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THE DISK AND HALO DENSITIES AT THE PLANE FROM STAR COUNTS IN THE GALACTIC POLES

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ABSTRACT

Four determinations of the density distribution $D(Z)$ with height from the plane at the solar circle for dwarfs with absolute magnitudes of $\langle M_V \rangle \simeq +4$ to $+5$ are combined. The four determinations are in good agreement. The data extend to $Z = 10$ kpc. The gradient, $d \log D(Z)/dZ$, changes continuously over the observed range of Z; the gradient flattens with increasing Z. Calculated density distributions for discrete kinematic samples with $\sigma(W)$ values of 17, 42, and 90 km s^{-1} provide a unique fit to the data, giving density normalizations at $Z = 0$ of 1:0.11:0.005 (or 200:22:1). We identify the $\sigma(W) = 17$ km s^{-1} component with the old thin disk (scale height ~ 270 pc), the $\sigma(W) = 42$ km s^{-1} component with the thick disk (scale height ~ 940 pc), and the $\sigma(W) = 90$ km s^{-1} component with the extreme halo (scale height at ~ 3.2 kpc). The derived density normalizations agree with the independent data from the unbiased radial-velocity sample (Sandage and Fouts 1987) but are larger by a factor of ~ 5 from the Gilmore-Reid normalization. The mass of the stellar component in the halo is estimated to be $\sim 3 \times 10^9$ $M_\odot$, comprising less than $\sim 2\%$ of the total Galactic mass. From two normalizations the local density of halo stars is between 1×10^5 $M_\odot$ kpc^{-3} and 2.5×10^5 $M_\odot$ kpc^{-3}, agreeing well with previous values by Schmidt and by Eggen from widely different methods.

I. INTRODUCTION

The local density ratio of the galactic thick disk and halo components to the old thin disk obtained in the unbiased radial-velocity sample reported in the previous paper (Sandage and Fouts 1987b) is so high (90:10:0.5) that a determination by independent means is necessary. The problem is considered in this paper by analyzing the star-count data in the galactic poles, from which the density distribution of stars perpendicular to the galactic plane can be determined to 10 kpc.

The density distribution of each of the galactic components is related directly to the velocity dispersion $\sigma(W)$ of that component if we are dealing with what can be approximated by a discrete Lindblad (1959) kinematic subsystem with a discrete value of $\sigma(W)$. To be sure, the Galaxy is a continuum structure formed by a process which had a variety of rates (star-formation rates, collapse rates, gas-dissipation rates, etc.), yet it is convenient to approximate this continuum by division into the various segments of the process, separated by particular values of the ratios of the relevant rates. It is, of course, generally understood by astronomers who discuss disks (old, young, thick, or thin), bulges, halos, etc., as separate structures that they are approximating this continuum by a series of discrete boxes according to these ratios. This is, in fact, at the heart of the description of the population concept.

The three components whose density normalization we are seeking will be assigned discrete and separate $\sigma(W)$ values, based on the results reported earlier in this series (Sandage and Fouts 1987a,b). Each separate $\sigma(W)$ leads to a particular run of density $D(Z)$ with height. The sum of the separate densities gives the observed total density distribution $D(Z)$ for the total sample considered.

We seek here to turn the problem around: From the observed $D(Z)$ for the total sample, we wish to find the separate $D_1(Z)$, $D_2(Z)$, and $D_3(Z)$ density distributions for the old thin disk, the old thick disk, and the halo, from which the density normalizations in the plane $D_1(0)$, $D_2(0)$, and $D_3(0)$ are sought. The data to be used are the observed summed density $D_T(Z) = D_1(Z) + D_2(Z) + D_3(z)$, and the three velocity dispersions $\sigma_1(W)$, $\sigma_2(W)$, $\sigma_3(W)$.

