of Jeffries 4.5 to 5.3 $M_\odot$. Second, if we however want an $M_{\rm i}/M_{\rm f}$ relation valid for $Z = 0.02$, we had to correct the data points for NGC 2516 by the difference between $Z = 0.008$ and $Z = 0.02$ models which amounts to an increase of about 0.05 $M_\odot$ i.e. we would expect a range of 5.7 to 6.7 $M_\odot$ for the typical white dwarf masses, $M_{\rm f} \approx 0.9$ to 1.05 $M_\odot$ of NGC 2516, which is exactly the range which Koester & Reimers 1996 obtained with their assumption of normal metal abundance. One thus cannot uphold Jeffries claim that the $M_{\rm i}/M_{\rm f}$ relation runs to 5-6 $M_\odot$ instead of 8 $M_\odot$. However a more serious problem arises from the fact that indeed all recent calculations of evolutionary models predict a smaller value than 8 $M_\odot$ for carbon ignition. It occurs already at 6 $M_\odot$ for the Girardi 2000 population I models and between 6 and 7 Mo for Frascati and Potsdam models (Mazzitelli et al. 1999, Herwig & Blöcker 2000, private communication) with exponential overshoot (see Fig. 1.) The core masses at the beginning of carbon burning are also smaller (0.96, 1.0 and 1.04 $M_\odot$ for Girardi et al. 2000, Forestini & Charbonnel 1997 and Mazzitelli et al. 1999 respectively). If correct this would imply that the white dwarfs in NGC 2516 are just at the limit of carbon ignition. A linear extrapolation of the semi-empirical $M_{\rm i}/M_{\rm f}$ relation from $M_{\rm f} \approx 1\,M_\odot$ for $M_{\rm i} = 6\,M_\odot$ for NGC 2516 to higher masses, e.g. to $M_{\rm f} = 1.2\,M_\odot$ for $M_{\rm i} = 8\,M_\odot$ (Koester & Reimers 1996) would then not be permitted for carbon-oxygen white dwarfs. The question of the upper limit for C/O white dwarfs will be further discussed in Sect. 5.

3.4. M67

A white dwarf cooling sequence has been detected with deep imaging down to V = 25 in this cluster (Richer et al. 1998) however only one DA star, G152, seems as yet to be spectroscopically confirmed (Fleming et al. 1997). The DA star is hot, 68000 K and of normal mass (0.57 $M_\odot$), according to Fleming et al. With a turn-off age of 4 Gyr the turn-off mass is around 1.25 $M_\odot$, and since the cooling time of the hot white dwarf is negligible compared to the total age the initial mass should be around 1.35 $M_\odot$ (from Girardi et al. 2000 models). If confirmed – there remained some doubt about the membership of G152 (Fleming et al. 1997) – this would provide a point for the low mass range of the $M_{\rm i}/M_{\rm f}$ relation in agreement with theoretical expectations. Since many more fainter white dwarfs with larger cooling ages are expected and may be made out in the color-magnitude diagram of Richer et al. 1998 – who fit their cooling sequence with white dwarfs of 0.7 $M_\odot$ – additional spectroscopic efforts should be made to obtain more reliable data.

