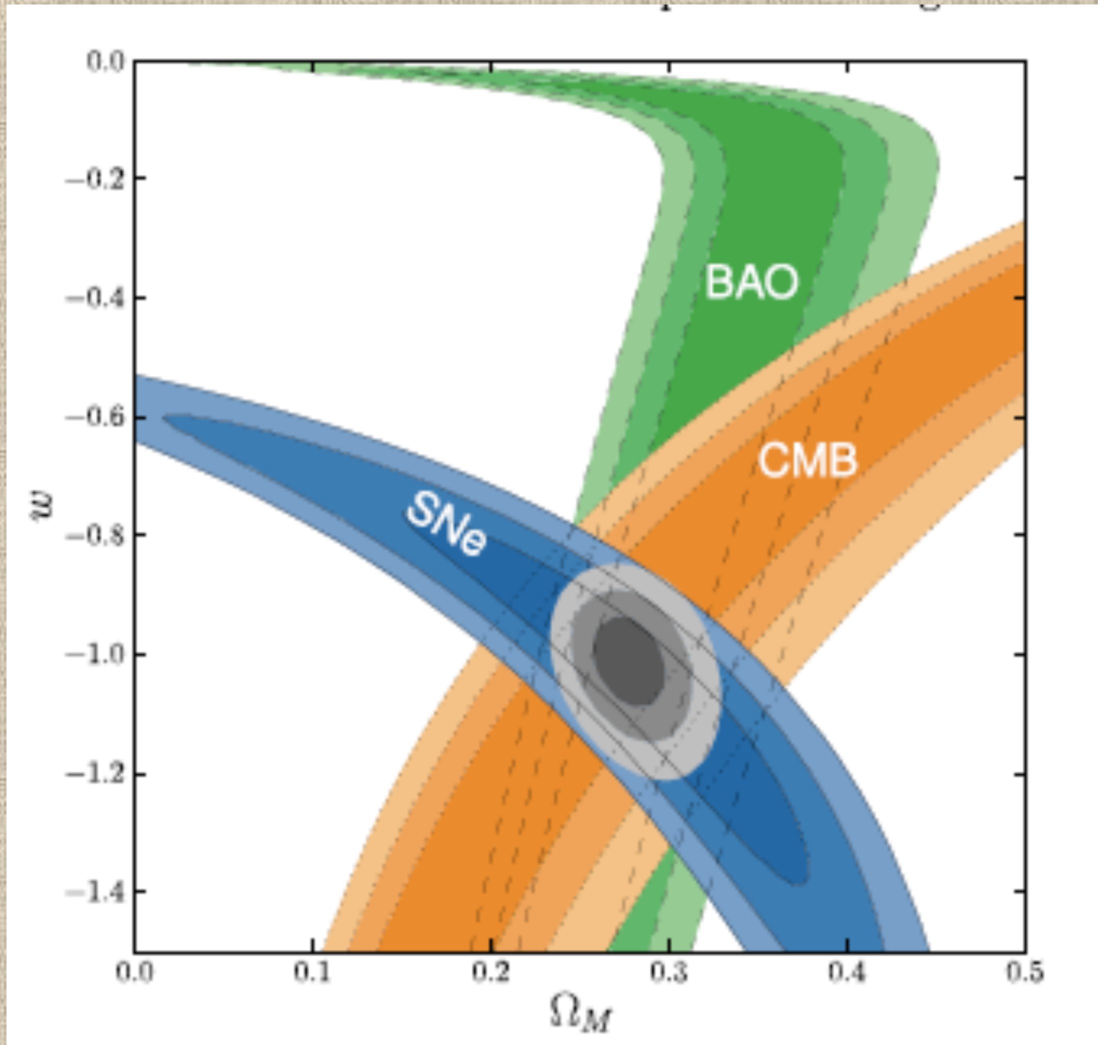
Lecture 2

Summary of Lecture 1

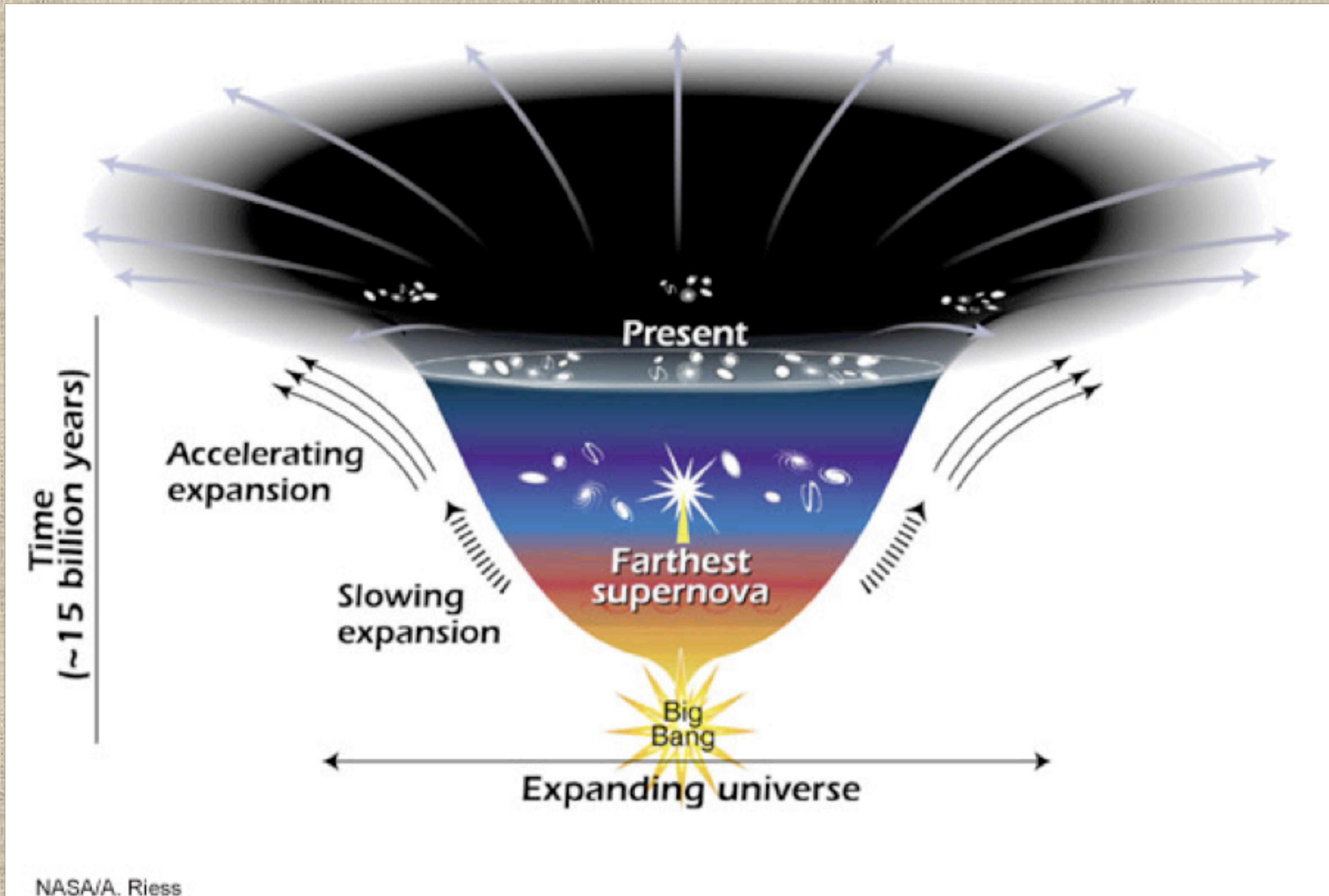
- Hubble diagram of distant standard candles (Sn Ia) plus Friedmann equation requires extra w=-1fluid in the cosmic budget $H^2(z) = H_0^2\{\Omega_m(1+z)^3 + \Omega_\Lambda\}$
- Formally, this coincides with Einstein's cosmological constant Λ
- This corresponds to energy density remaining constant with expansion, i.e. negative pressure
- Value of density in Λ is enough to make Universe accelerate in recent times
- Value of density in Λ is right what we need to make geometry flat
- It's the missing piece of the puzzle
- Large-scale structure contains imprint of cosmological parameters: we can extract them through statistical analyses of clustering in large surveys
- Relic feature of early Universe baryon-photons interaction provides standard ruler: Baryonic Acoustic Oscillations in the galaxy power spectrum
- First detection of BAO in currently largest surveys provides confirmation of SnIa results

