Cosmological information from large-scale structure

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Overview

1. Cosmic structure
   - Structure formation in linear perturbation theory
   - LSS statistics
   - Baryon acoustic oscillations
2. Gravitational lensing by large-scale structure
3. Combining CMB with the LSS
   - Integrated Sachs-Wolfe effect
   - Gravitational lensing of the CMB

1. Structure formation

Background cosmology

Expansion rate:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2(z) = H_0^2\left[(1-\Omega_m-\Omega_\Lambda)(1+z)^2+\Omega_m(1+z)^3+\Omega_\Lambda\right] \quad (1)$$

Angular diameter distance:

$$D_A(z) = \frac{c}{1+z}\int_0^z \frac{dz}{H(z)} \quad (2)$$

Equation of state:

$$w_X(z) = \frac{p_X(z)}{\rho_X(z)} \Rightarrow \rho_X(z) = \rho_X(0)\exp\left[3\int_0^z \frac{1+w_X(z)}{1+z}dz\right] \quad (3)$$

Density field

The cosmic density field can be modeled as:

$$\rho(\vec{x}) = \bar{\rho}\,(1 + \delta(\vec{x})) \tag{4}$$

As long as:

$$\delta \ll 1 \tag{5}$$

we can use linear perturbation theory to compute the time evolution of $\rho(\vec{x})$.

The density perturbation δ is a random variable, usually assumed to follow a Gaussian distribution.

Linear perturbation theory

Unperturbed flat Friedman-Robertson-Walker metric element:

$$ds^2 = a^2 \left[d\eta^2 - dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \tag{6}$$

(with $d\eta = dt/a$, conformal time). Perturbed metric element in Newtonian gauge (i.e. observers follow the unperturbed flow):

$$ds^2 = a^2 \left[(1+2\psi) d\eta^2 + (1-2\phi) dx_i dx^i \right] \tag{7}$$

with $|\phi|$, $|\psi| \ll 1$.
For perturbations in an ideal fluid, $\phi = \psi$ is the gravitational potential.

Peculiar velocities

Let $\vec{r}$ be a comoving coordinate and $\vec{x} = a\vec{r}$ the corresponding proper coordinate. Then:

$$\dot{\vec{x}} = \dot{a}\vec{r} + a\dot{\vec{r}} = Ha\vec{r} + a\dot{\vec{r}} \tag{8}$$

where $H = \dot{a}/a$ is the Hubble parameter. Then, the proper velocity is

$$\dot{\vec{x}} = H\vec{x} + \vec{v} \tag{9}$$

where the first contribution is the motion due to the Hubble flow, and $\vec{v} \equiv a\dot{\vec{r}} \equiv a\vec{u}(\ll 1)$ is the peculiar velocity, i.e. the motion with respect to the unperturbed background, due to density perturbations.

Perturbed equations

The linearized continuity, Euler and Poisson equations are:

$$\begin{aligned}
&\dot{\delta} = -\frac{1}{a}\nabla\cdot\vec{v} \\
&\dot{\vec{v}} + H\vec{v} = -\frac{1}{\bar{\rho}a}\nabla p - \frac{1}{a}\nabla\phi = -\frac{c_s^2}{a}\nabla\rho - \frac{1}{a}\nabla\phi \\
&\nabla^2\phi = 4\pi G a^2\bar{\rho}\delta
\end{aligned} \tag{10}$$

(gradients are with respect to comoving coordinates). In the second equation, we have used $\nabla p = c_s^2\nabla\rho$ where $c_s^2 \equiv \partial p/\partial\rho$ is the speed of sound and we assumed that p is a function of ρ alone.
These Newtonian equations are a good approximation to describe the evolution of small perturbations on scales which are much smaller than the Hubble length $d_H = H^{-1}$.

Plane wave decomposition

In general, we can think of a density perturbation as a superposition of Fourier modes (plane waves):

$$\delta(\vec{r}, t) = \frac{1}{(2\pi)^3} \int d^3\vec{r}\, e^{-i\vec{k}\cdot\vec{r}} \delta(\vec{k}, t) \tag{11}$$

and similarly for the other perturbed quanties. The previous equations become:

$$\begin{aligned} \dot{\delta} &= \frac{i}{a}\vec{k} \cdot \vec{v} \\ \dot{\vec{v}} + H\vec{v} &= -\frac{i}{a}\vec{k}\left(c_s^2\delta + \phi\right) \\ k^2\phi &= 4\pi G a^2 \bar{\rho}\delta \end{aligned} \tag{12}$$

(here we dropped the explicit k dependence). These are valid in the limit $k \gg H$.

Growing mode

Let us consider a non-collisional component ($p \sim 0$, $c_s \sim 0$) and combine the previous equations into a single, second-order one:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = 0 \tag{13}$$

We can then use the Friedman equation $H^2 = 8\pi G\bar{\rho}/3$ to write:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta = 0 \tag{14}$$

which is easily solved in the case $\Omega_m = 1$ (for which $3H^2/2 = 2/3t^2$), giving two solutions:

$$\delta_+ \propto t^{2/3} \propto a; \qquad \delta_- \propto t^{-1} \tag{15}$$

The second solution is clearly uninteresting. The first one is the growing mode, describing gravitation instability for a collisionless ideal fluid.

Peculiar velocities

We can use the continuity equation to write the peculiar velocity as:

$$\vec{v} = -ia\frac{\vec{k}}{k^2}\dot{\delta} = -ia\frac{\vec{k}}{k^2}Hf\delta \tag{16}$$

where we have defined

$$f \equiv \frac{\dot{\delta}}{\delta}\frac{a}{\dot{a}} = \frac{\dot{\delta}}{\delta}\frac{1}{H} \tag{17}$$

a good fitting formula for this function is

$$f \approx \Omega_m^{0.6} + (\Omega_\Lambda/70)(1 + \Omega_m/2) \sim \Omega_m^{0.6}$$

Note also that $\vec{v}$ can be related to the gravitational potential through the Poisson equation, as:

$$\vec{v} = -i\frac{2}{3}\vec{k}\frac{f}{aH}\frac{\phi}{\Omega_m} \tag{18}$$

Jeans length

If we cannot neglect pressure (i.e. collisional fluid), the equation has an extra-term:

$$\ddot{\delta} + 2H\dot{\delta} - \left(4\pi G\bar{\rho} - \frac{k^2 c_s^2}{a^2}\right)\delta = 0 \tag{19}$$

This leads to the definition of a critical wavelength (the Jeans length)

$$\lambda_J \equiv \frac{2\pi}{ak_J} = c_s\sqrt{\frac{\pi}{G\bar{\rho}}} \tag{20}$$

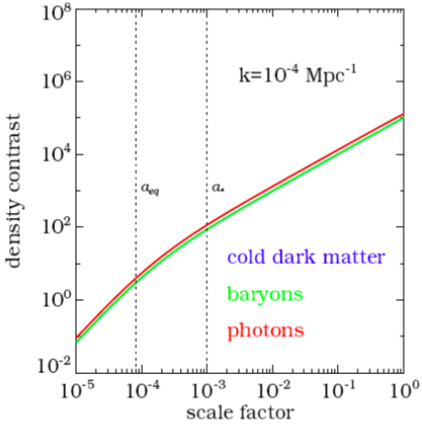
that separates the growing regime (gravity dominates for $\lambda \gg \lambda_J$) from the acoustic oscillation regime (pressure dominates for $\lambda \ll \lambda_J$).
The acoustic oscillation phase is $kc_s\eta$, so that we have a signature from the sound horizon scale $c_s\eta$.