3.5. Pleiades

The single white dwarf, LB1497, detected in the Pleiades, has been of special importance since it gave a strong argument for the extension of the $M_{\rm i}/M_{\rm f}$ relation towards higher initial masses than 5 $M_\odot$ (Weidemann 1977). With an initial mass around 6 $M_\odot$ and a spectroscopically determined mass of 0.9 $M_\odot$ (Schulz & Wegner 1981) it provided empirical evidence for an increasing $M_{\rm i}/M_{\rm f}$ relation before the detection of the white dwarfs in NGC 2516. Since Greenstein & Trimble 1972 had derived a redshift mass of only 0.52 $M_\odot$, efforts were made at a redetermination. The last one, made by Wegner et al. 1991, indeed lead to increase the redshift mass to 1.02 $M_\odot$ (using a Hamada Salpeter zero temperature mass-radius relation) or to even 1.05 $M_\odot$ (using Wood thick hydrogen surface layer mass-radius relations for the high temperature of 32000 K), whereas spectroscopic redeterminations (Bergeron et al. 1995) resulted in $M = 1.08\,M_\odot$. Herwig 1995 determined $M_{\rm i}$ to be between 5.1 and 5.7 $M_\odot$ and thus raised the $M_{\rm i}/M_{\rm f}$ relation to differ strongly from the generally accepted Weidemann 1987 relation which predicted $M_{\rm f}$ values between 0.8 and 0.9 $M_\odot$ only for this range of $M_{\rm i}$. However a change appears necessary in view of recent results as discussed by Ventura et al. 1998. Whereas the age of the cluster taken by Herwig had been 150 Myr (from Gianuzzi 1995) the formerly adopted age of 100 Myr (Meynet et al. 1993) had to be increased by the lower Hipparcos distance, $m - M = 5.33$ instead of 5.6, to about 120 Myr – a value which was also found from the transition to lithium-rich brown dwarfs in the cluster (Basri et al. 1996). Reid 1996 in his reduction had used also 100 Myrs. With the same cooling age, 52 Myr, and a cluster age of 100 -120 Myr we now obtain a range of 6.2 to 7 $M_\odot$ for the initial mass of LB1497 – practically at the same location as Reid 1996.

3.6. PG 0922+162

Aside from the open cluster method Finley & Koester (1997) have evaluated observations of a pair of young double degenerates PG 0922+162. High quality spectra allowed an accurate determination of surface gravities and effective temperatures. The corresponding masses are 0.79 ± 0.03 and $1.10 \pm 0.03\,M_\odot$ for PG 0922+162A and B respectively. Finley & Koester derive cooling times of 90 and 260 Myr and demonstrate how the difference translates in differences for the initial masses. Assuming a M_i-range for the high mass white dwarf between 5.5 and 7.5 $M_\odot$ (see discussion for NGC 2516) the initial mass range for the 0.79 $M_\odot$ white dwarf becomes narrow and must be between 3.6 and 4 $M_\odot$ thereby providing a comparatively well defined point for the $M_\mathrm{i}/M_\mathrm{f}$ relation which perfectly fits between the data for NGC 3532 and again is located exactly at the 1.TP core mass relation. The M_f value should be reduced from 0.79 to 0.78 $M_\odot$ since the Wood 1995 cooling tracks do not incorporate the fact that the hydrogen surface layer thickness is smaller for higher masses (Blöcker et al. 1997).

We plot the empirical results of this section in Fig. 2, together with the former $M_\mathrm{i}/M_\mathrm{f}$ relations by Weidemann 1987 and Herwig 1995 and notice that both have to been revised: whereas the 1987 relation runs too flat, not showing the theoretically predicted and now empirically confirmed upturn betwenn 3 and 4 $M_\odot$ the 1995 relation has to be changed due to the shift of the Hyades, and probably also the Pleiad, towards higher initial masses.

4. Comparison of $M_\mathrm{i}/M_\mathrm{f}$ relations. End of the AGB

Dominguez et al. 1999 as well as Girardi et al. 2000 present $M_\mathrm{i}/M_\mathrm{f}$ relations which were obtained by synthetic AGB calculations following the method of Groenewegen & de Jong 1994 with different mass loss prescriptions. As shown in Dominguez et al. (Fig. 16) up 30 to 60 TPs were necessary to reach the final positions plotted in Fig. 3. The final masses listed in the Padua tables were obtained by a similar simple prescription (mass loss according to Vassiliadis & Wood 1993, assumption of the classical core mass luminosity relation), they are presented in Girardi et al. 2000 (Fig. 4) and also plotted in our Fig. 3. Third dredge-up and hot bottom burning for the more massive stars were not included, thus core mass reductions like the ones found by the Potsdam group or deviations from the classical Mc-L relation (Blöcker & Schönberner 1991, Blöcker 1995, Herwig et al. 1998) as investigated by Marigo 1998 and Marigo et al. 1999 change the AGB evolution. Exponential diffusive overshoot tends to increase third dregde-up and core reduction and thus keeps the final mass closer to the core mass at 1.TP. Still the mass loss prescription determines the end of the AGB, as clearly demonstrated by Blöcker 1995 who used a strong mass loss law and reached the end of the AGB after only a few TPs with a resulting $M_\mathrm{i}/M_\mathrm{f}$ relation (his Fig. 4) practically following the first-TP relation from 5 to 7 $M_\odot$. New calculations with a realistic third-dredge up and hot bottom burning and different mass loss laws should give similar results: final core masses not much larger than 1.TP core masses – in the whole initial mass range from 3 $M_\odot$ to the upper limit M_up for off-center carbon ignition (Cb). At present there remains some discrepancy even if exponential diffusive overshooting is included: whereas Mazzitelli et al. 1999 find that for a 6 $M_\odot$ star dredge-up becomes so strong that the whole helium buffer layer is mixed-up and TPs are quenched, the Potsdam group cannot reproduce this result (Blöcker et al. 2000). Evidently the question what