Cosmic concordance

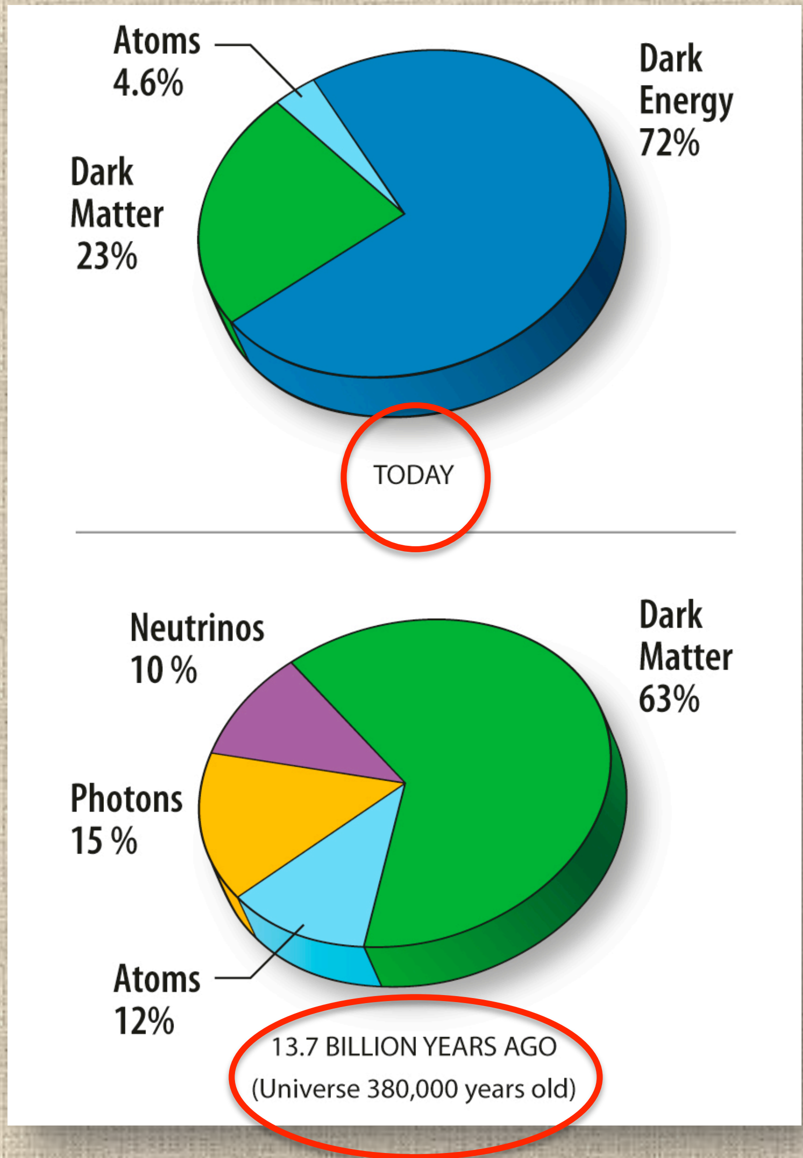


Amanullah et al. 2010 (Union supernovae)

A standard cosmological model



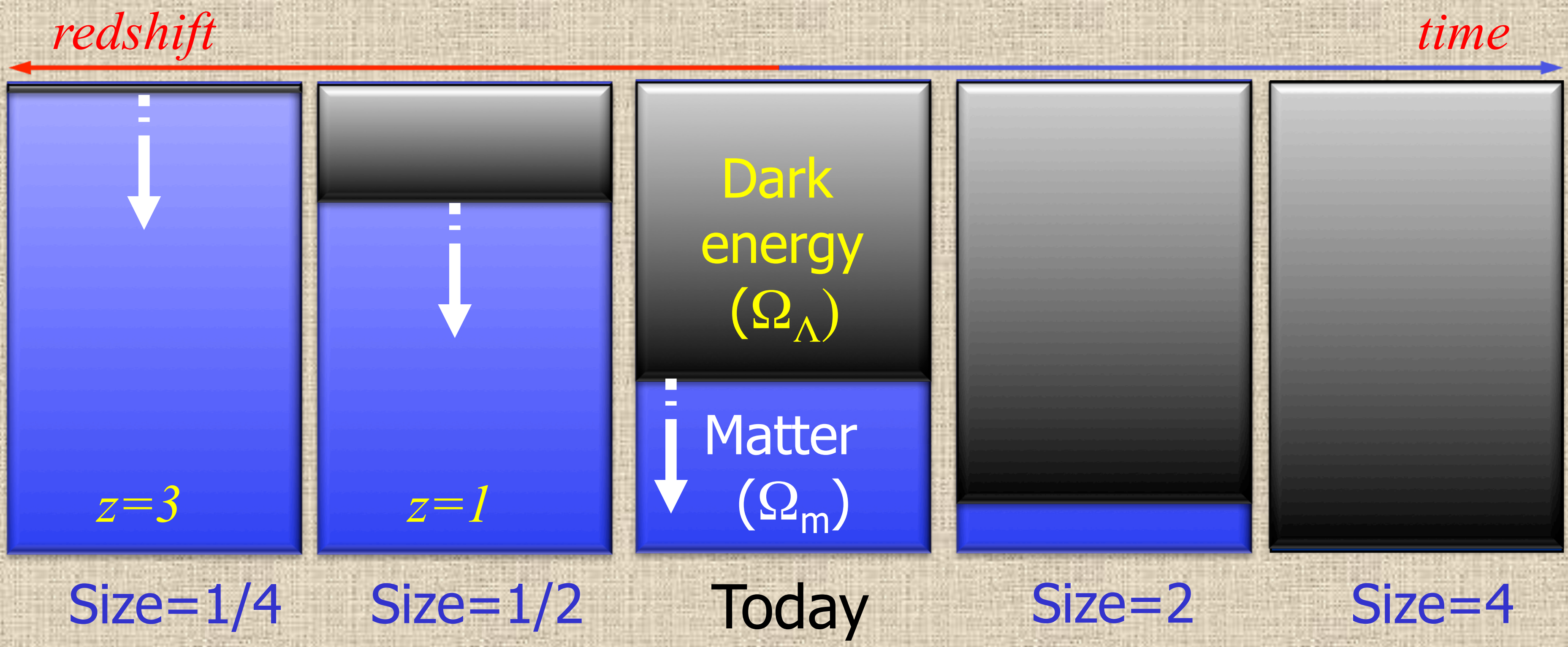
The cosmological constant (or DE) is dominant today



Fine-tuning and Cosmic Coincidence

• If Λ has something to do with vacuum energy (a "quantum zero-point"), then it is a factor ~10^{120} too small (Weinberg 1989)

• Also, it is suspiciously fine-tuned: why Ω_m and Ω_Λ so similar today?



• Anthropic solution?

• Or perhaps *w = w(z)* ? --> e.g. *quintessence*, a cosmic scalar field (Wetterich 1988), or some complex interaction between DM and DE? (see L. Amendola lectures)

DOES THEN UNDERSTANDING COSMIC ACCELERATION SIMPLY MEAN FINDING WHETHER $w=w(z)$?

Not only: we need to look at both sides of the story…

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^2} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

?

Modify gravity theory [e.g. ***R* → *f(R)***]

Add dark energy

"…the Force be with you"

And there is also a third possibility….

→ There is no dark energy or modification of GR whatsoever: we are just applying GR the wrong way to the inhomogeneous Universe (*backreaction*, e.g. Buchert 2008, GeRGr, 40, 467)

$$\left\langle G_{\mu\nu}\left(T_{\mu\nu}\right)\right\rangle \neq G_{\mu\nu}\left(\left\langle T_{\mu\nu}\right\rangle\right)$$

$$\left(G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)$$

→ Very difficult to test observationally

In any case, measuring $w(z)$ is not the end of the story…

→ How could we distinguish between (at least) the "dark energy" (cosmological constant, quintessence, …) and "modified gravity" options?

→ We need to look at how density fluctuations grow in the expanding Universe

Describing the growth of density fluctuations

Standard equations (Jeans theory) in inertial frame, neglecting pressure gradients

$$\text{Continuity equation}: \quad \left(\frac{\partial \rho}{\partial t}\right)_{\mathbf{r}} + \vec{\nabla}_{\mathbf{r}} \cdot (\rho \mathbf{u}) = 0 \quad \text{(mass conservation)}$$

$$\text{Euler equation}: \quad \left(\frac{\partial \mathbf{u}}{\partial t}\right)_{\mathbf{r}} + (\mathbf{u} \cdot \vec{\nabla}_{\mathbf{r}})\mathbf{u} = -\vec{\nabla}_{\mathbf{r}} \Phi \quad \text{(momentum conservation)}$$

$$\text{Poisson equation}: \quad \nabla_{\mathbf{r}}^2 \Phi = 4\pi G \rho.$$

Transforming these to *comoving coordinates* ***x*** (and *peculiar velocities* ***v***) and linearizing

$$\text{Continuity}: \quad \frac{\partial \delta}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot [(1+\delta)\mathbf{v}] = 0$$

$$\text{Euler}: \quad \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{a}(\mathbf{v} \cdot \vec{\nabla})\mathbf{v} + \frac{\dot{a}}{a}\mathbf{v} = -\frac{1}{a}\vec{\nabla}\phi$$

$$\text{Poisson}: \quad \nabla^2 \phi = 4\pi G \rho_b a^2 \delta \quad \text{with} \quad \phi \equiv \Phi - \frac{2}{3}\pi G \rho_b a^2 x^2.$$

$$\delta = (\rho - \bar{\rho}) / \bar{\rho} << 1$$

Describing the growth of density fluctuations

Combining these equations one gets

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}\delta$$

or

$$\ddot{\delta} + 2H(t)\dot{\delta} = 4\pi G\bar{\rho}\delta$$

which has a growing solution (plus a decaying one, which we can ignore after sufficient time)

$$\delta^{+}(\bar{x},t) = \hat{\delta}(\bar{x})D(t)$$

D(t) normalized to a reference redshift (e.g. CMB) is the GROWTH FACTOR

$$G(t) = \frac{D(t)}{D(t_{CMB})}$$

and we define the GROWTH RATE

$$f \equiv \frac{d\ln G}{d\ln a}$$

• The growth equation (and thus the growth rate) depends not only on the expansion history $H(t)$ (and thus on w) but also on the gravitation theory (e.g. Lue et al. 2004)