Complications

The general treatment is more complicated. One has to consider:

- The role of dark energy (i.e. we cannot assume $\Omega_m \sim 1$ and the growth of perturbation is suppressed with respect to $\delta \propto a$)
- The role of radiation in the early universe (before matter domination, the growing mode is $\delta \propto a^2$; this only applies to wavelengths larger than the Hubble radius, while below the horizon the growth is frozen by the so called Mészáros effect)
- The coupling between species (i.e. baryons and photons before recombination)
- Possible contributions from entropy perturbation (i.e. isocurvature modes)

Numerical solutions



The transfer function

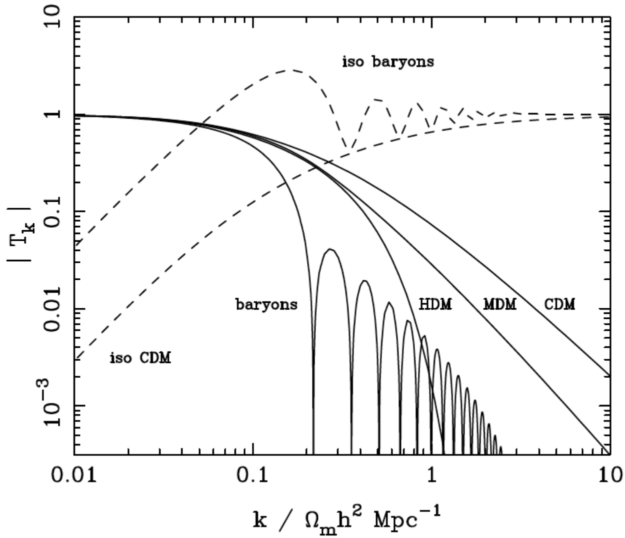
Since each mode evolves independently (in linear approximation) we can model the evolution of perturbations at different scales by a transfer function:

$$T(k) \equiv \frac{\delta(k, t = t_0)}{\delta(k, t = 0)} \tag{21}$$

This will depend on different scales:

- The horizon scale at equivalence (dark matter perturbations entering the horizon before equivalence are suppressed): $D_H(z_{eq}) \propto (\Omega_m h^2)^{-1}$Mpc;
- The free streaming comoving length (particles are relativistic when $kT \gg mc^2$ so that at that time they rapidly abandon any perturbed scale smaller than ct/a): $L_{fs} = 112(m/\mathrm{eV})^{-1}$ Mpc;
- Acoustic length (baryon perturbations oscillate before recombination).

The numerical transfer function



Statistics

The density fluctuation $\delta(\vec{x})$ is taken as a random variable. The ensemble average, $\langle\cdot\rangle$, is replaced by an average over large disconnected volumes (ergodicity).
Correlation function:

$$\xi(\vec{r}) \equiv \langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle = \frac{V}{(2\pi)^3}\int |\delta_{\vec{k}}|^2 e^{-i\vec{k}\cdot\vec{r}} d^3k \tag{22}$$

Power spectrum:

$$P(k) \equiv \langle|\delta_k|^2\rangle \tag{23}$$

Assuming isotropy:

$$\xi(r) = \frac{V}{(2\pi)^3}\int P(k)\frac{\sin kr}{kr}4\pi k^2\,dk \tag{24}$$

Variance per $\ln k$:

$$\Delta^2(k) \equiv \frac{V}{(2\pi)^3}4\pi P(k) = \frac{2}{\pi}k^3\int \xi(r)\frac{\sin kr}{kr}r^2\,dr \tag{25}$$

Correlation function

If δ follows a Gaussian stastistics (i.e., $P(\delta) \propto \exp\left(-\delta^2/2\sigma^2\right)$) the two-point correlation function (or the power spectrum) is all we need to fully characterize the density field. Otherwise, we need higher order correlation (three-point, and so on).
The observed correlation function of galaxies is:

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \tag{26}$$

with $\gamma \simeq 1.8$ and $r_0 \simeq 5h^{-1}$ Mpc. This is valid approximately for $2h^{-1}$ Mpc $\leq r \leq 30h^{-1}$ Mpc.

Primordial power spectrum

A common assumption is an initially scale-free power spectrum :

$$P(k) \propto k^n \tag{27}$$

Suppose $n = 1$ (Harrison-Zeldovich spectrum). Then $\Delta^2 \propto k^4$ and

$$k^2\phi \propto \delta \Rightarrow \Delta\phi \propto \Delta/k^2 = \text{cost.} \tag{28}$$

so that there is the same amount of fluctuation in the gravitational potential at any scale.
Inflation predicts $n \approx 1$, plus a certain amount of model dependent logarithmic running, $d \ln \Delta^2 / d \ln k$, which can (in principle) be tested.
Note that a given primordial power spectrum will evolve according to the transfer function of the model, as:

$$P(k, t_0) = P(k)T^2(k) \tag{29}$$

Non-linearity and bias

As the evolution enters the non-linear regime, different Fourier modes get mixed, and the density contrast departs from Gaussian statistics. Furthermore, when bound structures form, other physical processes take place (shocks, heating, cooling, and so on) so that the object distribution does not necessarily follow the mass distribution.
This is quantified by the linear bias parameter b, so that, e.g.:

$$\frac{\delta n(\vec{r})}{\bar{n}} = b\frac{\delta\rho(\vec{r})}{\bar{\rho}} \tag{30}$$

Accurate treatment must be done numerically, through n-body simulations.

Bulk flows

The average square peculiar velocity on a scale R is

$$\langle v^2 \rangle_R = \frac{H_0^2 f^2}{2\pi^2} \int P(k) W^2(kR) dk \tag{31}$$

where W is the window function used to identify the scale R. The quantity $\bar{v} \equiv \langle v^2 \rangle_R^{1/2}$ is called the bulk flow. If one could measure $\bar{v}$ and $P(k)$ separately, one could estimate f, that depends on the model, and in particular on Ω_m.
In practice, since we measure

$$P_g(k) = b^2 P(k) \tag{32}$$

the estimated quantity is $f/b \sim \Omega_m^{0.6}/b$.