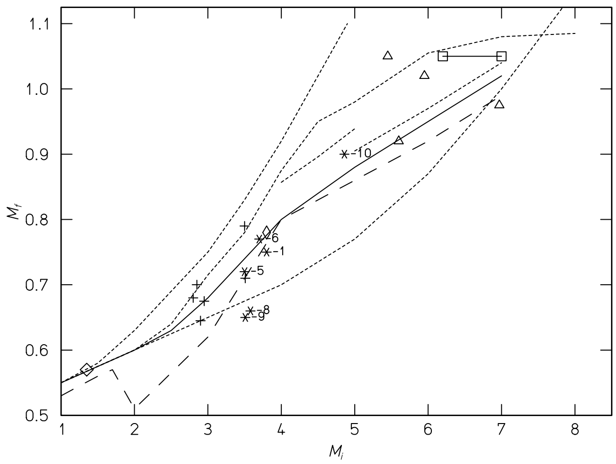

Fig. 3. Initial-to-final mass relations. – (from top to bottom): relations from Girardi et al. 2000, Herwig 1995, Marigo 1998, Dominguez et al. 1999, Weidemann 1987. Long dashed: 1.TP relation from Herwig & Blöcker (priv. comm). Full line: new $M_\mathrm{i}/M_\mathrm{f}$ relation as tabulated in Table 3

Table 3. Revised Initial-to-Final Mass Relation

M_i	1	2	2.5	3	4	5	6	7
M_f	0.55	0.60	0.63	0.68	0.80	0.88	0.95	1.02

terminates the AGB evolution for stars with higher masses, i.e. the interaction of core mass growth and dredge-up and carbon ignition needs further study. However as outlined above the coincidence of semi-empirical and theoretical data especially in the range from 3 to 4 $M_\odot$ strongly suggests a new $M_\mathrm{i}/M_\mathrm{f}$ relation intermediate between Weidemann 1987 and Herwig 1995 as tabulated in Table 3 and plotted in Fig. 3. The new relation is anchored at $M_\mathrm{i} = 4\,M_\odot$, $M_\mathrm{f} = 0.8\,M_\odot$ in view of the agreement of the data points for PG0922+162A, NGC3532 and the theoretical predictions of the Potsdam group. For M_i larger than 4 $M_\odot$ it is assumed to be in between the 1.TP relation and the Dominguez et al. 1999 M_f-relation which was reached only after many thermal pulses (calculated with the synthetic method) whereas a stronger mass loss law produces a relation closer to the 1.TP relation (see Blöcker 1995).

5. Upper mass limit for white dwarf production

Of more general interest then the details of the $M_\mathrm{i}/M_\mathrm{f}$ relation may be the upper mass limit for the production of white dwarfs above which supernova explosions or collapse to neutron stars does occur. This limit, M_up, has been taken to be located at $M_\mathrm{i} = 8\,M_\odot$ up to now, in agreement with an extrapolation of the semi-empirical $M_\mathrm{i}/M_\mathrm{f}$ relation to M_f at the Chandrasekhar limit, 1.4 $M_\odot$ (Weidemann & Koester 1983). However classical model calculations claimed that the upper mass limit for single C/O white dwarfs is about 1.1 $M_\odot$, since for higher masses carbon ignition will occur leading to Ne/O degenerates which either explode or else go to collapse by electron capture.