In the next section we compute a family of $D(Z)$ curves for particular $\sigma(W)$ values, using an assumed run of the gravitational acceleration $g(Z)$ perpendicular to the plane. In Sec. III the polar star-count data from the Basel, the Edinburgh, and the Tokyo surveys are analyzed and compared with the analysis of Fenkart (1966) in Selected Area 57 to obtain an adopted $D(Z)$ density run to heights of 10 kpc from the galactic plane. This total density is decomposed into the three density components of thin disk, thick disk, and halo in Sec. IV using the predicted individual functions from Sec. II. In this way we obtain the density normalization at $Z = 0$ to be 1:0.11:0.005, or 200:22:1. These values agree with those obtained from the unbiased radial-velocity sample discussed in the preceding paper.

Finally, with the density normalization and the density falloff of the halo now known, we compute in Sec. V the mass of the halo for the stars alone.

II. CALCULATED DENSITY DISTRIBUTIONS FOR THREE ASSUMED VELOCITY DISPERSIONS $\sigma(W)$

It is well known (Kapteyn 1922; see Lindblad 1959 for a concise derivation) that the density distribution perpendicular to the plane which a group of stars assumes, if their velocity distribution is Gaussian with dispersion $\sigma(W)$, is

$$D(Z) = D(0) \exp\left[-\sigma(W)^{-2} \int_0^Z g(Z) dZ \right], \tag{1}$$

where $g(Z)$ is the gravitational acceleration perpendicular to the plane.

Table I lists the calculation of this function from the galactic plane to $Z = 10$ kpc. The adopted $g(Z)$ is from Saio and Yoshii (1979, Fig. 1) based on combining the determination of Oort (1965) and Oort and van Woerkom (1941). Columns 4–6 are the resulting predicted density distributions for the old thin disk with $\sigma(W) = 17$ km s^{-1}, the thick disk

TABLE I. Adopted acceleration perpendicular to the plane at the solar circle and calculated density distribution with height.

Z	g(Z)	$\int_0^Z g(Z)dZ$	log D(Z)/D(0)		
kpc	10^9 cm s^{-2}	cm^2 s^{-2}	σ=17 km s^{-1}	σ=42 km s^{-1}	σ=90 km s^{-1}
(1)	(2)	(3)	(4)	(5)	(6)
0.0	0.0	0	0	0	0
0.1	1.8	5.58 x 10^{11}	-0.08	-0.01	0
0.2	3.4	1.61 x 10^{12}	-0.24	-0.04	-0.01
0.3	5.3	3.26 x 10^{12}	-0.49	-0.08	-0.02
0.4	6.2	5.18 x 10^{12}	-0.78	-0.13	-0.03
0.5	7.1	7.38 x 10^{12}	-1.11	-0.18	-0.04
0.6	7.3	9.64 x 10^{12}	-1.45	-0.24	-0.05
0.7	7.7	1.20 x 10^{13}	-1.80	-0.30	-0.06
0.8	7.8	1.44 x 10^{13}	-2.16	-0.36	-0.08
0.9	7.9	1.69 x 10^{13}	-2.54	-0.42	-0.09
1.0	8.0	1.94 x 10^{13}	-2.92	-0.48	-0.10
1.2	8.2	2.44 x 10^{13}	-3.67	-0.60	-0.13
1.4	8.4	2.96 x 10^{13}	-4.45	-0.73	-0.16
1.6	8.6	3.49 x 10^{13}	-5.24	-0.86	-0.19
1.8	8.8	4.03 x 10^{13}	-6.06	-0.99	-0.22
2.0	8.9	4.58 x 10^{13}		-1.13	-0.25
2.5	9.1	5.98 x 10^{13}		-1.48	-0.32
3.0	9.3	7.42 x 10^{13}		-1.83	-0.40
3.5	9.5	8.89 x 10^{13}		-2.19	-0.48
4.0	9.6	1.04 x 10^{14}		-2.57	-0.56
4.5	9.6	1.19 x 10^{14}		-2.94	-0.64
5.0	9.4	1.33 x 10^{14}		-3.27	-0.71
5.5	9.2	1.47 x 10^{14}		-3.63	-0.79
6.0	9.0	1.62 x 10^{14}		-3.98	-0.87
6.5	8.9	1.75 x 10^{14}		-4.32	-0.94
7.0	8.6	1.89 x 10^{14}		-4.66	-1.01
7.5	8.3	2.02 x 10^{14}		-4.98	-1.08
8.0	8.0	2.14 x 10^{14}		-5.28	-1.14
8.5	8.0	2.26 x 10^{14}		-5.58	-1.21
9.0	8.0	2.39 x 10^{14}		-5.90	-1.28
9.5	8.0	2.51 x 10^{14}		-6.19	-1.35
10.0	8.0	2.64 x 10^{14}		-6.51	-1.42