Redshift distorsions

The redshift of galaxies is affected by peculiar velocities:

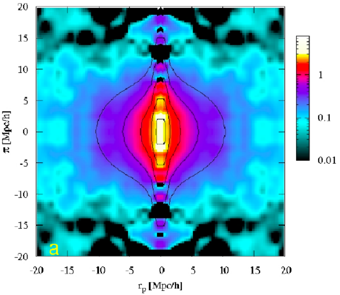
$$cz = H_0 d + v \tag{33}$$

If the position $\vec{r}$ is decomposed into a parallel ($\vec{\pi}$) and perpendicular ($\vec{r}_p$) contribution with respect to the line of sight, the correlation function $\xi(\vec{r}_p, \vec{\pi})$ will be distorted along the $\vec{\pi}$ direction.
The projected correlation function is:

$$w_p(r_p) = 2\int_0^\infty d\pi \xi(r_p, \pi) = 2\int_{r_p}^\infty dy \frac{y\xi_r(y)}{y^2 - r_p^2} \tag{34}$$

Comparing the real-space theoretical ξ_r with the projected one, the quantity f/b can be constrained.

Redshift distorsions



Guzzo et al. 2005, using $5,895$ galaxies at effective redshift 0.77 from the Vimos VLT Deep Survey, find $f = 0.91 \pm 0.36$. (Cfr. 2dF Galaxy Redshift Survey, $220,000$ galaxies at $z \approx 0.15$, find $f = 0.49 \pm 0.14$.) This is consistent with $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$.

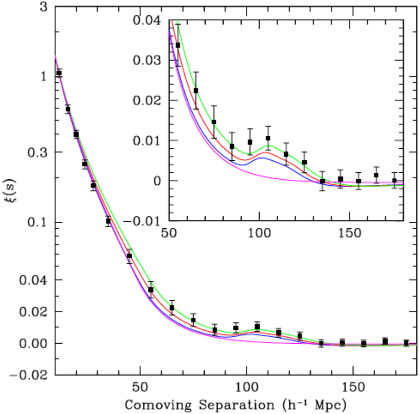
Baryon acoustic oscillations (BAO)

The transverse ($r_\perp$) and line-of-sight ($r_\parallel$) size of a certain object are related to its angular ($\Delta\theta$) and redshift (Δz) "size" by:

$$r_\parallel = \frac{c\Delta z}{H(z)}$$
$$r_\perp = (1+z)D_A(z)\Delta\theta \qquad (35)$$

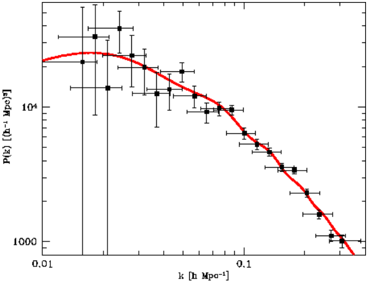
Then, if we observe a certain known scale at redshift z, we can reconstruct $H(z)$ and $D_A(z)$ (and the cosmological parameters).
One scale we know well is the sound horizon scale at recombination $c_s\eta_{rec}$.
This is left imprinted both in the CMB and in the matter power spectrum.

Detection of BAO from SDSS



(from Eisenstein et al 2005)

Matter power spectrum



The matter power spectrum has been measured, e.g. by SDSS, 2dF. We have a reliable estimate of the matter density from the shape:

$$0.24 \lesssim \Omega_m \lesssim 0.31$$

The amplitude is uncertain because of bias

2. Gravitational lensing

Why lensing?

The (dark) matter distribution in the universe perturbs the light path of distant sources.
This is the most direct way to probe the large-scale distribution of matter

- no bias effect, direct probe of the gravitational potential
- the expected deviation is small, calculations can be done in linear theory (just as the CMB)
- but: observationally challenging

Light propagation

Unperturbed spacetime metric, written as

$$ds^2 = cdt^2 - a^2(t)\left[dw^2 + f_K^2(w)(d\phi^2 + \sin^2\theta d\theta^2)\right] \tag{36}$$

with

$$f_K(w) = \begin{cases} \sin(\sqrt{K}w)/\sqrt{K} & (K > 0) \\ w & (K = 0) \\ \sinh(\sqrt{-K}w)/\sqrt{-K} & (K < 0) \end{cases} \tag{37}$$

and $K \equiv (H_0/c)^2(\Omega_m + \Omega_\Lambda - 1)$ (curvature parameter). Light propagation is governed by the equation:

$$\frac{d^2\vec{x}}{dw^2} + Kx = 0 \tag{38}$$

where $\vec{x}$ is the comoving separation between neighbouring rays. This equation has solutions $\vec{x} = \vec{\theta} f_K(w)$.

Light deflection in a perturbed universe

Let us introduce small inhomogeneities modeled by the gravitational Newtonian potential $|\phi| \ll c^2$, on scales much smaller than the curvature scale.
Then, the metric is approximately Minkowskian, with first-order Newtonian corrections, so that:

$$n = 1 - \frac{2\phi}{c^2} \tag{39}$$

is the effective local refraction index: then, Fermat's theorem $\delta \int n \, dl = 0$ yields the expression for light deflection near a perturbation:

$$\frac{d^2\vec{x}}{dw^2} = -\frac{2}{c^2} \nabla_\perp \phi \tag{40}$$

Perturbed propagation equation

Now, the initial equation for the unperturbed light paths will get an extra contribution describing the light deflection. We just have to incorporate the relative deviation between adjacent rays, so that:

$$\frac{d^2\vec{x}}{dw^2} + K\vec{x} = -\frac{2}{c^2}\Delta(\nabla_\perp\phi) \tag{41}$$

The solution is obtained using Green's function method, and reads:

$$x(\vec{\theta}, w) = f_K(w)\vec{\theta} - \frac{2}{c^2}\int_0^w dw' f_K(w-w')\Delta\left[(\nabla_\perp\phi(\vec{x}(\vec{\theta}, w'), w'))\right] \tag{42}$$

Born approximation

Assuming a small deflection with respect to the unperturbed path, we can take $\vec{x}(\vec{\theta}, w') \approx f_K(w')\vec{\theta}$ in the integrand, and use the Born approximation $\Delta(\nabla_\perp \phi) \approx \nabla_\perp(\Delta\phi)$. Defining the deflection angle

$$\vec{\alpha}(\vec{\theta}, w) \equiv \frac{f_K(w)\vec{\theta} - \vec{x}(\vec{\theta}, w)}{f_K(w)} \ll 1 \tag{43}$$

we obtain:

$$\vec{\alpha}(\vec{\theta}, w) = \frac{2}{c^2} \int_0^w dw' \frac{f_K(w - w')}{f_K(w)} \nabla_\perp \phi(f_K(w')\vec{\theta}, w') \tag{44}$$

(where we renamed $\Delta\phi \to \phi$).