Recently however Garcia-Berro, Ritossa and Iben (1997) have demonstrated that a 9 $M_\odot$ star (calculated with solar composition, without mass loss, and the old Iben code) does indeed go to off-center carbon ignition in the E-AGB stage, burning carbon in the core by several flashes to Ne/O, but afterwards starts thermal pulsing in a super-AGB which produces new carbon/oxygen and leads to a final white dwarf of 1.15 $M_\odot$ with an 1.07 $M_\odot$ Ne/O core and an outer 0.08 $M_\odot$ C/O shell which due to cooling by neutrinos will not explode or collapse. We thus should be more specific about $M_{\rm up}$: either define it as is usually done as the lower mass limit for off-center partly degenerate carbon ignition, $M_{\rm up}$(Cb), or take it as the upper limit for the production of single white dwarfs even after carbon burning in the way demonstrated by Garcia-Berro et al. 1997. These authors also investigated a 10 $M_\odot$ star (Ritossa et al. 1996) and obtained a final single white dwarf of 1.26 $M_\odot$ with a 1.19 $M_\odot$ NeO core and a 0.07 $M_\odot$ C/O outer shell. Spectroscopically would such a post super-AGB star not necessarily appear different from normal C/O white dwarfs with masses below the classical limit of 1.1 $M_\odot$. We shall call this final mass limit for single white dwarfs $M_{\rm up}$(Wd). Further investigations by (Ritossa et al. 1999) showed that a 11 $M_\odot$ star would undergo central carbon ignition and after the super-AGB phase produce a degenerate Ne/O core mass of 1.368 $M_\odot$ with a C/O layer of 0.014 $M_\odot$ which however – being that close to Chandrasekhar limit – collapses due to electron capture and produces a weak type II supernova. We thus with the present knowledge would have to put $M_{\rm up}$(Wd) between 10 and 11 Mo with an upper limit for the white dwarf mass around 1.3 $M_\odot$.

More information is available about $M_{\rm up}$(Cb). We check the recent data given in the literature: Umeda et al. 1999 find 8 $M_\odot$ for $Z = 0.02$, 7.5 $M_\odot$ for $Z = 0.008$; Dominguez et al. 1999 7.5 $M_\odot$ for $Z = 0.02$, 7 $M_\odot$ for $Z = 0.008$; Mazzitelli et al. 1999 6.5 $M_\odot$ for $Z = 0.02$; Girardi et al. 2000 5-6 $M_\odot$ for $Z = 0.019$, 4.5 – 5.0 $M_\odot$ for $Z = 0.008$ (the latter range was based on the run of the 1.TP relation as listed in Table 2 of Girardi & Bertelli 1998. But if plotted according to the electronic tables of Girardi et al. 2000 the range is shifted to also above 5 $M_\odot$). It is remarkable that Girardi et al. 2000 obtain off-center carbon burning already at $M_{\rm i} = 6\,M_\odot$ for a core mass of 0.97 $M_\odot$, far below the classical limit of 1.1 $M_\odot$. For Mazzitelli et al. 1999 the core mass at 1.TP for $M_{\rm i} = 6\,M_\odot$ is 1.01 $M_\odot$ and no carbon burning occurs. The same is true for Dominguez et al. 1999 at $M_{\rm i} = 7\,M_\odot$. Herwig and Blöcker (priv. comm.) find off-center carbon burning at $M_{\rm i} = 7\,M_\odot$, $M_{\rm c}$ (1.TP) at 0.99 $M_\odot$. To summarize: recent model calculations cover a large predicted range of $M_{\rm up}$ (Cb) (see Fig. 1) for solar composition from 8 $M_\odot$ down to 5 $M_\odot$, but if the classical upper mass limit for the white dwarf masses, 1.1 $M_\odot$ is reached depends on the duration of AGB evolution after the 1.TP. Here we try to explain the differences between the predicted $M_{\rm up}$ of Girardi et al. 2000 and Dominguez et al. 1999. On comparison of the evolutionary tracks in the central temperature/central density plane (Fig. 5 in Dominguez et al.) with the data given in the Padua tables one notices that a Girardi 7 $M_\odot$ star evolves like a Dominguez 9 $M_\odot$ star towards central carbon ignition, whereas Girardi 6 $M_\odot$ like Dominguez 8 $M_\odot$ and Garcia-Berro et al. 1997 9 Mo stars evolve towards off-center carbon ignition. The lowering of $M_{\rm up}$(Cb) can partly be understood by the fact that Garcia-Berro et al. used an old Iben code, constant mass and no overshoot whereas Dominguez et al. used modern input data (e.g.OPAL opacities) which increase luminosities and core masses for the same initial mass.