using $\sigma(W) = 42$ km s^{-1} from Paper VI of the subdwarf series (Sandage and Fouts 1987b), and the halo using $\sigma(W) = 90$ km s^{-1}.

Figure 1 shows the three predicted distributions. Very near the plane the log $D(Z)$ function decreases approximately as Z^2 (because $g(Z)$ is linear in Z for the first few hundred parsecs). At larger distances, $D(Z)$ is closely exponential. The resulting e^{-1} scale height and the naperian log density gradient are listed beside each of the calculated curves in Fig. 1.

III. THE DENSITY DISTRIBUTION $D(Z)$ FOR STARS WITH $+4 \lesssim M_V \lesssim +6$ DERIVED FROM POLAR STAR COUNTS

Counts of the number of stars in given apparent magnitude and spectral class or color intervals per unit area on the sky have been the traditional data from which to derive the density distribution along any given line of sight. There are two methods of solution. One is to solve the fundamental integral equation of stellar statistics for $D(r)$, given the observed count data $A(m, \text{color})$ (which is the number of stars per square degree at magnitude m in interval dm within a given color interval). The other method as used in the extensive Basel galactic structure program is to determine the distance of each star in the program from its magnitude and $U-G$ and $G-R$ colors and then bin the sample by distance. Each method requires information on the absolute luminosity of individual stars, invariably obtained either from the spectral or the color data. The two methods are, of course, equivalent.

Three large surveys at bright magnitudes of the galactic polar regions exist. The polar region of the Basel survey is

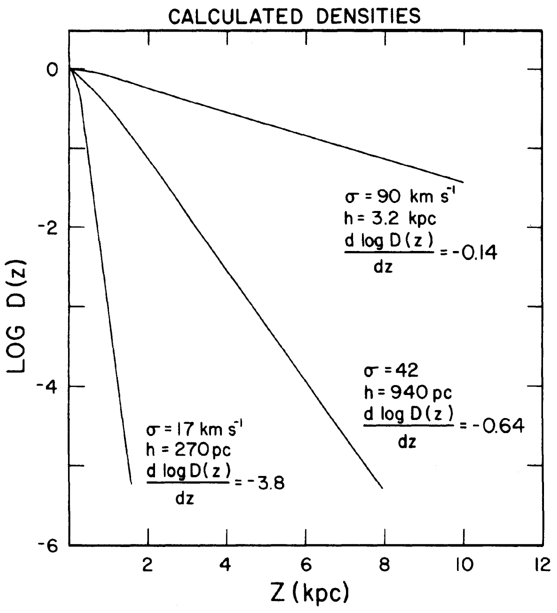

FIG. 1. Calculated densities from Eq. (1) using the acceleration $g(Z)$ listed in Table I. Three values of $\sigma(W)$ are adopted. The scale heights and density gradients are marked for each curve.

Selected Area 57. Fenkart (1966) has derived $D(Z)$ for various absolute magnitude and distance intervals, and for a division based on the $U-G$, $G-R$ colors of disk and halo stars. A summary of Fenkart's density determination is set out in Table II, where his disk and halo sample has been combined, and where we restrict his total sample to stars between M_G of $+4$ to $+6$. In Table II the distance interval and mean distance are in columns 1 and 2. The log of the densities (in units of stars per pc^3) for stars between $M_G = 4$–5, 5–6, and 4–6 are in columns 3–5. The largest distance sampled is the interval 7.9–10.0 kpc for a mean distance of $\langle Z \rangle = 8.95$ kpc. The data in Fenkart's analysis are from the Basel galactic program where three colors and magnitudes of every star in given fields were measured. The area of the SA 57 Basel catalog is 2.61 square degrees.