Deflection angle in flat models

When $f_K(w) = w$, defining $y = w'/w$ we obtain

$$\vec{\alpha}(\vec{\theta}, w) = \frac{2w}{c^2} \int_0^1 dy(1-y) \nabla_\perp \phi(wy\vec{\theta}, wy) \tag{45}$$

Summarizing, this is the deflection from the unperturbed light ray in the $\vec{\theta}$ direction at comoving distance w, under the assumptions:

- localized inhomogeneities (i.e. on small scales with respect to the curvature scale) to a homogeneous and isotropic flat background;
- weak field, $\phi \ll c^2$;
- small deflection angles.

Distorsion

For a thin lens (located at w =const.), the relation between the unperturbed angle $\vec{\beta}$ and the the deflection $\vec{\alpha}$ in the direction θ is

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) \tag{46}$$

The net effect of lensing on an extended source is the distorsion of its shape. This can be quantified by the Jacobian matrix:

$$A \equiv \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \tag{47}$$

It can be shown that A can be rewritten as

$$A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} \tag{48}$$

Convergence and shear

The scalar quantity

$$\kappa(\vec{\theta}) \equiv \frac{1}{2}\nabla_{\vec{\theta}} \cdot \vec{\alpha}(\vec{\theta}) \tag{49}$$

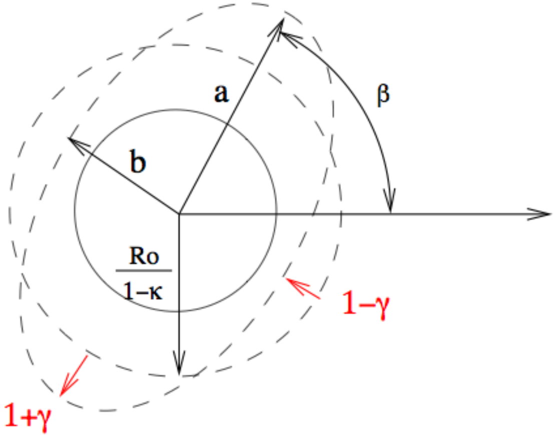
is called convergence. The pseudo-vector (γ_1, γ_2) is called shear. A circular source of radius r is mapped into an ellipse having axes:

$$a = \frac{r}{1-\kappa-\gamma}; \qquad b = \frac{r}{1-\kappa+\gamma} \tag{50}$$

(where $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$). The magnification is defined by:

$$\mu(\vec{\theta}) \equiv \frac{1}{\det\left[A(\vec{\theta})\right]} = \frac{1}{(1-\kappa)^2 - \gamma^2} \tag{51}$$

Observations



(from Munshi et al 2006)

Effective convergence

In analogy with the thin lens definition, we can define

$$\kappa_{\rm eff}(\vec{\theta}, w) \equiv \frac{1}{2}\nabla_{\vec{\theta}} \cdot \vec{\alpha}(\vec{\theta}, w) \tag{52}$$

so that

$$\kappa_{\rm eff}(\vec{\theta}, w) = \frac{1}{c^2}\int dw' \frac{f_K(w')f_K(w-w')}{f_K(w)}\nabla_{\perp}^2 \phi(f_K(w')\vec{\theta}, w') \tag{53}$$

we can replace $\nabla_{\perp}^2 \to \nabla^2$ since $\partial^2\phi/\partial z^2 = 0$ (i.e. the derivative along the light path is zero on average). We can then use the Poisson equation $\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta = (3H_0^2\Omega_m/2a)\delta$ to write

$$\kappa_{\rm eff}(\vec{\theta}, w) = \frac{3H_0^2\Omega_m}{2c^2}\int dw' \frac{f_K(w')f_K(w-w')}{f_K(w)} \frac{\delta(f_K(w')\vec{\theta}, w')}{a(w')} \tag{54}$$

Projected convergence

We can average over distances (or redshifts)

$$\bar{\kappa}_{\text{eff}}(\vec{\theta}) \equiv \int dw G(w) \kappa_{\text{eff}}(\vec{\theta}, w) \tag{55}$$

where $G(w)dw = p(z)dz$ is the redshift probability distribution of sources. Then

$$\bar{\kappa}_{\text{eff}}(\vec{\theta}) = \frac{3H_0^2 \Omega_m}{2c^2} \int dw W(w) f_K(w) \frac{\delta(f_K(w')\vec{\theta}, w')}{a(w')} \tag{56}$$

with

$$W(w) \equiv \int dw' G(w') \frac{f_K(w' - w)}{f_K(w')} \tag{57}$$

Limber's equation

As usual, we are interested in the correlation function

$$\xi_\kappa(\vec{\gamma}) = \langle \kappa(\vec{\theta} + \vec{\gamma})\kappa(\vec{\theta})\rangle \tag{58}$$

or equivalently, the projected power spectrum

$$P_\kappa(l) = \int d^2\gamma \xi_\kappa(\gamma) \exp{(il\gamma)} \tag{59}$$

After some algebra, this can be related to the projected power spectrum of density fluctuations P_δ by

$$P_\kappa(l) = \frac{9H_0^4\Omega_m^2}{4c^2} \int dw \frac{W^2(w)}{a^2(w)} P_\delta\left(\frac{l}{f_K(w)}, w\right) \tag{60}$$

Magnification correlation

Using the definition of magnification we get

$$\mu(\theta, w) \equiv \frac{1}{\det\left[A(\theta, w)\right]} \tag{61}$$

$$\approx 1 + \nabla_\theta \cdot \alpha = 1 + 2\kappa_{\rm eff}(\theta, w) \tag{62}$$

Then, the correlation function of the magnification is simply:

$$\xi_\mu(\gamma) = 4\xi_\kappa(\vec{\gamma}) \tag{63}$$

and the typical magnification can be estimated by the r.m.s value

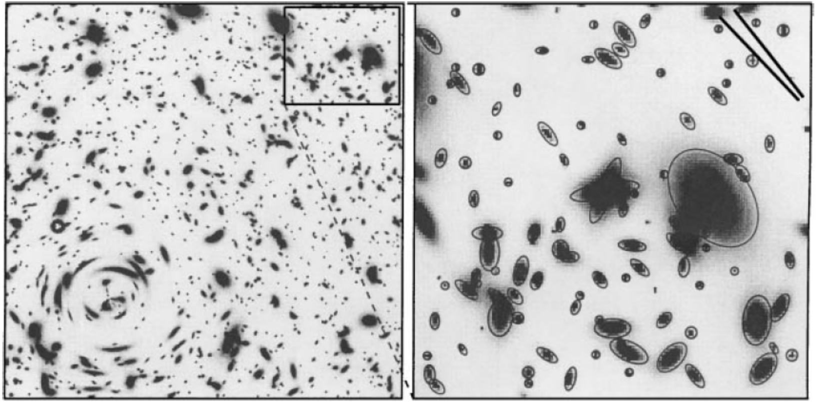
$$\langle \delta\mu^2 \rangle^{1/2} = \xi_\mu^{1/2}(0) = \left[\int \frac{l\,dl}{2\pi} P_\kappa(l)\right]^{1/2} \tag{64}$$