The calculations of the Padua group on the other hand include relatively strong overshoot which again leads to increases in the core masses and thus steeper $M_{\rm c}/M_{\rm i}$ relations and a correspondingly lowering of $M_{\rm up}$. In view of the fact that the calculations of the Potsdam group with the most modern prescription of exponential diffusive overshoot (cf.Sect. 2) do predict $M_{\rm up}$(Cb) to be between $M_{\rm i} = 6$ and 7 $M_\odot$ (see Fig. 1 and Table 1) we are inclined to prefer this higher value compared to the lower $M_{\rm up}$(Cb) derived by the Padua group. If the Padua limit, at $M_{\rm i}$ below 6 $M_\odot$, $M_{\rm c}$ below 0.97 $M_\odot$ were realistic we would have to claim that the more massive white dwarfs observed in NGC 2516 and the Pleiades were indeed not C/O but Ne/O white dwarfs. The question if field white dwarfs with masses larger than 1 $M_\odot$ (see e.g. Napiwotzki et al. 1999, Vennes 1999) with C/O cores are possible depends on the AGB core mass growth for initial masses smaller than $M_{\rm up}$(Cb), i.e. essentially on the mass loss law. It is however improbable that core masses will be growing that far on the AGB since our investigation has shown that at least for $M_{\rm i} = 3 - 4\,M_\odot$ the final mass is close to the first-TP core mass. Even by extension of the AGB evolution with synthetic calculations including hot-bottom burning Marigo et al. 1998 obtain for $M_{\rm i} = 4\,M_\odot$ $(Z = 0.008)$ a final mass of only 0.94 $M_\odot$ after 174 TPs and for $M_{\rm i} = 5\,M_\odot$ $(Z = 0.02)$ the same value after 83 thermal pulses. More massive field white dwarfs as recently included in the studies of Napiwotzki et al. (1999) and Vennes (1999) should thus be produced by more massive progenitors and have Ne/O cores or are the result of mergers as considered by Segretain et al. (1997).

6. Differential mass loss

There is no reason to assume the existence of a single-valued initial-to- final mass relation in view of the fact that there are additional parameters besides of initial mass and composition which determine the final outcome, e.g. rotation, magnetic fields and binarity. The lifting effect of rotation as been shown to be able to increase the core masses of massive white dwarfs in the AGB phase (Dominguez et al. 1996) e.g. to $M_{\rm c}$(1.TP) between 1.1 and 1.4 $M_\odot$ for $M_{\rm i} = 6.5\,M_\odot$ calculated in an approch similar to Sackmann & Weidemann (1972) who demonstrated that carbon detonation could be avoided if the angular momentum remaining in the core was large enough to slow down the increase of central density to critical values for deeply degenerate carbon ignition. Magnetic fields are of importance as evident by the fact that magnetic white dwarfs are more massive than nonmagnetic (Ferrario et al. 1998). Binarity of course is of uppermost importance if interactions produce and determine mass exchange, leading e.g. to cataclysmic variables or even to mergers. Strong evidence for differential mass loss in low mass stars is given by the fact that the horizontal branch can be understood only if mass loss on the first giant branch differs for stars which come from the same turn-off (Iben & Rood 1970). If differential mass loss occurs on the first giant branch there is no reason not to expect the same to occur on the AGB (Weidemann 1977). The fact that white dwarf masses in clusters scatter considerably in the same region of initial masses has been taken by Reid (1996) to conclude that mass loss depends more on individual stellar properties than on a global mechanism and that mass loss (predominantely on the AGB) is essentially a stochastic phenomenon. He is lead to this conclusion especially by his result that the redshift masses for three Praesepe white dwarfs are 0.42, 0.66 and 0.91 $M_\odot$ for the same initial mass, $\approx 2.45\,M_\odot$. A similar conclusion had been possible from the investigation of Herwig 1995 whose Fig. 1 showed final masses for NGC 3532 between 0.65 and 1.05 $M_\odot$ for initial masses around 4 $M_\odot$ and between 0.67 for NGC 2168 and 1.03 $M_\odot$ for the Pleiad at $M_{\rm i} \approx 5.5\,M_\odot$. However the data for NGC 3532 have been revised according to more recent observations by Koester and Reimers – as discussed above – and now coincide with the theoretical expectation for a single valued $M_{\rm i}/M_{\rm f}$ relation (Fig. 3). This suggests that also the white dwarfs in NGC 2168 should be reobserved, as well as the rather faint Praesepe members ($V \approx 17.5$) for which effective temperatures have only been estimated from (B-V) color and spectroscopic gravity determinations are lacking. The only case in which a cluster white dwarf gives a reliably different mass from the other members is the Hyades white dwarf 0406+1659, (LB 227, EG 29): with $M_{\rm f} \approx 0.8\,M_\odot$, it could be a merger product as suggested by Weidemann 1997. The evidence for differential mass loss thus is rather weak and future observations should be awaited to settle this question. Also for field white dwarfs differential mass loss cannot be stronger than about $\pm\,0.1\,M_\odot$ since otherwise the existence of the narrow white dwarf mass distribution around 0.6 $M_\odot$ could not be reproduced by synthetic galactic evolution models (Yuan 1992).