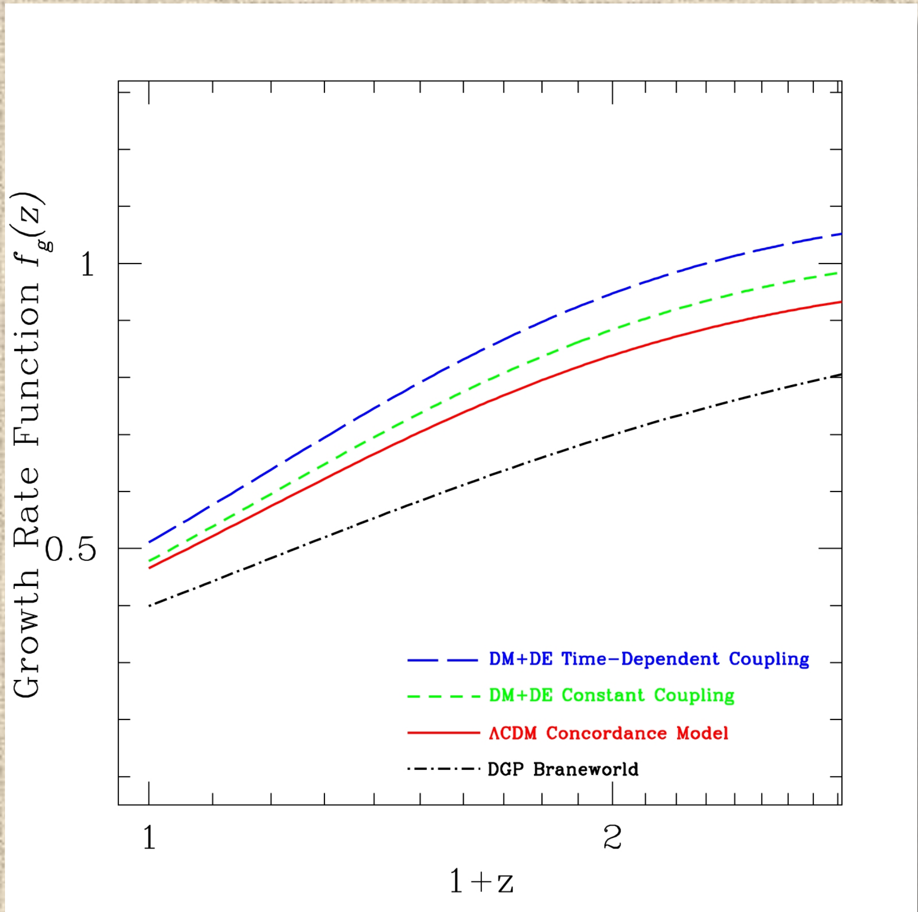
• Measuring the growth rate $f(z)$ can break the degeneracy between dark energy and modified gravity (however, see Kunz & Sapone 2007)

For a wide variety of models

$$f(z)=[\Omega_m(z)]^\gamma$$

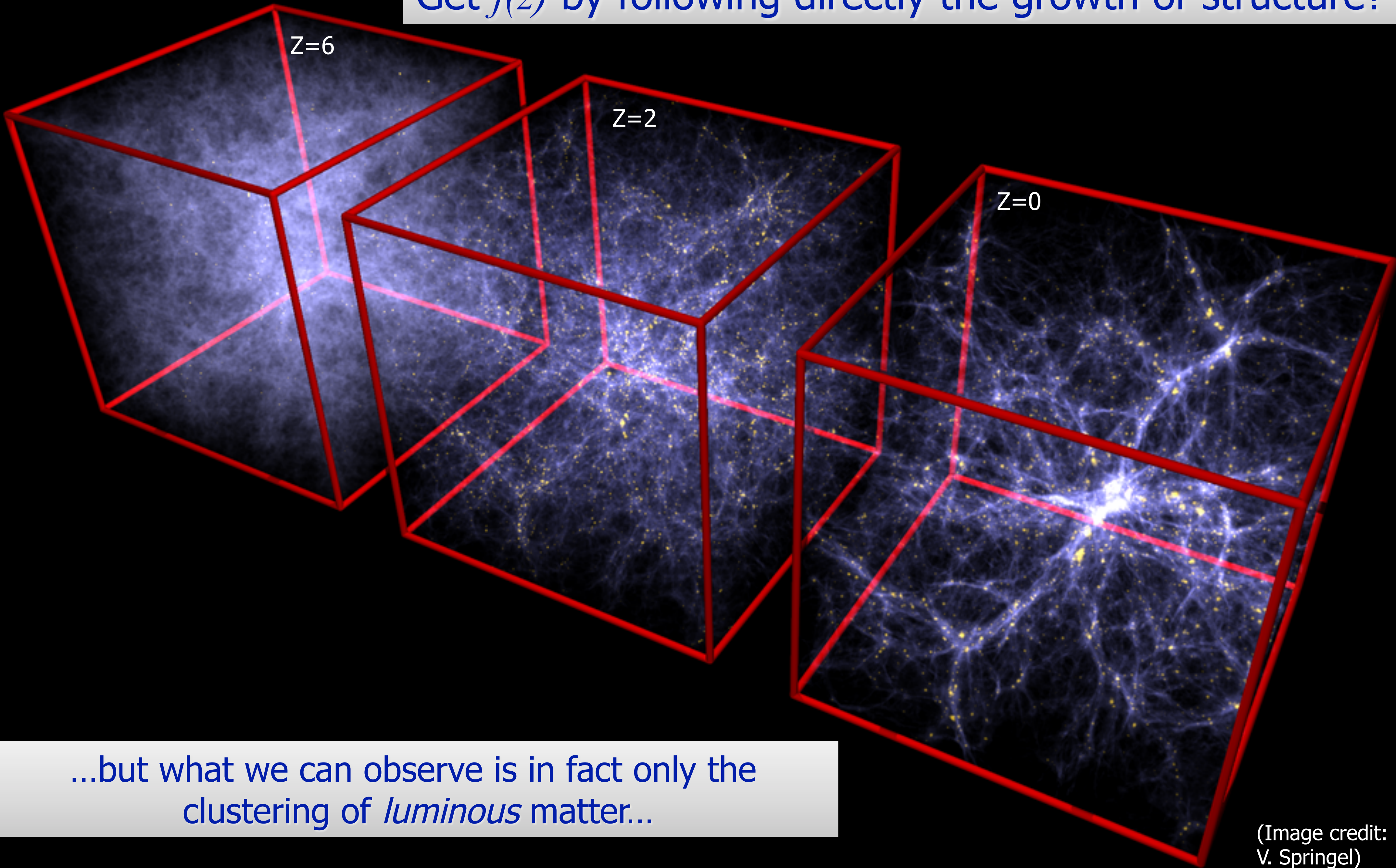
(Wang & Steinhardt 1998, Amendola et al. 2005, Linder 2005)

e.g.
$\gamma=0.55$ for standard Λ
$\gamma=0.68$ for DGP braneworld



How can we measure $f(z)$?