Observations

Some key features of weak lensing of galaxies:

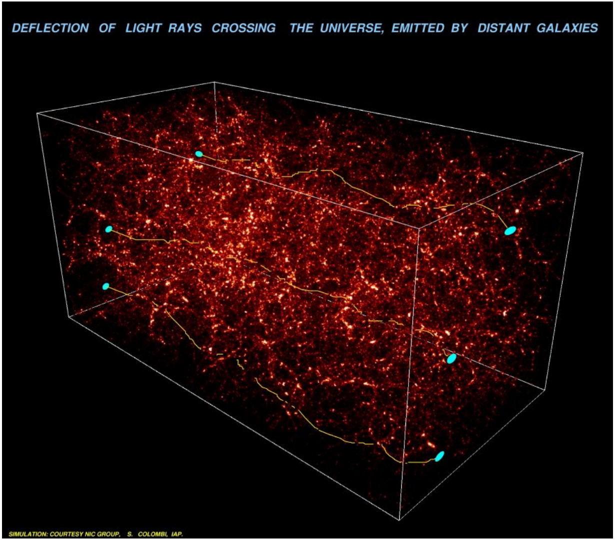
- weak lensing cannot be observed on single galaxies: one cannot separate intrinsic ellipticity from lens-induced distorsion;
- however, the distorsion on nearby galaxies should be correlated, while their intrinsic ellipticity should be randomly oriented;
- we need high galaxy density, and very deep surveys, to average over large number of faint galaxies:
- control of systematics (atmosphere, anisotropy of the point-spread function, and so on);
- typical cosmic shear value is 1% on scales of a few arcminutes
- number of galaxies to average over is roughly $N > (\sigma_\epsilon/\epsilon)^2$, where ϵ is the ellipticity to measure and σ_ϵ the dispersion.

Convergence and shear



(from Mellier 1999)

Observations



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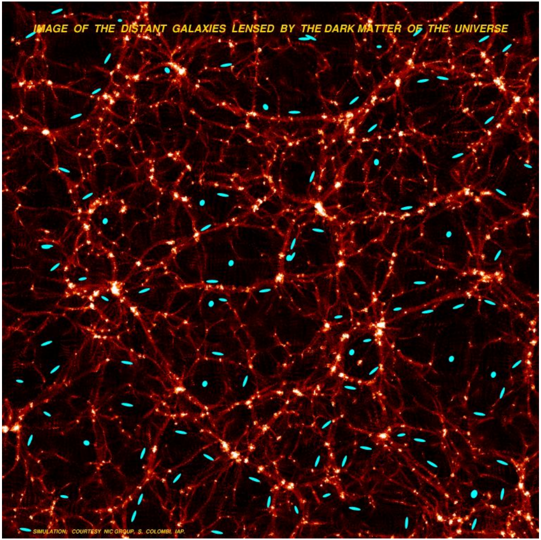

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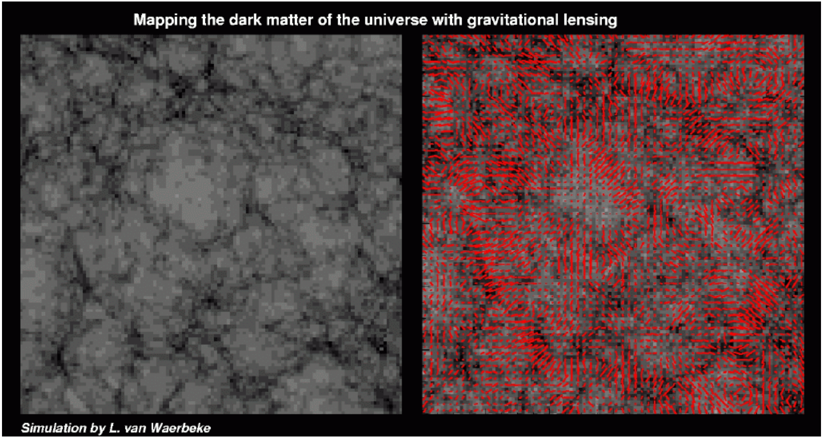

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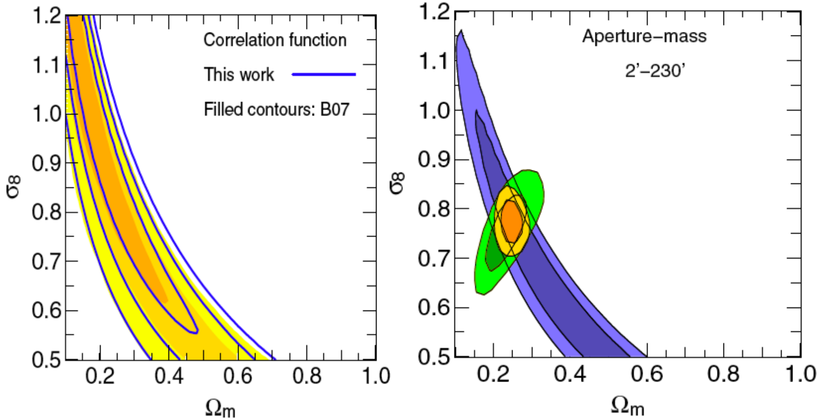


(from Fu et al. 2008 - CFHTLS data)

3. Combining CMB with the LSS

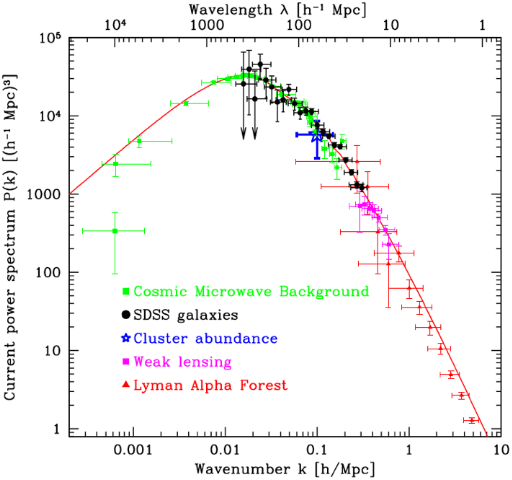
The importance of complementarity

We can probe the evolution of the universe and the growth of cosmic structures at different epochs using different observables