7. Conclusions

The present situation justifies empirically as well as theoretically the concept of a universal single-valued initial-to-final mass relation and therefore its application to calculations of stellar and galactic evolution. However this relation has to be revised compared to the smoothly increasing $M_{\rm i}/M_{\rm f}$ relation of Weidemann 1987 or to the strongly increasing relation of Herwig 1995. New empirical data especially for the open cluster NGC 3532 and the white dwarf pair PG 0922+162 give a reliable fixpoint at $M_{\rm i} \approx 4, M_{\rm f} \approx 0.8\,M_\odot$, whereas the initial masses for the Hyades white dwarfs have to be raised to $\approx 3\,M_\odot$ (at $M_{\rm f} \approx 0.68\,M_\odot$) due to a smaller Hyades age derived from HIPPARCOS data by Perryman et al. 1998. These empirically revised data coincide almost exactly with new theoretical predictions of the core masses at the beginning of the thermal pulsing AGB. One has thus to conclude that the following TPAGB phase does not lead to further increase of the core masses. This is predicted only for stellar models which include diffusive exponentially declining overshoot which produce a strong third dredge-up (leading to carbon stars) and even core mass reduction. Although not yet calculated to the end of the AGB with different mass loss laws one can already expect that the final masses will be reached very closely to the first-TP relation. It is presumed and supported weakly by the empirical data that this closeness of the final mass to the first-TP core mass relation continues also to higher initial masses which go through hot bottom burning until an upper mass limit for AGB evolution and C/O white dwarfs is reached by off-center carbon ignition around $M_{\rm i} \approx 6--7.5\,M_\odot$ and $M_{\rm f} \approx 1\,M_\odot$. However Ne/O white dwarfs with outer layers of C/O could be produced beyond this limit up to $M_{\rm f} \approx 1.3\,M_\odot$ according to recent calculations by Garcia-Berro et al. 1997. Otherwise supermassive white dwarfs can be produced by the lifting effect of rotation (Dominguez et al. 1996) or by mergers (Segretain et al. 1997). Since these alternatives are only partly explored, and the outcomes are strongly dependent on initial conditions, a reliable new $M_{\rm i}/M_{\rm f}$ relation can only be proposed up to initial masses of $\approx 7\,M_\odot$. It is tabulated for normal composition in Table 3.

Acknowledgements. I thank Falk Herwig and D. Schönberner, Potsdam, for clarifying remarks and S. Jordan, Kiel for help at the preparation of the figures and of the manuscript and the referee for critical comments.