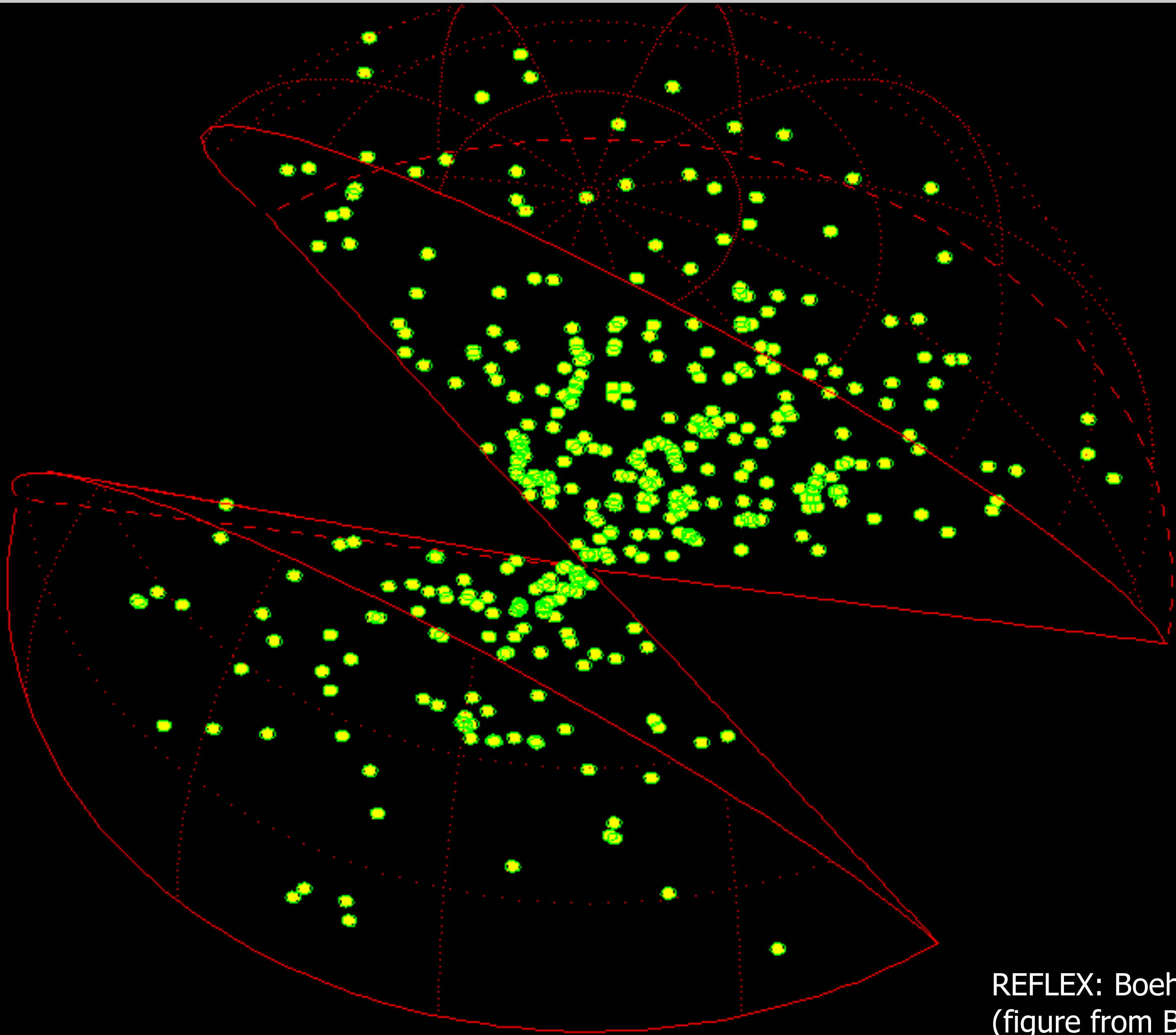
Get $f(z)$ by following directly the growth of structure?



…but what we can observe is in fact only the clustering of *luminous* matter…

(Image credit: V. Springel)

Or we can count the number density of X-ray clusters…

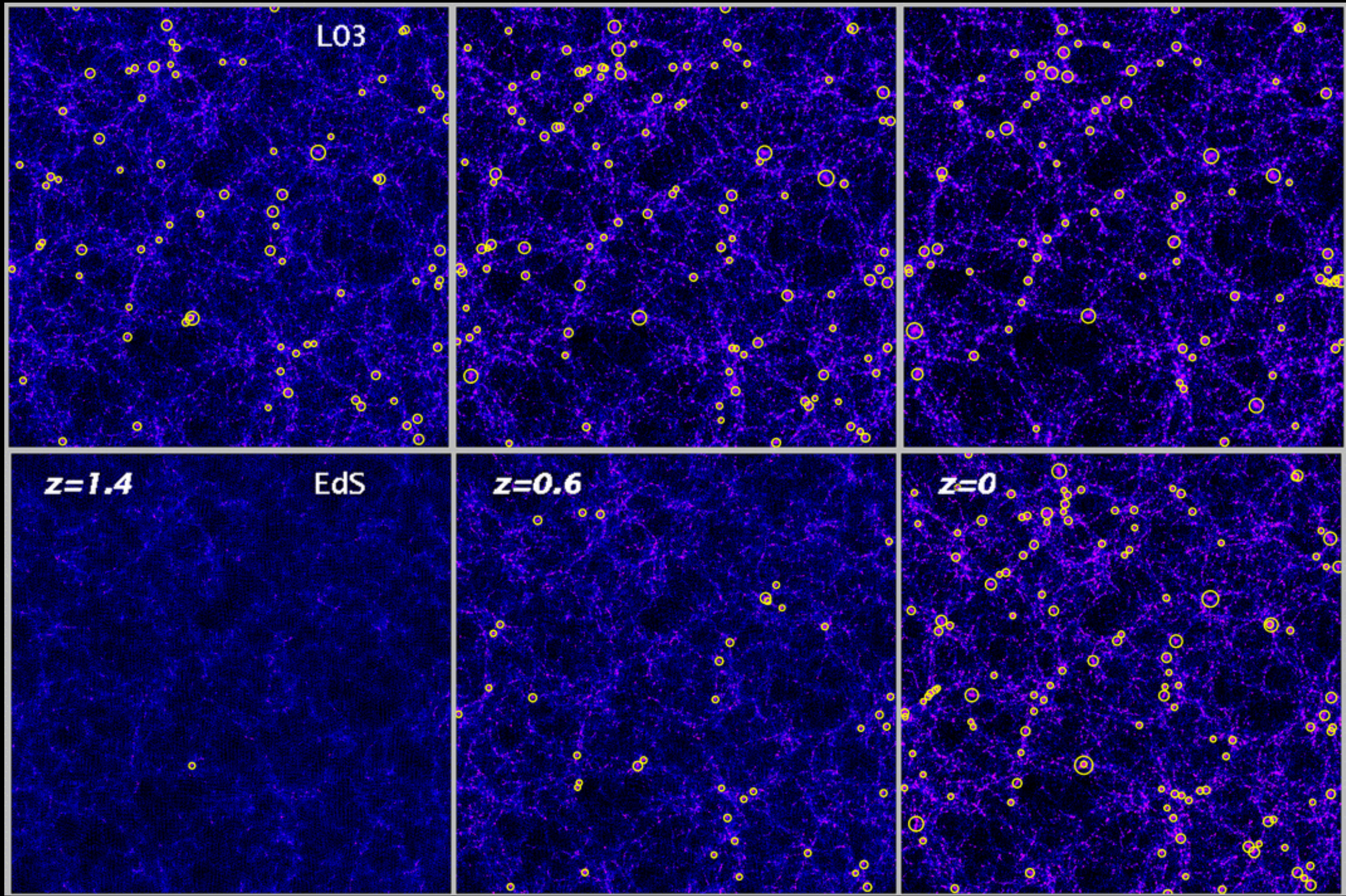


REFLEX: Boehringer et al. 2004
(figure from Borgani & Guzzo 2001)

…and how it evolves with redshifts: probing a combination of the growth rate f(z) and the expansion rate H(z)