- the CMB ($z \sim 1000$)
- the 21 cm emission ($z \sim 10 - 20$)
- the Ly-α forest ($z \sim 2 - 4$)
- weak lensing ($z \sim 1$)
- galaxy surveys ($z \sim 1$)

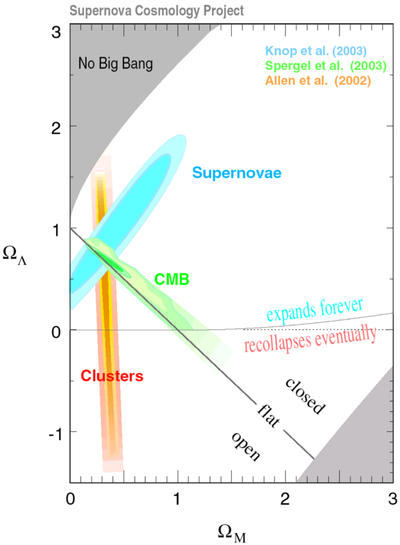
Putting together different probes is crucial, for example to understand the nature of dark energy, which dominates only very recently.

The matter power spectrum

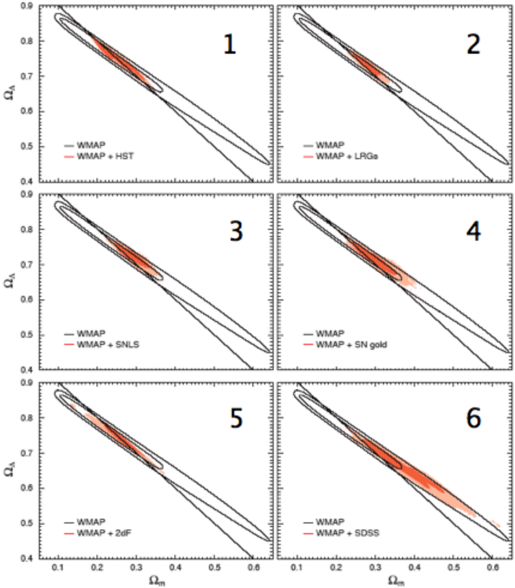


(Tegmark)

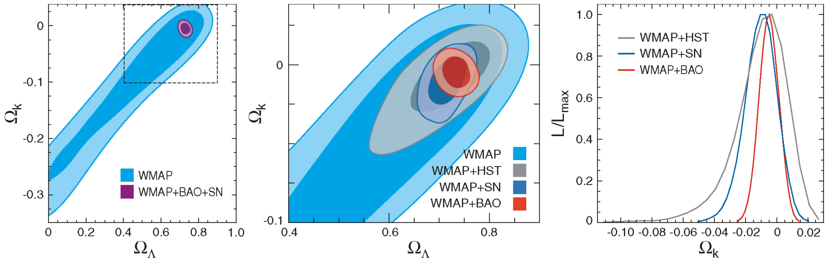
The matter-energy budget



The matter-energy budget



The matter-energy budget



(Komatsu et al)

Dark energy models

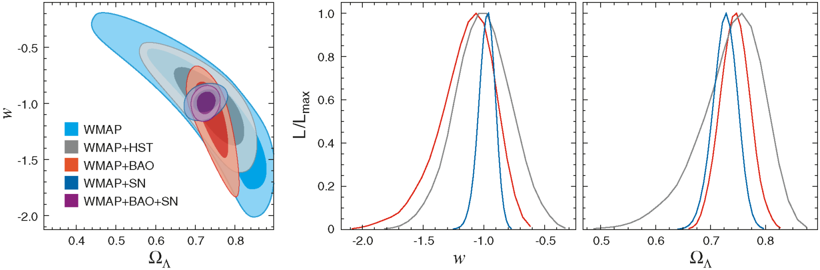
We can obtain an accelerated expansion by means of a generic nearly-homogeneous component with $w \equiv p/\rho < -1/3$.
This can be, for example, a generic scalar field with

$$w(z) = \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

similar to the one that might have driven inflation (albeit with a different vacuum energy scale).
The dynamic of the field might alleviate the fine-tuning problem.
Also, the speed of sound and clustering properties are different in different models, e.g.:

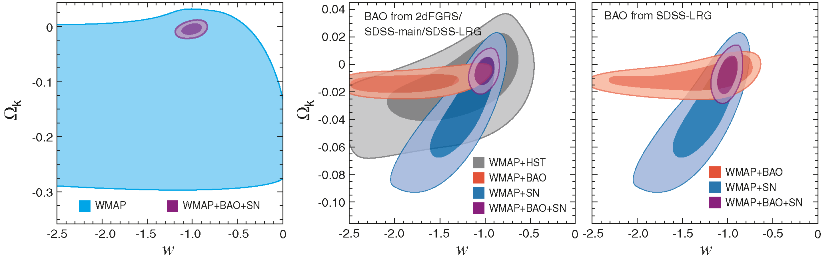
- quintessence: $c_s^2 = 1$
- Chaplygin gas: $c_s^2 = -\alpha w$
- unified dark matter: $c_s^2 = 0$

Constraints on dark energy



(Komatsu et al)

Dark energy and curvature



(Komatsu et al)

The integrated Sachs-Wolfe effect

The CMB photons are affected by changes of the gravitational potential along the line of sight (integrated Sachs-Wolfe, ISW)

$$\frac{\Delta T}{T} = 2\int_{\eta_\star}^{\eta_0} d\eta \frac{\partial \phi(\eta)}{\partial \eta} \qquad (65)$$

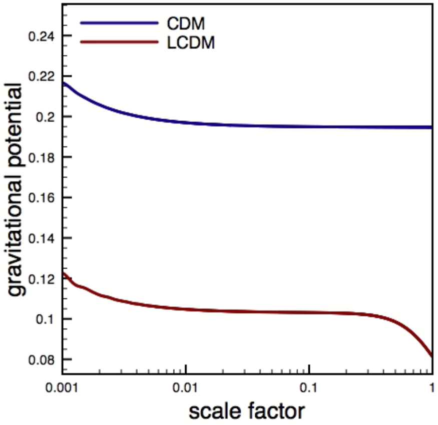
Remember that

$$k^2\phi = 4\pi G a^2 \bar{\rho}\delta \Rightarrow \phi \propto \delta/a \qquad (66)$$

Thus, ϕ =const. during matter domination ($\delta \propto a$) and there is no ISW effect. But we have