References

Basri G., Marcy G., Graham J.R., 1996, ApJ 458, 600
Becker S.A., Iben Jr.I., 1979, ApJ 232, 831
Bergeron P., Saffer R.A., Liebert J., 1992, ApJ 394, 228
Bergeron P., Liebert J., Fulbright M.S., 1995, ApJ 444, 810
Blöcker T., 1995, A&A 297, 727
Blöcker T., Schönberner D., 1991, A&A 244, L43
Blöcker T., Herwig F., Driebe T., Bramkamp H., Schönberner D., 1997, In: Isern J., Hernanz M., Garcia-Berro E. (eds.) White Dwarfs. Proc. 10.European Workshop, Kluwer, Dordrecht, p. 57
Blöcker T., Herwig F., Driebe T., 2000, Mem. Soc. Astron. Ital. in press, astro-ph/0002455
Boothroyd A.D., Sackmann I.J., 1988, ApJ 328, 653
Bowen G.H., 1988, ApJ 329, 299
Bressan A., Fagotto F., Bertelli G., Chiosi C., 1993, A&AS 100, 647
Bryan G.L., Volk K., Kwok S., 1990, ApJ 365, 301
D'Antona F., Mazzitelli I., 1996, ApJ 470, 1093
Dominguez I., Straniero O., Tornambe A., Isern J., 1996, ApJ 472, 783
Dominguez I., Chieffi A., Limongi M., Straniero O., 1999, ApJ 524, 226
Dreizler S., Heber U., 1998, A&A 334, 618
Ferrario L., Vennes S., Wickramasinghe D.T., 1998, MNRAS 299, L1
Finley D.S., Koester D., 1997, ApJ 489, L79
Fleming T.A., Liebert J., Bergeron P., Beauchamp A., 1997, In: Isern J., Hernanz M., Garcia-Berro E. (eds.) White Dwarfs. Proc. 10.European Workshop, Kluwer, Dordrecht, p. 91
Forestini M., Charbonel C., 1997, A&AS 123, 241
Garcia-Berro E., Ritossa C., Iben Jr. I., 1997, ApJ 485, 765
Gianuzzi M.A., 1995, A&A 297, 727
Girardi L., Bertelli G., 1998, MNRAS 300, 533
Girardi L., Bressan A., Bertelli G., Chiosi C., 2000, A&AS 141, 371
Greenstein J.L., Trimble V.L., 1972, ApJ 175, L1