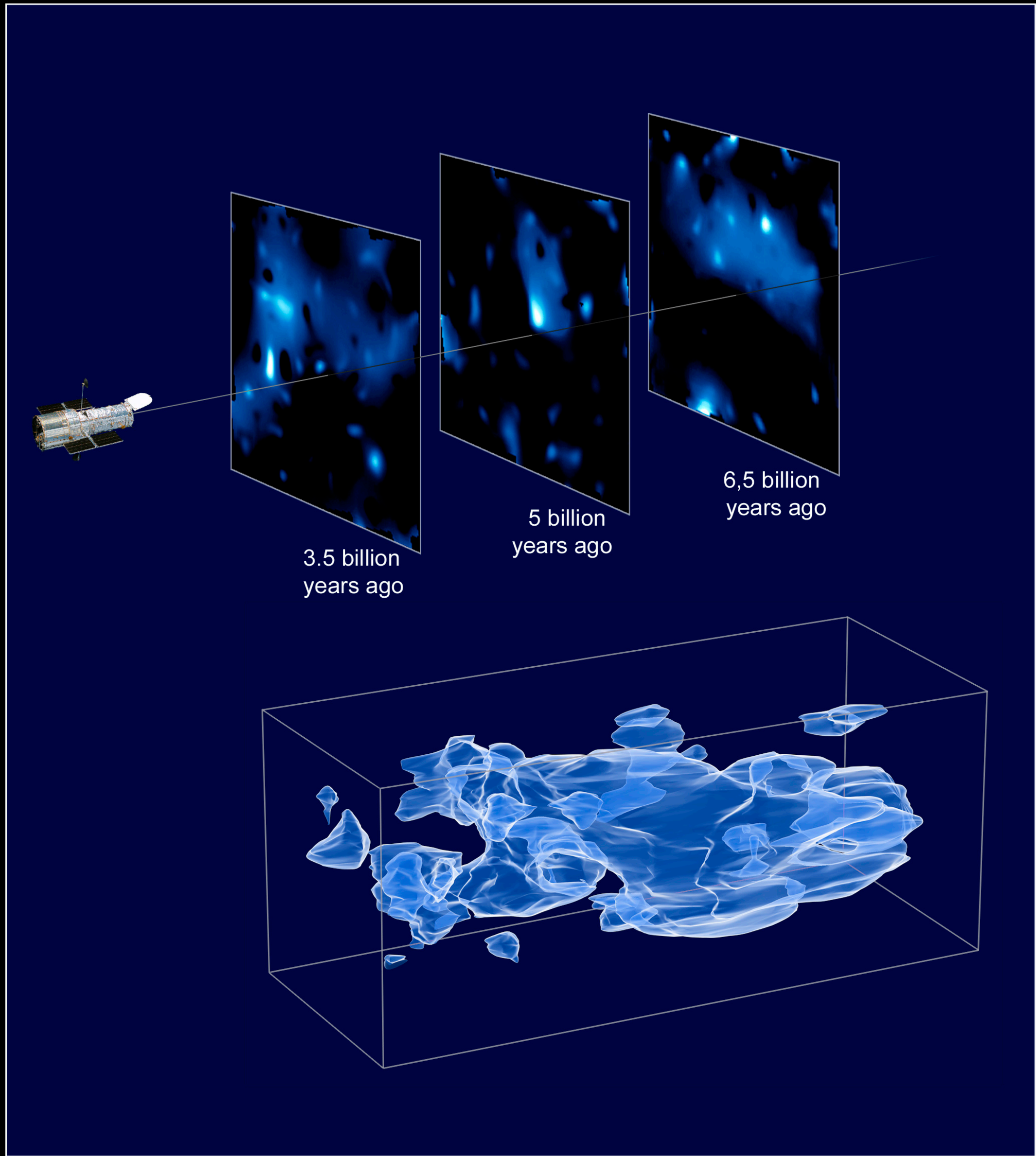
Lambda model

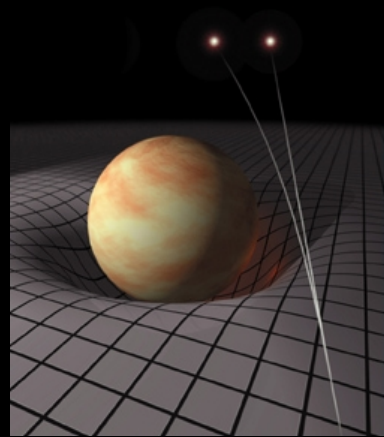
Einstein-DeSitter model



Borgani & Guzzo 2001

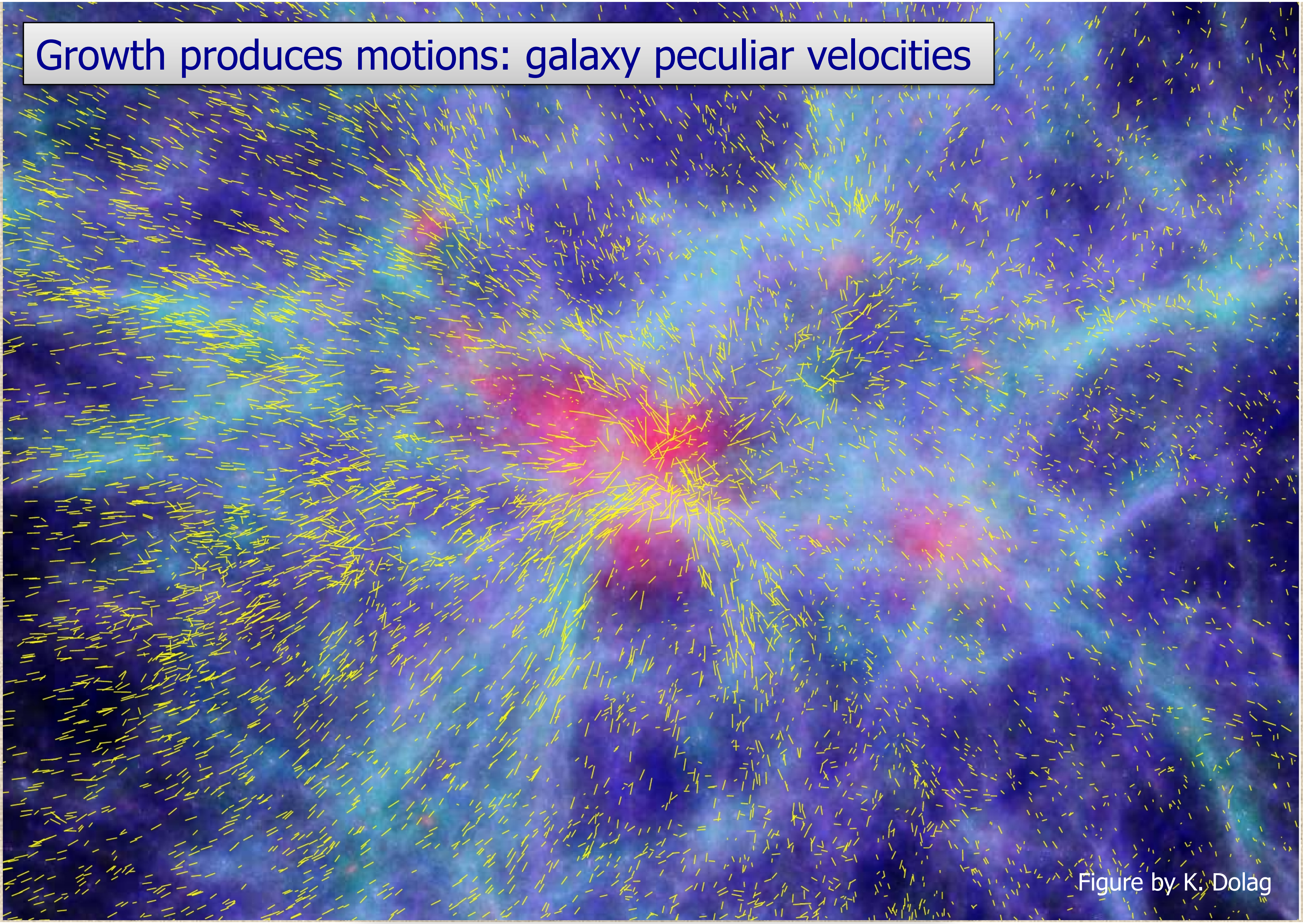
…or use weak gravitational lensing “tomography”





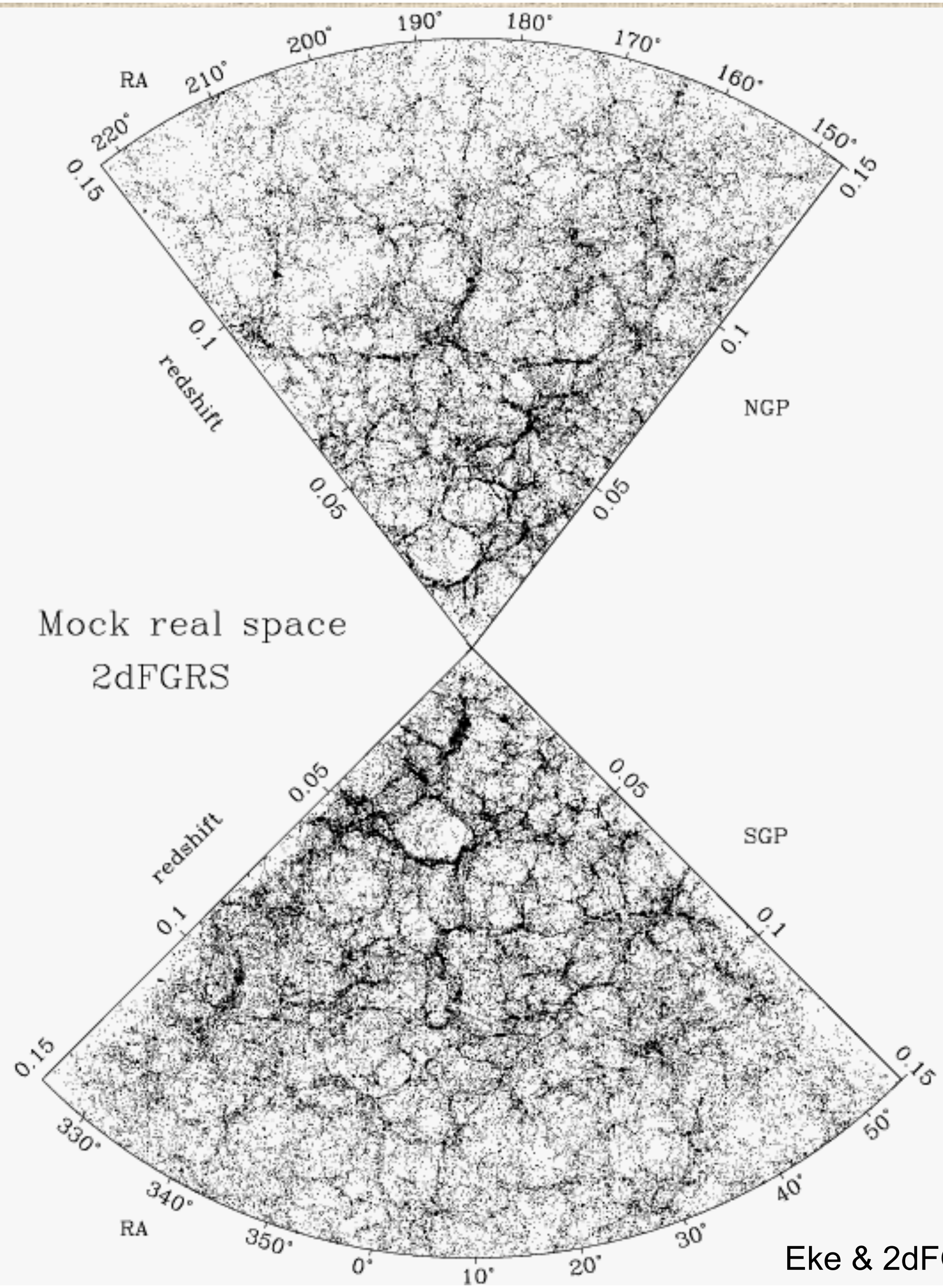
COSMOS: Massey et al. 2007

Growth produces motions: galaxy peculiar velocities



Peculiar velocities manifest themselves in galaxy redshift surveys as *redshift-space distortions* (Kaiser 1987)