A two-color survey of 18.24 square degrees at the south Galactic pole for stars brighter than $V=19$ was made by Gilmore and Reid (1983). They summarize their extensive count data in their Tables 1 and 2 as the number of stars per 18.24 square degrees in the absolute-magnitude interval $M_V \pm 0.5$ in the distance interval of $\log Z \pm 0.1$. To obtain densities, their listed numbers must be divided by the volume contained within each distance interval. This volume element is

$$\Delta \mathrm{Vol} = 1/3\omega(r_1^3 - r_2^3), \tag{2}$$

where r_1 and r_2 are the inner and outer boundaries of the particular volume shell and ω is the solid angle (in square radians) of the 18.24 square degree field. For the Gilmore-Reid sample, their volume elements within distance shells bounded by distances r_1 and r_2, reduced to a unit square degree area, are

$$\Delta \mathrm{Vol} = 1.02 \times 10^{-4}(r_1^3 - r_2^3)\ \mathrm{pc}^3\ \mathrm{deg}^{-2}, \tag{3}$$

where r_1 and r_2 are in parsecs.

Using the volume elements calculated in this way gives the derived densities, in units of stars per pc^3, listed in Table III for the absolute-magnitude intervals of M_V between 4 and 5 and between 5 and 6. The Gilmore-Reid data extend to a maximum distance of $Z = 4.5$ kpc.

A three-color survey of 21.46 square degrees of the north galactic polar region has been made by Yoshii, Ishida, and Stobie (1987), also to $V=18$. They list their binned data in 1 mag intervals from $V=10$ to $V=18$ and in intervals of ± 0.05 mag of $B-V$ and $U-B$ color. From these listings, an analysis can be made via the principal equation of stellar statistics

$$A(m) = \omega \int_0^\infty r^2 D(r)\phi(M)dr, \tag{4}$$

TABLE II. Fenkart's density distribution in SA 57 for stars with M_G between $+4$ and $+6$ (disk + halo).

Z kpc (1)	<Z> kpc (2)	log $D(Z)_{M_G}$ 4 to 5 (3)	5 to 6 (4)	4 to 6 (5)
0 - 0.5	0.25	-3.26	-2.92	-2.76
0.5 - 0.6	0.5	-3.83	-3.81	-3.52
0.6 - 0.8	0.7	-3.61	-3.31	-3.13
0.8 - 1.0	0.9	-4.15	-3.60	-3.49
1.0 - 1.3	1.15	-4.28	-3.79	-3.67
1.3 - 1.6	1.45	-4.52	-4.20	-4.03
1.6 - 2.0	1.8	-4.76	-4.44	-4.27
2.0 - 2.5	2.25	-5.50	-4.65	-4.59
2.5 - 3.2	2.85	-5.37	-4.99	-4.84
3.2 - 4.0	3.6	-5.93	-5.18	-5.11
4.0 - 5.0	4.5	-5.94	-5.66	-5.48
5.0 - 6.3	5.65	-5.87	-6.50	-5.78
6.3 - 7.9	7.10	-6.13	----	----
7.9 -10.0	8.95	-6.92	----	----

*Unit on the densities is number of stars per pc^3.

TABLE III. Gilmore and Reid densities derived from their star counts in 18.24 square degrees of the SGP (their Table 2 using the metallicity correction).

