Lévy distribution

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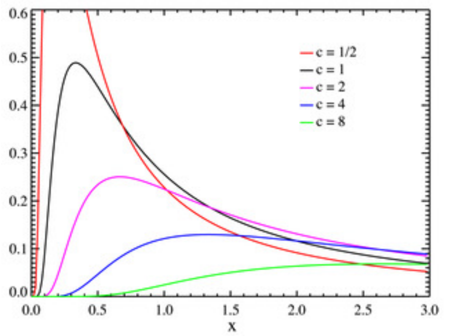
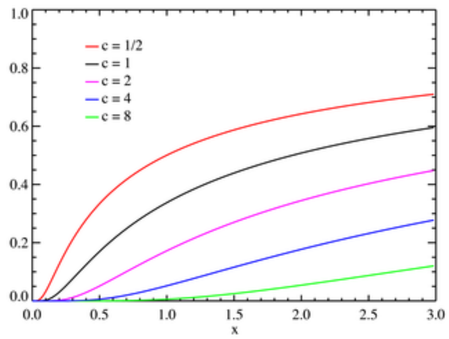
In probability theory and statistics, the **Lévy distribution**, named after Paul Pierre Lévy, is one of the few distributions that are stable and that have probability density functions that are analytically expressible. The others are the normal distribution and the Cauchy distribution. All three are special cases of the Lévy skew alpha-stable distribution, which does not generally have an analytically expressible probability density. In spectroscopy this distribution, with frequency as the dependent variable, is known as a **Van der Waals profile**.

Lévy (unshifted)

Probability density function



Cumulative distribution function



The probability density function of the Lévy distribution over the domain $x \geq 0$ is

$$f(x;c) = \sqrt{\frac{c}{2\pi}}\,\frac{e^{-c/2x}}{x^{3/2}}$$

where c is the scale parameter. The cumulative distribution function is

$$F(x;c) = \textrm{erfc}\left(\sqrt{c/2x}\right)$$

where erfc(z) is the complementary error function. A shift parameter μ may be included by replacing each occurrence of x in the above equations with $x - \mu$. This will simply have the effect of shifting the curve to the right by an amount μ, and changing the support to the interval [μ, ∞). The characteristic function of the Lévy distribution (including a shift μ) is given by

$$\varphi(t;c) = e^{i\mu t - \sqrt{-2ict}}.$$

Note that the characteristic function can also be written in the same form used for the Lévy skew alpha-stable distribution with α = 1 / 2 and β = 1:

$$\varphi(t;c) = e^{i\mu t - |ct|^{1/2}~(1 - i~\textrm{sign}(t))}.$$

The nth moment of the unshifted Lévy distribution is formally defined by:

$$m_n \stackrel{\mathrm{def}}{=} \sqrt{\frac{c}{2\pi}} \int_0^\infty \frac{e^{-c/2x}\,x^n}{x^{3/2}}\,dx$$

which diverges for all $n > 0$ so that the moments of the Lévy distribution do not exist. The moment generating function is formally defined by:

$$M(t;c) \stackrel{\mathrm{def}}{=} \sqrt{\frac{c}{2\pi}} \int_0^\infty \frac{e^{-c/2x+tx}}{x^{3/2}}\,dx$$

which diverges for $t > 0$ and is therefore not defined in an interval around zero, so that the moment generating function is not defined *per se*. In the wings of the distribution, the PDF exhibits heavy tail behavior falling off as:

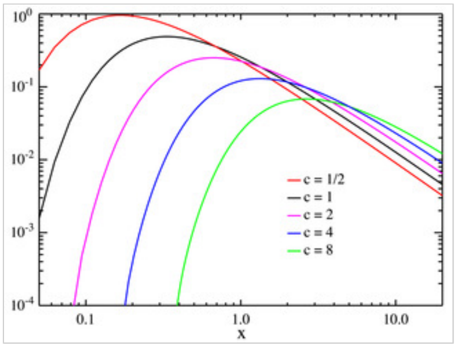
$$\lim_{x\rightarrow\infty} f(x;c) = \sqrt{\frac{c}{2\pi}}\,\frac{1}{x^{3/2}}.$$

This is illustrated in the diagram below, in which the PDF's for various values of c are plotted on a log-log scale.