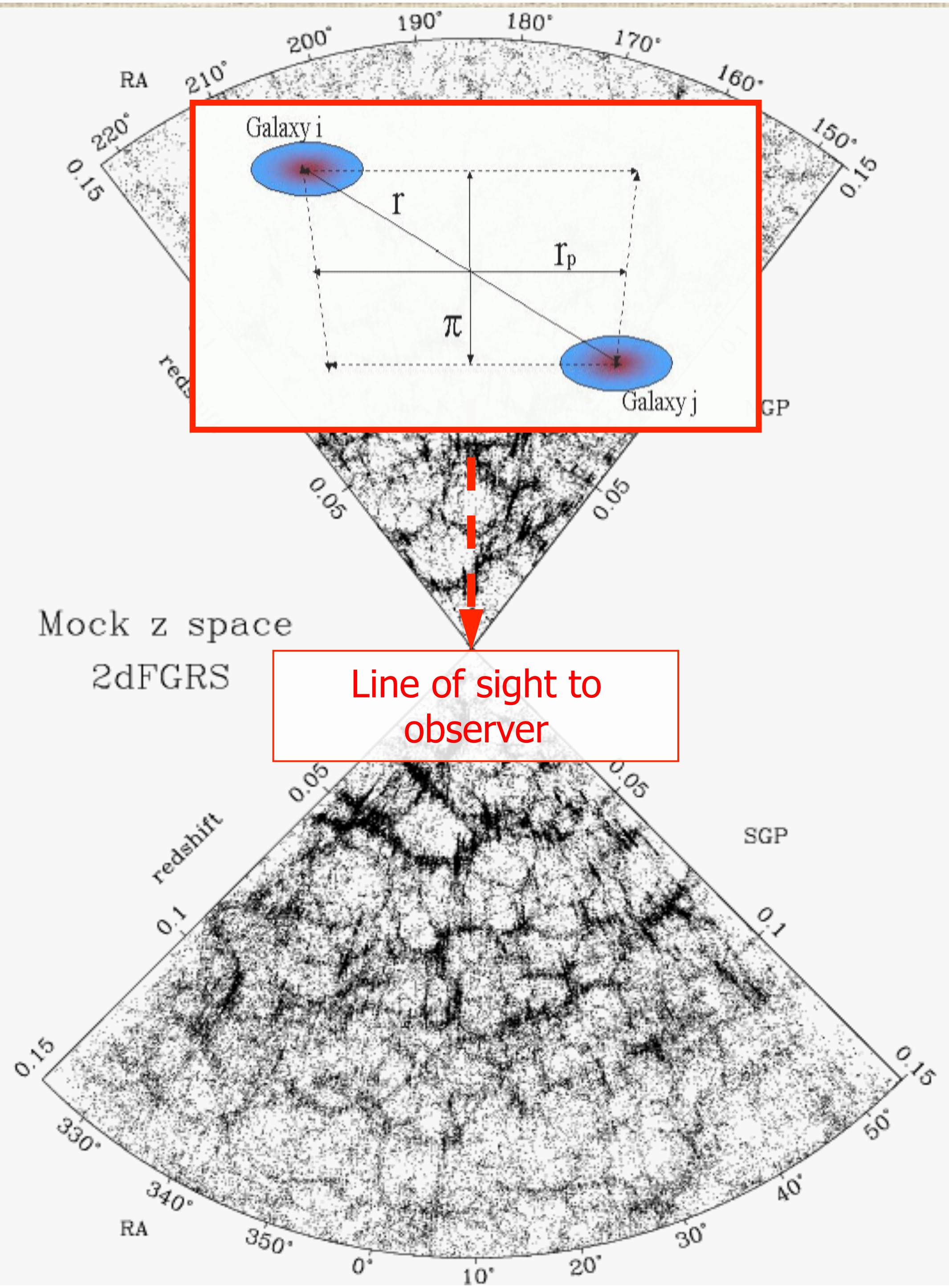
real space



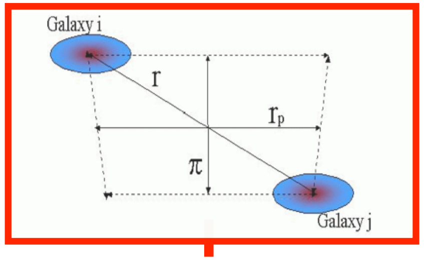
Eke & 2dFGRS 2003

Peculiar velocities manifest themselves in galaxy redshift surveys as *redshift-space distortions* (Kaiser 1987)

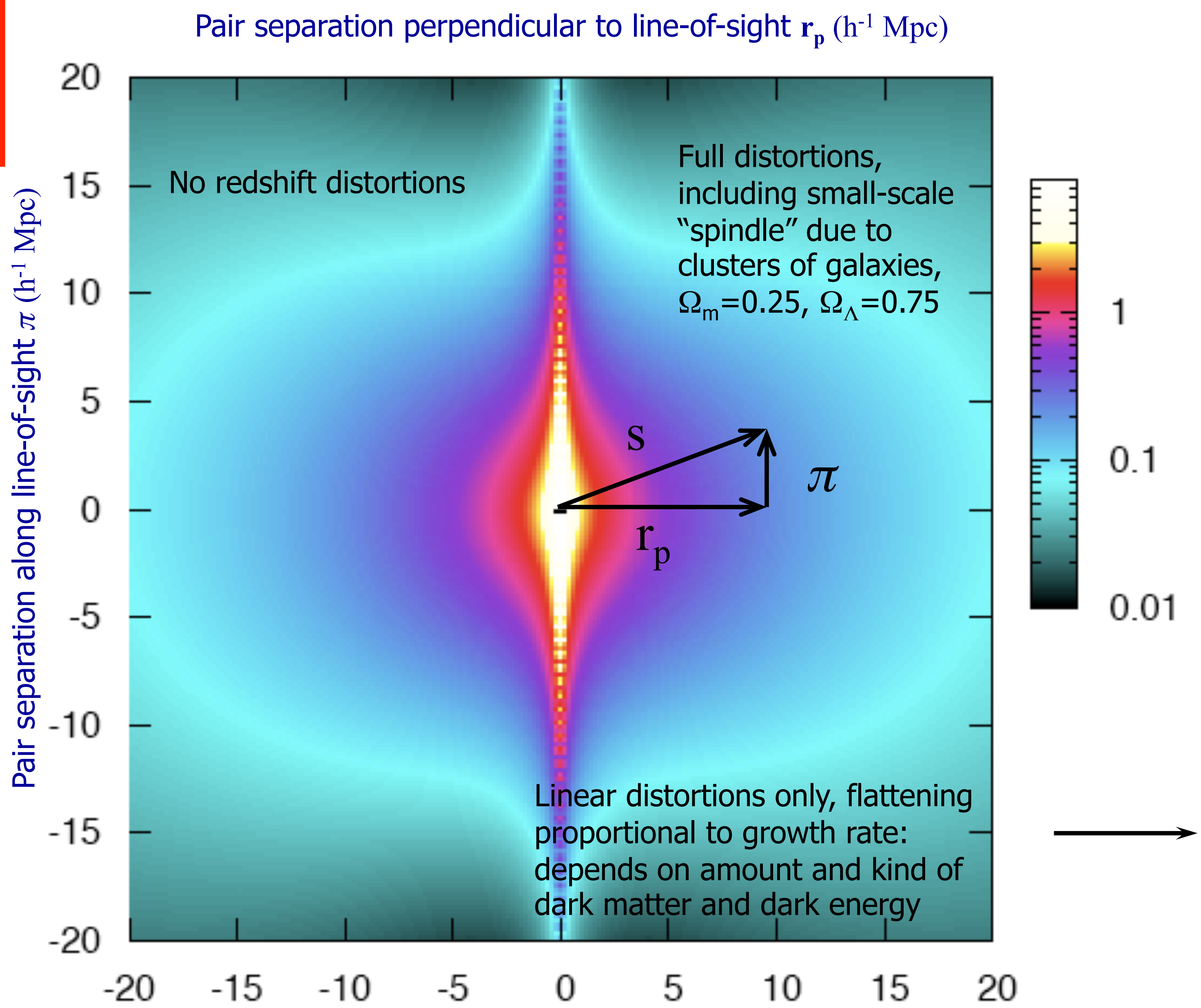
redshift space



Redshift-space galaxy-galaxy correlation function $\xi(r_p,\pi)$



Line of sight to observer



Compression parameter β (Kaiser 1987)