log <Z>	<Z> kpc	log $D(Z)_{M_V}$ 4 to 5	log $D(Z)_{M_V}$ 5 to 6	log <Z>	<Z> kpc	log $D(Z)_{M_V}$ 4 to 5	log $D(Z)_{M_V}$ 5 to 6
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
2.05	0.11	-2.79	-2.67	2.95	0.89	-3.75	-3.65
2.15	0.14	-2.67	-2.45	3.05	1.12	-3.98	-3.91
2.25	0.18	-2.57	-2.47	3.15	1.41	-4.25	-4.18
2.35	0.22	-2.81	-2.72	3.25	1.79	-4.46	-4.47
2.45	0.28	-2.80	-2.79	3.35	2.24	-4.74	-4.68
2.55	0.35	-3.04	-2.95	3.45	2.82	-4.95	-4.89
2.65	0.45	-3.19	-2.99	3.55	3.55	-5.20	----
2.75	0.56	-3.28	-3.19	3.65	4.47	-5.47	----
2.85	0.71	-3.49	-3.50				

where $A(m)$ is the number of stars per unit area of solid angle ω at m in interval dm, $\phi(M)$ is the luminosity function of absolute magnitudes, and m, M, and r are related by $m - M = 5 \log r - 5$.

By restricting the analysis to a narrow range of color, we restrict the data to a narrow range of absolute magnitude, and can thereby simplify the solution of Eq. (4) for $D(r)$. We have analyzed the Yoshii, Ishida, and Stobie data by using only the counts between $0.3 \leqslant B - V \leqslant 0.6$ and have assumed that stars in this color range have an absolute magnitude near the globular cluster turnoff at $M_V = +4$. This replaces the $\phi(M)$ distribution by a delta function. Furthermore, by choosing distance intervals separated by $\Delta \log r = 0.2$ and apparent-magnitude intervals of ± 0.5 mag (i.e., 1 mag for the total interval), the integral of Eq. (4) can be replaced by a single calculation, giving

$$D(\langle r \rangle) = A(m)/\Delta \text{Vol}, \qquad (5)$$

where ΔVol is given by Eq. (2), $\langle r \rangle$ is $(r_1 + r_2)/2$, and r and m are related (assuming $M_V = +4$) by $\log r = 0.2m + 0.2$. The method is standard and follows directly from Kapteyn's m, $\log \pi$ method of solving Eq. (4), as described in Bok's (1937) textbook, for example.

Preparatory to applying the method, we list the observed $A(m)$ values for the Yoshii, Ishida, and Stobie data in Table IV. The units of $A(m)$ are the number of stars per square degree between $B - V$ of 0.3 and 0.6 within the listed V magnitude interval.

We have analyzed the Basel data in the same way by working directly with the Basel catalog of SA 57 (Becker and Fenkart 1976). The RGU data were transformed to the UBV system by Steinlin's (1968) equations, and a V, $B - V$ color-magnitude diagram was constructed. Counts were made in this diagram to give the $A(m)$ values also listed in Table IV.

Table V lists the derived $D(Z)$ densities from both the Basel and Tokyo data of Table IV. The midpoint of the distance intervals is in column 1, with its corresponding mean distance in column 2. The volume element between the inner and outer boundaries of the distance shell considered is in column 3, calculated from Eq. (2) for 1 square degree. Columns (4) and (5) give the log of the derived densities calculated from Eq. (5) using the $A(m)$ values from Table IV and the volume elements of column 3.

IV. THE COMPOSITE DENSITY DISTRIBUTION DECOMPOSED INTO THREE COMPONENTS

a) Our Density Normalizations

The four density determinations for $D(Z)$ agree remarkably well, as shown in the top panel of Fig. 2. Aside from a slight normalization adjustment, the run of $D(Z)$ with Z is indistinguishable between the four determinations. In plot-

TABLE IV. Number of stars per square degree $A(m)$ between $B - V = 0.3$ and 0.6 in the Basel Catalog of SA 57 and the catalog of Yoshii, Ishida, and Stobie in the NGP.[a,b]

V	A(m) stars deg^{-2} $0.3 \leq B-V \leq 0.6$ Basel	Tokyo	V	A(m) $0.3 \leq B-V \leq 0.6$ Basel	Tokyo
10.5 - 11.5	2.31	2.62	15.5 - 16.5	20.1	23.3
11.5 - 12.5	4.59	4.66	16.5 - 17.5	27.1	30.9
12.5 - 13.5	7.22	7.56	17.5 - 18.5	34.3	37.0
13.5 - 14.5	10.43	11.18	18.5 - 19.5	43.0	
14.5 - 15.5	14.74	16.16			

[a] Basel counts based on complete sample in 2.61 square degrees.