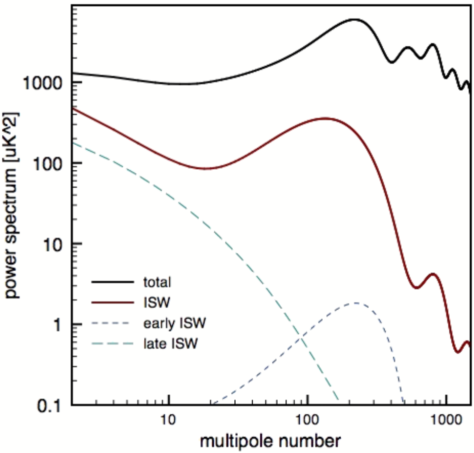
- early ISW (at equivalence, $z \sim 10^4$ during the transition between radiation and matter domination
- late ISW (at $z \lesssim 1$) when dark energy starts dominating and the expansion starts accelerating: this is an important tool to probe DE models

Gravitational potential decay

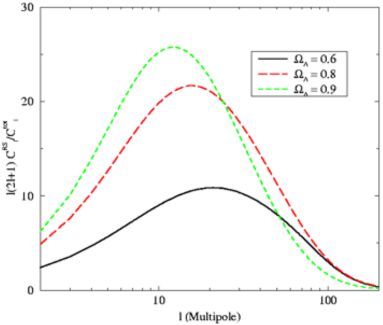


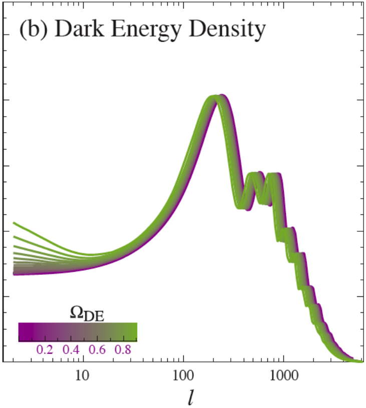
The ISW signal

- subdominant contribution (only about 10% of the total)
- peaks at low multipoles, where cosmic variance is bigger

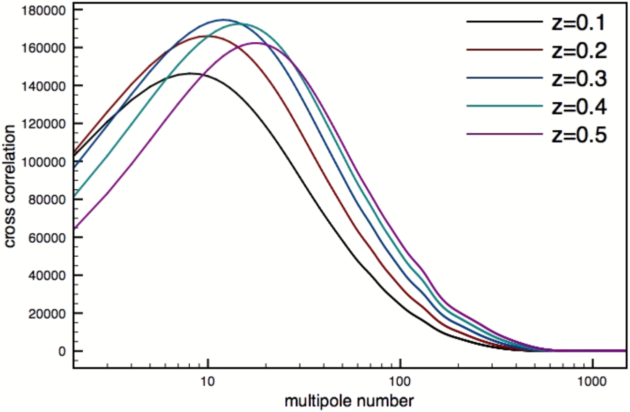


The ISW and dark energy

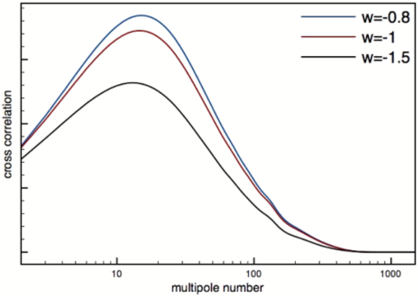




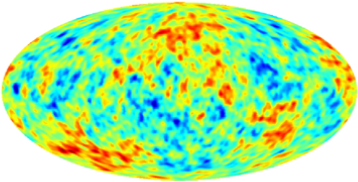
ISW vs redshift



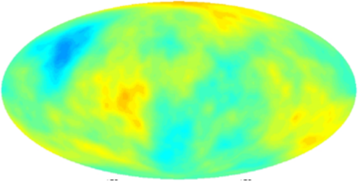
The ISW and dark energy



Primary CMB signal vs ISW



CMB signal at $z \sim 1000$



ISW signal at $z \sim 1$

How can the subdominant ISW signal be disentangled from the primary CMB signal?
Cross-correlate the CMB with a tracer of dark matter (tracking the gravitational potential) at low redshift:

$$C_{TM}(\theta) = \langle \Delta T(\hat{\gamma}_1) \delta(\hat{\gamma}_2) \rangle$$

Density tracers

We can trace the underlying density by some (projected) source count map

$$\delta n(\hat{\gamma}) = \int dz\, b(z) \frac{dN}{dz} \delta(r(z)\hat{\gamma}, z) \tag{67}$$

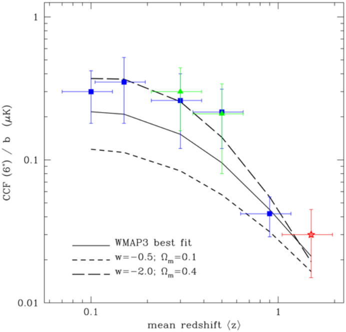
where b is the bias and dN/dz is the catalogue selection function. Both quantities has to be known (or estimated) in order to analyze the data.

ISW detection

As of today, the ISW signal has been detected (at $\gtrsim 3\sigma$) using a number of low redshift LSS tracers

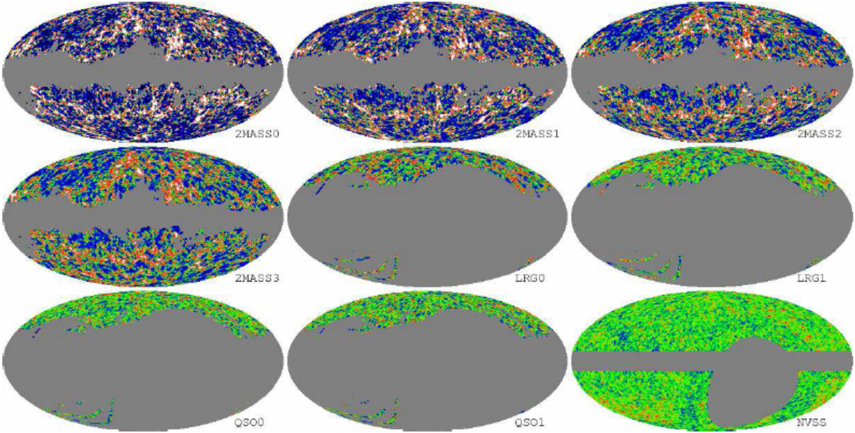
- X-ray background
- QSO's (SDSS)
- Radio galaxies (NVSS, FIRST)
- Infrared galaxies (2MASS)
- Optical galaxies (APM, SDSS)