Kaiser model of linear redshift-distortions (1987)

• Easier to describe in Fourier space:

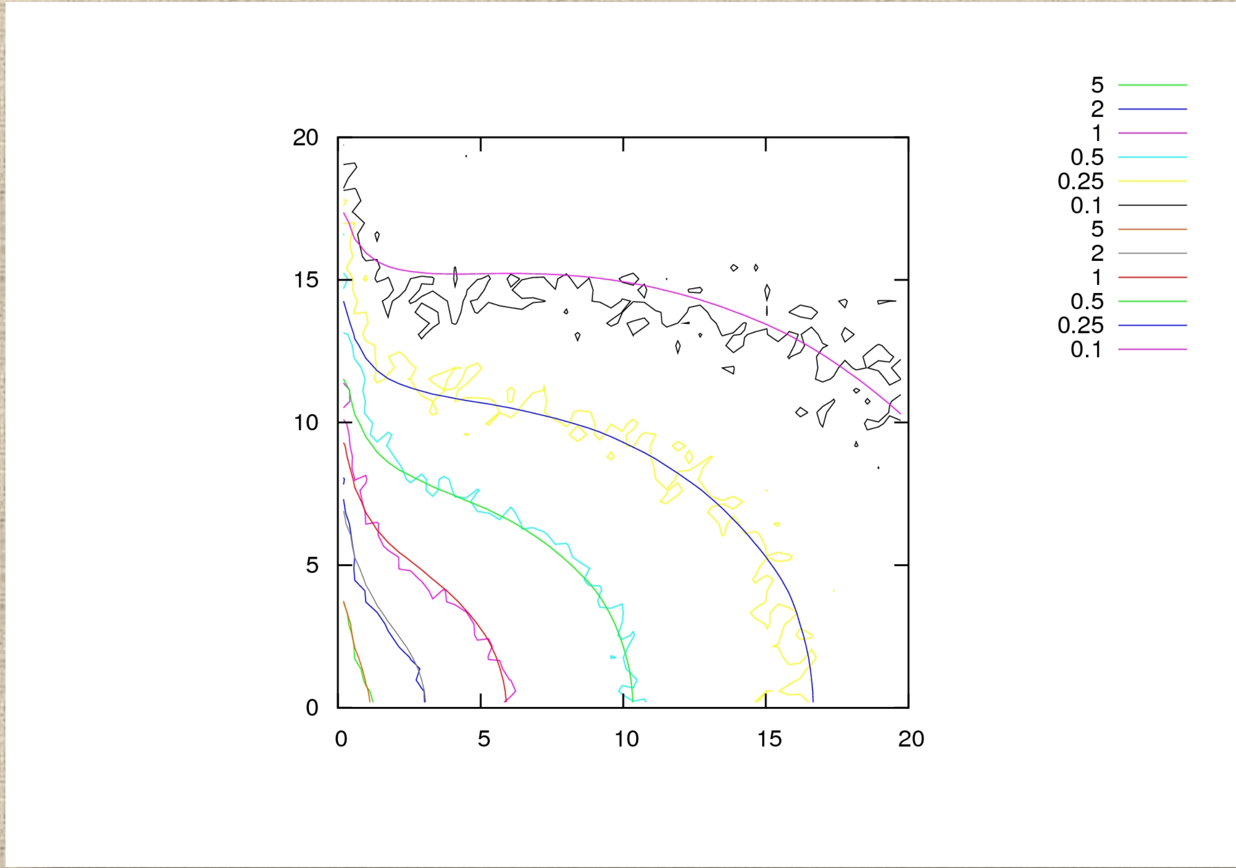
$$P(k,\mu) = P(k)\left[1 + \frac{f}{b}\mu^2\right]$$

where f is growth rate and b is the *linear bias* of the class of galaxies that we are using. If galaxies trace the mass $b=1$.

• It is usually expressed in terms of the parameter $\boldsymbol{\beta = f/b}$

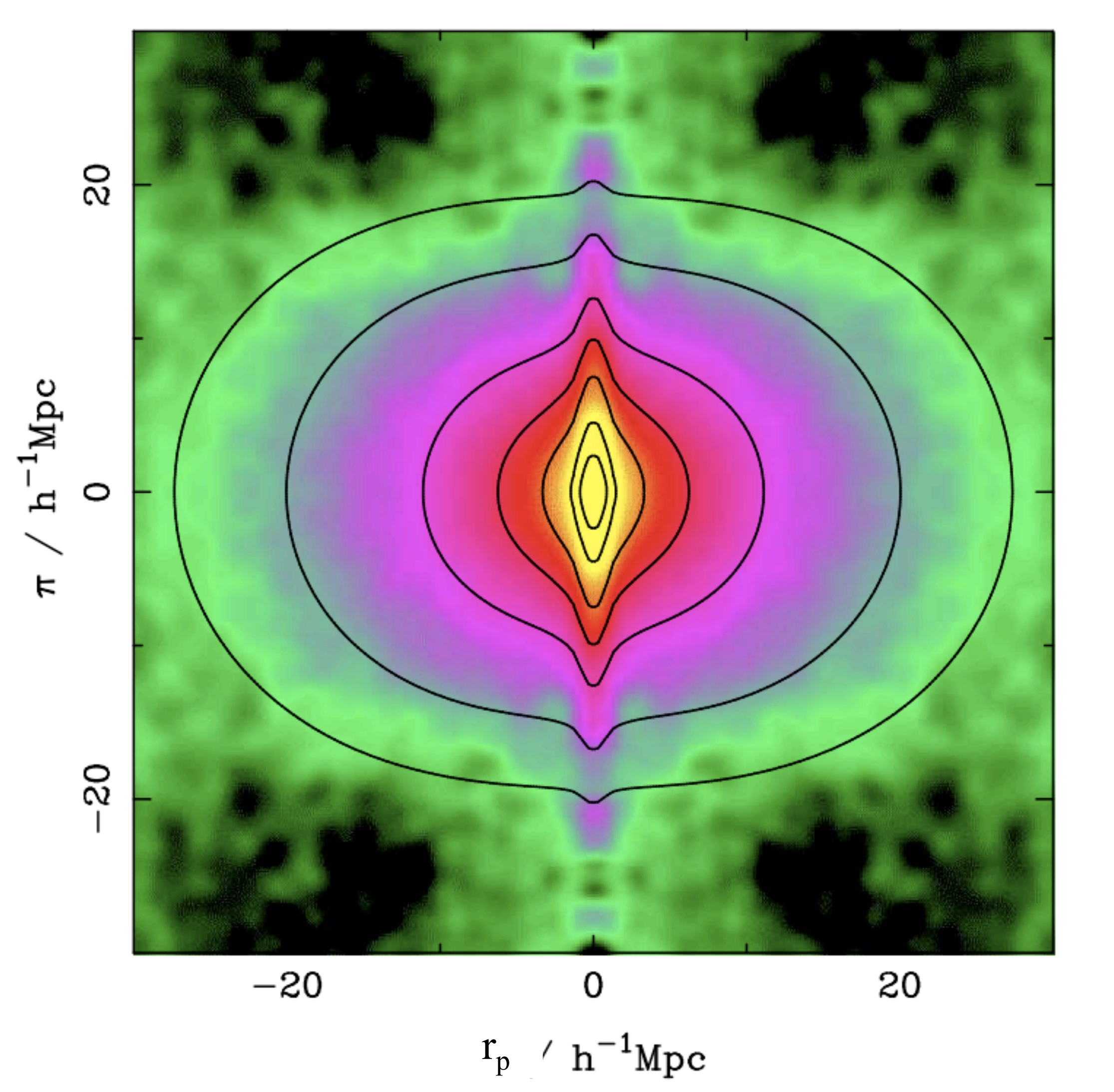
• One has also to account for the nonlinear effects on small scales, so a more complete description is given by

$$P(k_{||}, k_{\perp}) = P(k)\left(1 + \beta\mu^2\right)^2 D(k\mu\sigma_p).$$



Redshift-space galaxy-galaxy correlation function $\xi(r_p,\pi)$

2dFGRS, z~0.1



Compression parameter
$\beta=0.49\pm0.09$

$$f = b_L\beta$$

Peacock et al. 2001,
Hawkins et al. 2003

How to estimate β from observed $\xi(r_p,\pi)$?