Parameters	$c > 0$
Support	$x \in [0, \infty)$
Probability density function (pdf)	$\sqrt{\frac{c}{2\pi}}\,\frac{e^{-c/2x}}{x^{3/2}}$
Cumulative distribution function (cdf)	$\textrm{erfc}\left(\sqrt{c/2x}\right)$
Mean	infinite
Median	$c/2(\textrm{erf}^{-1}(1/2))^2$
Mode	$\frac{c}{3}$
Variance	infinite
Skewness	undefined
Excess kurtosis	undefined
Entropy	$\frac{1+3\gamma+\ln(16\pi c^2)}{2}$
Moment-generating function (mgf)	undefined
Characteristic function	$e^{-\sqrt{-2ict}}$

Related distributions

- Relation to Lévy skew alpha-stable distribution: If $X \sim \textrm{Levy}(c)$ then $X \sim \textrm{Levy-S}\alpha\textrm{S}(1/2,1,c,0)$
- Relation to Scale-inverse-chi-square distribution: If $X \sim \textrm{Levy}(c)$ then $X \sim \textrm{Scale-inv-}\chi^2(1,c)$
- Relation to inverse gamma distribution: If $X \sim \textrm{Levy}(c)$ then $X \sim \textrm{Inv-Gamma}(1/2,c/2)$



Probability density function for the Lévy distribution

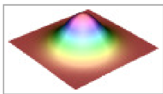
Relevance

- It is claimed that fruit flies follow a form of the distribution to find food. [1]

References and external links

- Information on stable distributions (http://academic2.american.edu/~jpnolan/stable/stable.html). Retrieved on July 13, 2005. - John P. Nolan's introduction to stable distributions, some papers on stable laws, and a free program to compute stable densities, cumulative distribution functions, quantiles, estimate parameters, etc. See especially An introduction to stable distributions, Chapter 1 (http://academic2.american.edu/~jpnolan/stable/chap1.pdf)

^ The Lévy distribution as maximizing one's chances of finding a tasty snack (http://www.livescience.com/animalworld/070403_fly_tricks.html). Retrieved on April 7, 2007.

	**Probability distributions [ ]**	
	Univariate	**Multivariate**
Discrete:	Benford • Bernoulli • binomial • Boltzmann • categorical • compound Poisson • discrete phase-type • degenerate • Gauss-Kuzmin • geometric • hypergeometric • logarithmic • negative binomial • parabolic fractal • Poisson • Rademacher • Skellam • uniform • Yule-Simon • zeta • Zipf • Zipf-Mandelbrot	Ewens • multinomial • multivariate Polya
Continuous:	Beta • Beta prime • Cauchy • chi-square • Dirac delta function • Coxian • Erlang • exponential • exponential power • F • fading • Fisher's z • Fisher-Tippett • Gamma • generalized extreme value • generalized hyperbolic • generalized inverse Gaussian • Half-Logistic • Hotelling's T-square • hyperbolic secant • hyper-exponential • hypoexponential • inverse chi-square (scaled inverse chi-square)• inverse Gaussian • inverse gamma (scaled inverse gamma) • Kumaraswamy • Landau • Laplace • **Lévy** • Lévy skew alpha-stable • logistic • log-normal • Maxwell-Boltzmann • Maxwell speed • normal (Gaussian) • normal-gamma • normal inverse Gaussian • Pareto • Pearson • phase-type • polar • raised cosine • Rayleigh • relativistic Breit-Wigner • Rice • shifted Gompertz • Student's t • triangular • type-1 Gumbel • type-2 Gumbel • uniform • Variance-Gamma • Voigt • von Mises • Weibull • Wigner semicircle • Wilks' lambda	Dirichlet • inverse-Wishart • Kent • matrix normal • multivariate normal • multivariate Student • von Mises-Fisher • Wigner quasi • Wishart
Miscellaneous:	Cantor • conditional • exponential family • infinitely divisible • location-scale family • marginal • maximum entropy • posterior • prior • quasi • sampling • singular	

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