Giannantonio et al



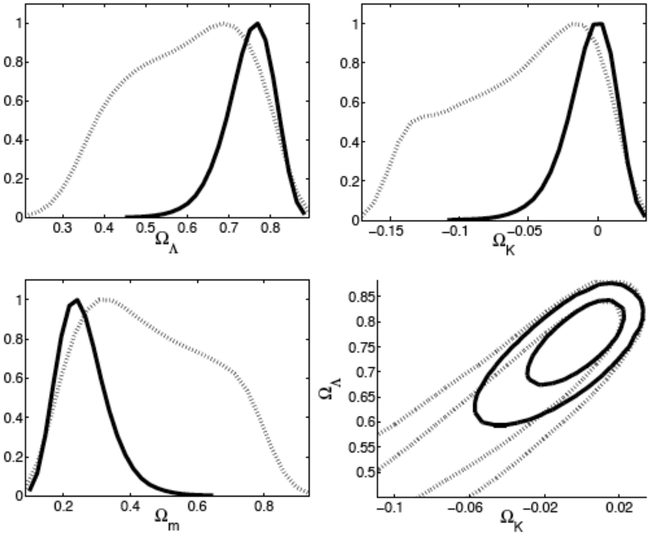
The results are consistent with the concordance LCDM model

Examples of density tracers



(from Ho et al 2008)

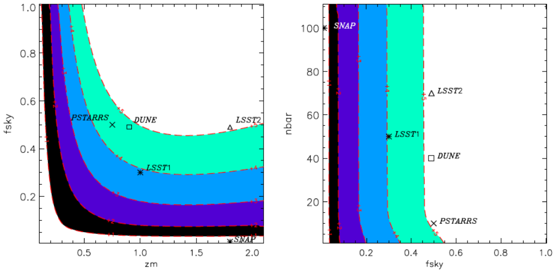
Cosmological constraints



(from Ho et al 2008)

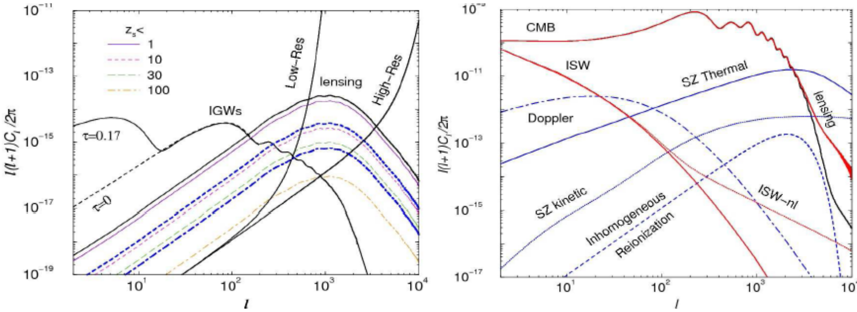
Optimizing surveys

	f_{sky}	z_{med}	nbar (arcmin^{-2})
DUNE(1)	49%	0.9	40
LSST-1(2)	30%	1	50
LSST-2(3)	49%	1.8	70
SNAP(4)	1 %	1.8	100
PanSTARRS(5)	50%	0.75	5
PLANCK	f_{sky} = 80%	θ_{beam} = 7 arcmin	ω_T^{-1} = 4e^{-17}



(from Douspis et al 2008)

LSS effects on the CMB



(Munshi et al)

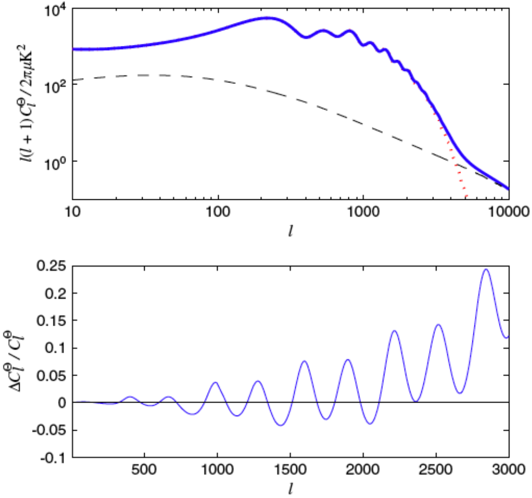
Gravitational lensing of the CMB

The intervening distribution of matter also leaves an imprint on the CMB by weak gravitational lensing. The main features of these effect are:

- The source is at very well-known redshift ($z \sim 1000$) and gaussian: this provides a test of high redshifts (where no galaxy can be observed) and a is free from intrinsic alignments
- Smoothing of acoustic oscillation in T and E modes
- Generation of spurious B modes from E polarization
- Generation of large-scale non-Gaussianity
- May help reducing some degeneracies (e.g. geometrical) leading to better constraints on cosmological parameters (e.g. dark energy)

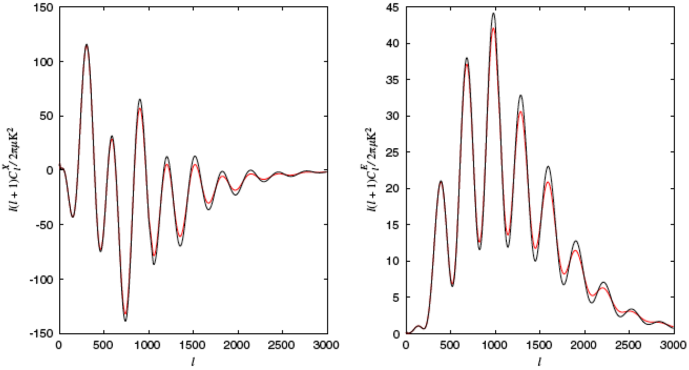
But: observationally very challenging.

Gravitational lensing of the CMB (temperature)



(Lewis & Challinor)

Gravitational lensing of the CMB (polarization)



(Lewis & Challinor)

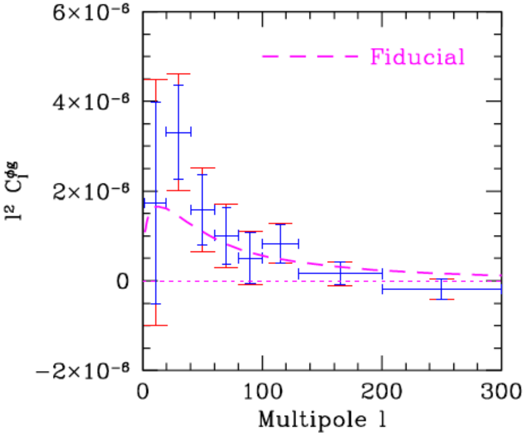
Current status

To extract the lensing signal from a CMB maps, one needs very high S/N at high l's ($l \gtrsim 1000$) which is not currently available. Going to higher order statistics can help.
However, with present CMB data, only two attempts were made, by cross-correlating CMB lensing with LSS

- Hirata et al 2008 used SDSS data
- Smith et al 2008 used NVSS data

they both find only marginal evidence for lensing.

Current status



(Smith et al)