[b] Tokyo counts based on complete sample in 21.46 square degrees.

TABLE V. Derived densities for stars with $B-V$ between 0.3 and 0.6 in the NPC from the Basel and the Tokyo counts.

log <Z>	<Z> kpc	ΔVol pc^3 deg^{-2}	log D(Z) Basel	Tokyo
(1)	(2)	(3)	(4)	(5)
2.4	0.25	2.40×10^3	-3.02	-2.96
2.6	0.40	9.55×10^3	-3.32	-3.31
2.8	0.63	3.80×10^4	-3.72	-3.70
3.0	1.00	1.51×10^5	-4.16	-4.13
3.2	1.58	6.03×10^5	-4.61	-4.57
3.4	2.51	2.40×10^6	-5.08	-5.01
3.6	3.98	9.55×10^6	-5.55	-5.49
3.8	6.31	3.80×10^7	-6.04	-6.01
4.0	10.00	1.51×10^8	-6.55	----

ting the points in Fig. 2(a) we have normalized all of the data to the derived Basel densities (Table IV).

The Basel data extend to 10 kpc. This is more than twice as far as the last Gilmore-Reid point at 4.5 kpc, and it is this difference which is crucial in explaining our different density normalization of ~ 10 to 1 for the thin to thick disk compared to ~ 50 to 1 obtained by Gilmore and Reid.

The decomposition into the components is now simply

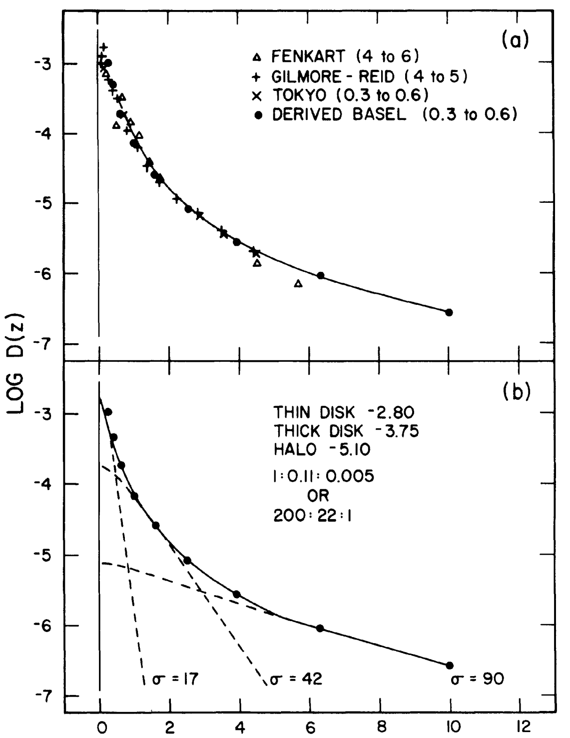

FIG. 2. (Top) The density $D(Z)$ perpendicular to the galactic plane for the four data sets listed in Table II, III, and V. (Bottom) Decomposition of the observed log $D(Z)$ function into three components using the calculated densities shown in Fig. 1.

made by fitting each of the three calculated relations from Fig. 1 to appropriate sections of Fig. 2(a). The fit is unique and is well determined in Fig. 2(b) to give the density ratios at $Z=0$ between the thin disk, the thick disk, and the halo to be 1:0.11:0.005.

b) Comparison with Gilmore and Reid

The procedure described above is the same in principle as used by Gilmore and Reid in the fitting of *two* exponentials to their data (the same as Table III here) and reading the difference in the intercepts at $Z=0$ to give the 50 to 1 ratio. However, their use of only two components neglects the halo contribution at their furthest point at $z=4.5$ kpc. Their resulting thick-disk component has then a necessarily large scale height of ~ 1.5 kpc corresponding to $\sigma(W) \simeq 60$ km s^{-1}, required by the data if the halo component is neglected.