Groenewegen M.A.T., de Jong T., 1993, A&A 267, 410
Groenewegen M.A.T., de Jong T., 1994, A&A 283, 463
Herwig F., 1995, In: Noels A., Fraipont-Caro D., Gabriel M., Grevesse N., Demarque P. (eds.) Proc. 32. Liege Int. Astrophys. Coll., p. 441
Herwig F., 1997, In: Isern J., Hernanz M., Garcia-Berro E. (eds.) White Dwarfs. Proc. 10.European Workshop, Kluwer, Dordrecht, p. 63
Herwig F., 2000, A&A 360, 952
Herwig F., Blöcker T., Schönberner D., El Eid M., 1997, A&A 324, L81
Herwig F., Schönberner D., Blöcker T., 1998, A&A 340, L43
Herwig F., Blöcker T., 1999, In: Proc. 35th Liege Int. Astr. Coll. in press
Herwig F., Blöcker T., Schönberner D., 1999a, In: Bertre T.L., Lebre A,, Waelkens C. (eds.) AGB stars. IAU Symp.191, ASP Conf.Ser. p. 41
Herwig F., Blöcker T., Driebe T., 1999b, Mem. Soc. Astron. Ital. in press, astro-ph/9912350
Iben Jr. I., 1981, ApJ 246, 278
Iben Jr. I., 1999, In: Bertre T.L., Lebre A,, Waelkens C. (eds.) AGB stars. IAU Symp.191, ASP Conf.Ser. p. 591
Iben Jr. I., Rood R.T., 1970, ApJ 161, 587
Iben Jr. I., Renzini A., 1983, ARA&A 21, 271
Jeffries R.D., 1997, MNRAS 288, 585
Jeffries R.D., James D.J., Thurston M.R., 1998, MNRAS 300, 550
Koester D., 1972, A&A 16, 459
Koester D., Schulz H., Weidemann V., 1979, A&A 76, 262
Koester D., Weidemann V., 1980, A&A 81, 141
Koester D., Reimers D., 1981, A&A 99, L8
Koester D., Reimers D., 1993, A&A 275, 479
Koester D., Reimers D., 1996, A&A 313, 810
Marigo P., 1998, A&A 340, 463
Marigo P., Bressan A., Chiosi C., 1998, A&A 331, 564
Marigo P., Girardi L., Bressan A., 1999, A&A 344, 123
Mazzitelli I., D'Antona F., Ventura P., 1999, A&A 348, 846
McCook G.P., Sion E.M., 1987, ApJS 65, 603
Meynet G., Mermillod J.-C., Maeder A., 1993, A&AS 98, 477
Napiwotzki R., Green P.J., Saffer R.A., 1999, ApJ 517, 399
Perryman M.A.C., Brown A.G.A., Lebreton Y., et al., 1998, A&A 331, 81
Reid I.N., 1992, MNRAS 257, 257
Reid I.N., 1996, AJ 111, 2000
Reimers D., 1975, Mem. Soc. R. d. Science Liege, VI/8, 369
Reimers D., Koester D., 1982, A&A 116, 341
Reimers D., Koester D., 1989, A&A 218, 118
Richer H.B., Fahlman G.G., Rosvick J., Ibata R., 1998 ApJ 504, L91
Ritossa C., Garcia-Berro E., Iben Jr. I., 1996, ApJ 460, 489
Ritossa C., Garcia-Berro E., Iben Jr. I., 1999, ApJ 515, 381
Sackmann I.-J., Weidemann V., 1972, ApJ 178, 427
Schaller G., Schaerer D., Meynet G., Maeder A., 1992 A&AS 96, 269
Schaerer D., Meynet G., Maeder A., Schaller G., 1993, A&AS 98, 523
Schmidt H., 1996, A&A 311, 852
Schulz H., Wegner G., 1981, A&A 94, 272
Segretain L., Chabrier G., Mochkovitch R., 1997, ApJ 481, 355
Straniero O., Chieffi A., Limongi M., et al., 1997, ApJ 478, 332
Umeda H., Nomoto K., Yamaoka H., Wanajo S., 1999, ApJ 513, 861
Vassiliadis E., Wood P.R., 1993, ApJ 413, 641
Vennes S., 1999, ApJ 525, 995
Ventura P., Zeppieri A., Mazzitelli I., D'Antona F., 1998, A&A 334, 953
Wagenhuber J., Groenewegen M.A.T., 1998, A&A 340, 183
Wegner G., Reid I.N., McMahan R.K., 1991, ApJ 376, 186
Weidemann V., 1977, A&A 59, 418
Weidemann V., 1987, A&A 188, 74
Weidemann V., 1993a, In: Schwarz H.E. (ed.) Mass loss on the AGB and beyond, Proc. ESO Conference La Serena 1992, No.46, p.55
Weidemann V., 1993b, In: Barstow M.A. (ed.) White dwarfs: Advances in Observation and Theory. Eighth European Workshop on White Dwarfs in Leicester 1992, Kluwer, p. 1
Weidemann V., 1993c, A&A 275, 158
Weidemann V., 1997, In: Rood R.T., Renzini A. (eds.) Advances in Stellar Evolution. Proc. Elba Workshop Stellar Ecology 1996. Cambridge Contemp. Astrophysics, Cambridge Univ. Press, p. 169
Weidemann V., Koester D., 1983, A&A 121, 77
Weidemann V., Jordan S., Iben Jr I., Casertano S., 1992, AJ 104, 1876
Werner K., 2000 In: Blöcker T., Waters L.B.F.M., Zijlstra A.A. (eds.) Low mass Wolf-Rayet stars: Origin and evolution. Ap&SS in press
Wood M.A., 1992, ApJ 286, 539
Wood M.A., 1995, In: Koester D., Werner K. (eds.) Proc. 9th. European Workshop on White Dwarfs. Lecture Notes in Physics, Springer, Berlin, p. 41
Yuan J.W., 1992, A&A 261, 105