However, our direct evidence from Paper VI (Sandage and Fouts 1987a) is that $\sigma(W)=42$ km s^{-1} for the thick-disk component. Hartkopf and Yoss (1982) obtain the same value. With this lower $\sigma(W)$ we are forced to the normalization for the thick disk which we have made in Fig. 2(b), showing that at the Gilmore-Reid last data point at 4.5 kpc the halo must dominate the counts so as to produce the observed flattening of the $D(Z)$ curve beyond $Z \simeq 4$ kpc. The resulting scale height is ~ 3.2 kpc.

We now ask whether $d \log_{10} D(Z)/dZ = -0.14$ is consistent with the density distribution $\rho(r)$ of known halo objects, measured relative to the center of the Galaxy, where r is the galactocentric distance. The globular cluster system is well represented by

$$\rho(r)/\rho(0) = (1+r)^{-3.5}, \qquad (6)$$

shown most recently by Harris (1976) and Zinn (1985). The RR Lyraes follow a similar distribution to $r \sim 20$ kpc (Kinman *et al.* 1966; Kinman 1972; Saha 1985), with possibly a steeper decay at larger galactocentric distances.

Numerical experiments show that a Hubble power law of the form $\rho(r) \sim (1+r)^{-n}$ in galactocentric distance r is very well approximated by an apparent exponential in Z for the observed density of the halo when it is sampled in a vertical cut above and below the Sun's position at the distance R_0 from the center. The numerical experiments which were made assume that $R_0=8$ kpc, $\rho(r)=\rho(0)/(1+r)^n$, of course $r^2=R_0^2+Z^2$, and a spherical halo. The run of $\rho(r)$ was computed for various values of n along the vertical cut in Z, perpendicular to the plane. These densities $D(Z)$ are what would be observed at height Z for the various n values, as seen from the Sun.

The resulting $D(Z)$, Z plots proved to be remarkably linear, showing that an exponential approximation to $D(Z)$ for $0<Z \leqslant 15$ kpc is an excellent fit to the $(1+r)^{-n}$ Hubble law relative to the Galactic center. In this way it is found that $d \log_{10} D(Z)/dZ = -0.05$ for $n=3$; -0.07 for $n=3.5$; -0.09 for $n=4$; -0.12 for $n=5$; and -0.15 for $n=6$. Hence, if we require $n=3.5$, our value of -0.14 in Figs. 1 and 2 is not steep enough. To obtain -0.07 requires that $\sigma(W)=130$ km s^{-1}. This is close to $\sigma(W)=120$ km s^{-1} which has been observed for the most metal-poor stars (i.e., [Fe/H] <-2.5) in the sample nearby subdwarfs studied by Sandage and Fouts (1987b, hereafter referred to as SF, Table VI).

We conclude from this that if the data in Fig. 2(a) at 10 kpc are correct, then the stars to which we fit a halo gradient of -0.14 are not the most metal poor, but rather have [Fe/H] $\simeq -1.4$ whose $\sigma(W) \simeq 90$ km s^{-1}, again reading from Table VI of SF. It should be emphasized that the consequences of such a relation of $\sigma(W)$, progressively increasing with decreasing metallicity, is, indeed, a hierarchy of $d \log D(Z)/dZ$ values that are progressive with [Fe/H]. This will, of course, set up a true chemical gradient in the halo—a still controversial result observationally.

The fit to the data in Fig. 2 has very little arbitrariness. It is not possible to fit a halo component with a slope of -0.07 to the last observed data point at $Z = 10$ kpc and still retain the gradient of $d \log_{10} D(Z)/dZ = -0.64$ for the thick-disk component, and thereby fit all the data points in Fig. 2(a) using *any* set of normalization factors. The conclusion is that if -0.07 is the required halo slope, then the last observed data point at $Z = 10$ kpc in Fig. 2 is too low and should be moved up by ~ 0.4 dex. This factor may be within the errors of the present count and resulting density data, but this remains a problem with the present material on the stated assumptions of the halo gradient.

If we arbitrarily move the last point up by 0.4 dex and fit a "halo" curve of slope -0.07 to the resulting equivalent of Fig. 2, we obtain an adequate fit to all the points with normalizations of 1:0.06:0.002 or 500:30:1 for the three components. These differ by a factor of ~ 2 from those in Fig. 2(b). We use these values as an illustration in the next section to calculate the mass of the halo assuming the self-consistent value of $d \log_{10} D(Z)/dZ = -0.07$ that applies to a spherical* halo with $n = 3.5$.

V. ESTIMATE OF THE MASS OF STARS IN THE GALACTIC HALO

Following the data in the last section, we assume a galactocentric halo density decay of Eq. (6) and a normalization at the Sun at $R_0 = 8$ kpc of 500 to 1 for the ratio of old disk to halo density. The mass of stars in a spherical halo structure of this kind is

$$M = 4\pi\rho(0) \int_0^\infty r^2(1+r)^{-3.5} dr, \tag{7}$$

where $\rho(0)$ is the mass density at the Galactic center of the halo component.

The central density $\rho(0)$ is obtained from $D(Z=0, R_0 = 8) \equiv \rho(8 \text{ kpc})$ from Eq. (6) using an assumed local old-disk density of 0.05 $M_\odot$ pc^{-3} (or 5×10^7 $M_\odot$ kpc^{-3}) and a disk-to-halo ratio of 500:1, giving $D_{\rm halo}(8) = 10^5 M_\odot$ kpc^{-3}. This local halo density should be compared with Schmidt's (1975) value of 1.7×10^5 $M_\odot$ kpc^{-3} and Eggen's (1987) higher value of 4×10^5 $M_\odot$ kpc^{-3}. The agreement of these three values to within a factor of ~ 4, each using widely different methods, seems quite satisfactory. From Eq. (6), $\rho(0)/\rho(8) = 2.2\times10^3$, giving $\rho(0) = 2\times10^8$ $M_\odot$ kpc^{-3} for the density of the halo component at the center.

Integration of Eq. (7) gives

$$M(\text{halo}) = 4\pi\rho(0)[2 - \tfrac{4}{3} + \tfrac{2}{5}] = 13\rho(0), \tag{8}$$

or

$$M(\text{halo}) \simeq 3\times10^9 M_\odot.$$

This is a very weak stellar halo, comprising only $\sim 2\%$ of the Galactic mass. This mass in stars is an order of magnitude lower than that proposed by Ostriker and Peebles (1973) for the *total* mass of the halo.

*A formal way to obtain the observed vertical gradient of $d \log_{10} D(Z)/dZ = -0.14$ for a galactocentric Hubble density decay with $n = 3.5$ is to severely flatten the halo. If we assume the halo to be an oblate ellipsoid (two equal axes in the plane and a flattened vertical axis) of axial ratio $b/a = (1-e^2)^{1/2}$, where e is the eccentricity by definition, a numerical experiment such as described above, reading at each r the equivalent R along the major axis (in the plane) that has the same $D(r)$, i.e., along the isodensity ellipse, steepens the calculated $d \log D(Z)/dZ$ gradient. The relation between the galactocentric radius r and the equivalent major-axis distance R is the equation of an ellipse as

$$\frac{r}{R} = \left(\frac{1-e^2}{1-e^2\cos^2\theta}\right)^{1/2},$$

where θ is the angle between the major axis and the radius vector r, evidently given by $\theta = \arcsin(Z/r)$, or $\theta = \arctan(Z/R_0) \equiv \arctan(Z/8)$ in our example. Experimentation shows that for $b/a = 0.6$, the vertical gradient at $R_0 = 8$ becomes $d \log D(Z)/dZ = -0.14$ for $n = 3.5$. The data in Fig. 2(b) could then be taken to mean that the halo is so flattened. However, the *actual observed* gradient for RR Lyrae stars in the pole by Kinman *et al.* (1966) *is* -0.07 rather than -0.14, and the explanation seems, therefore, unlikely because the Lick polar data for the variables are consistent with a spherical halo.

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