A natural way for this circularly (a)symmetric problem is to decompose ξ(s) in spherical harmonics (Hamilton 1992, 1997)

$$\xi(r_p,\pi) = \xi_0(s)P_0(\mu) + \xi_2(s)P_2(\mu) + \xi_4(s)P_4(\mu)$$

where P_n are *Legendre polynomials* and the moments ξ_n(s) are given by

$$\xi_0(s) = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right)\xi(r)\,,$$

$$\xi_2(s) = \left(\frac{4}{3}\beta + \frac{4}{7}\beta^2\right)\left[\xi(r) - \bar{\xi}(r)\right]\,,$$

$$\xi_4(s) = \frac{8}{35}\beta^2\left[\xi(r) + \frac{5}{2}\bar{\xi}(r) - \frac{7}{2}\,\bar{\bar{\xi}}\,(r)\right]\,;$$

$$\bar{\xi}(r) \equiv 3r^{-3}\int_0^r \xi(r')r'^2dr'\,,$$

$$\bar{\bar{\xi}}\,(r) \equiv 5r^{-5}\int_0^r \xi(r')r'^4dr'\,.$$

In general, β can be extracted in different ways from combinations of the moments or using the full relationship:

a) Ratio of redshift to real-space functions

$$\frac{\xi(s)}{\xi(r)} = \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right)$$

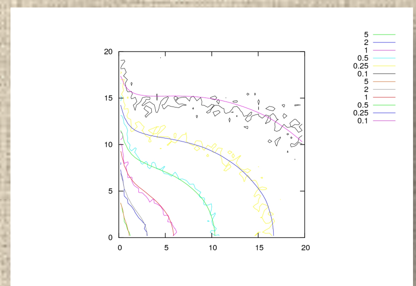
b) Quadrupole to monopole ratio [requires power-law assumption for ξ(r)]

$$\frac{\xi_2(s)}{\xi_0(s)} = \frac{\gamma - 3}{\gamma} \frac{\frac{4}{3}\beta + \frac{4}{7}\beta^2}{1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2}.$$

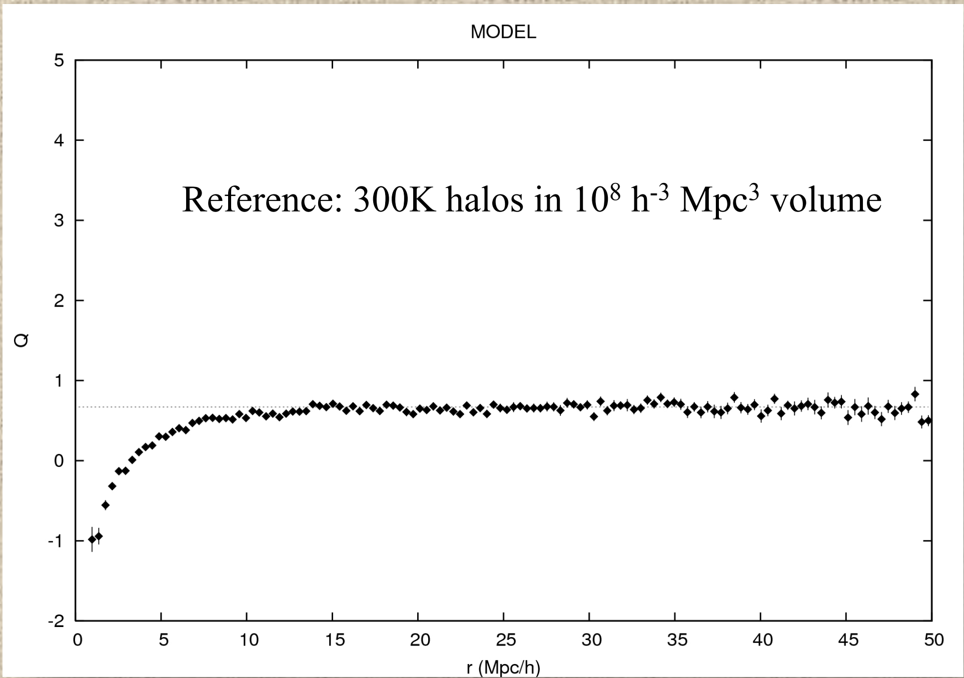
c) Q-statistics [totally independent from functional form of ξ(r)]

$$Q(s) = \frac{\frac{4}{3}\beta + \frac{4}{7}\beta^2}{1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2} = \frac{\xi(s)}{\xi_0(s) - \frac{3}{s^3}\int_0^s \xi_0(s')s'^2 ds}$$

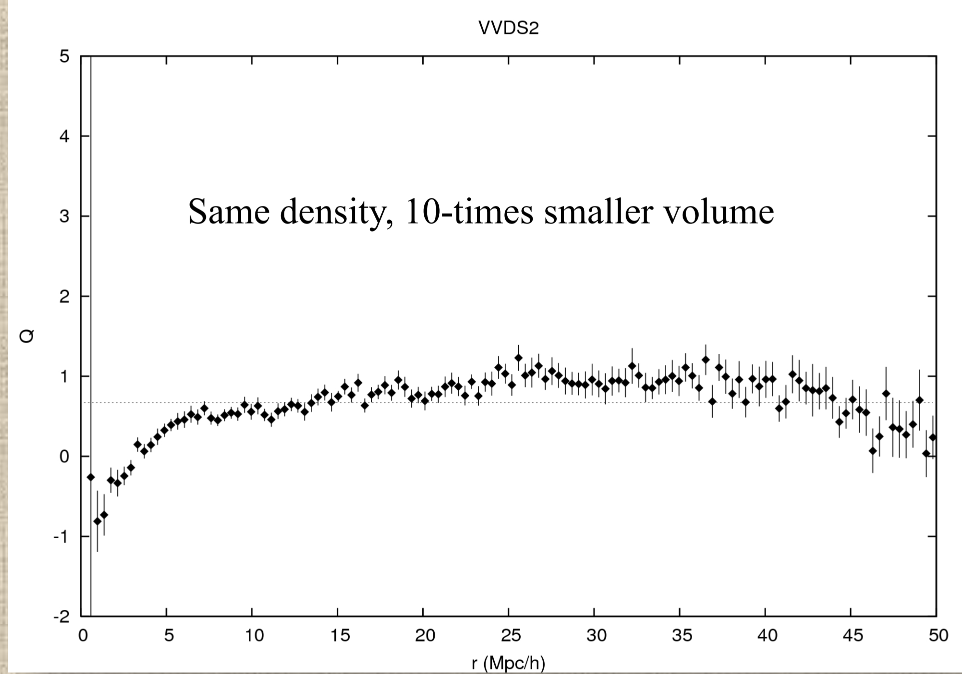
d) Full model in terms of moments of ξ(r) and Legendre polynomials

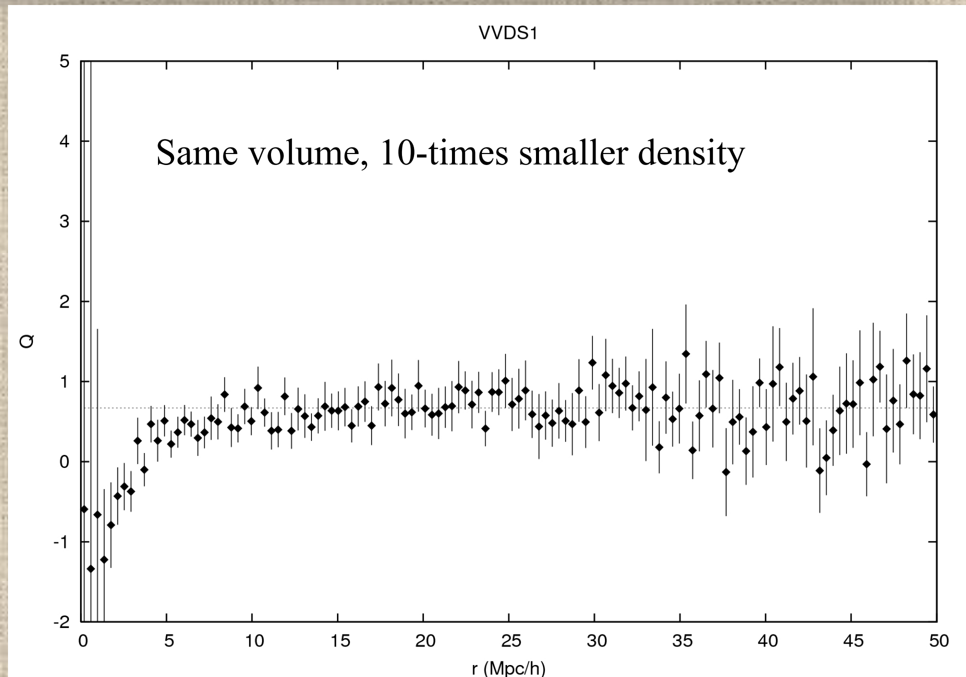


Q statistics for different sample volume/densities



M. Pierleoni, Laurea thesis, Bologna, 2006





The full 2D model is constructed as the convolution of the linearly-distorted $\xi_L(s)$

$$\xi_L(r_p,\pi) = \xi_0(s)P_0(\mu) + \xi_2(s)P_2(\mu) + \xi_4(s)P_4(\mu)$$

with the distribution of non-linear pairwise velocities, well described by an exponential (Peebles 1980)

$$f(v) = \left(\sigma_{12}\sqrt{2}\right)^{-1} \exp\left\{-\sqrt{2}|v|/\sigma_{12}\right\}$$

such that $\xi(r_p,\pi)$ is then modelled as

$$\xi(r_p,\pi) = \int_{-\infty}^{\infty} \xi_L\left[r_p, \pi - \frac{v(1+z)}{H(z)}\right] f(v)dv$$

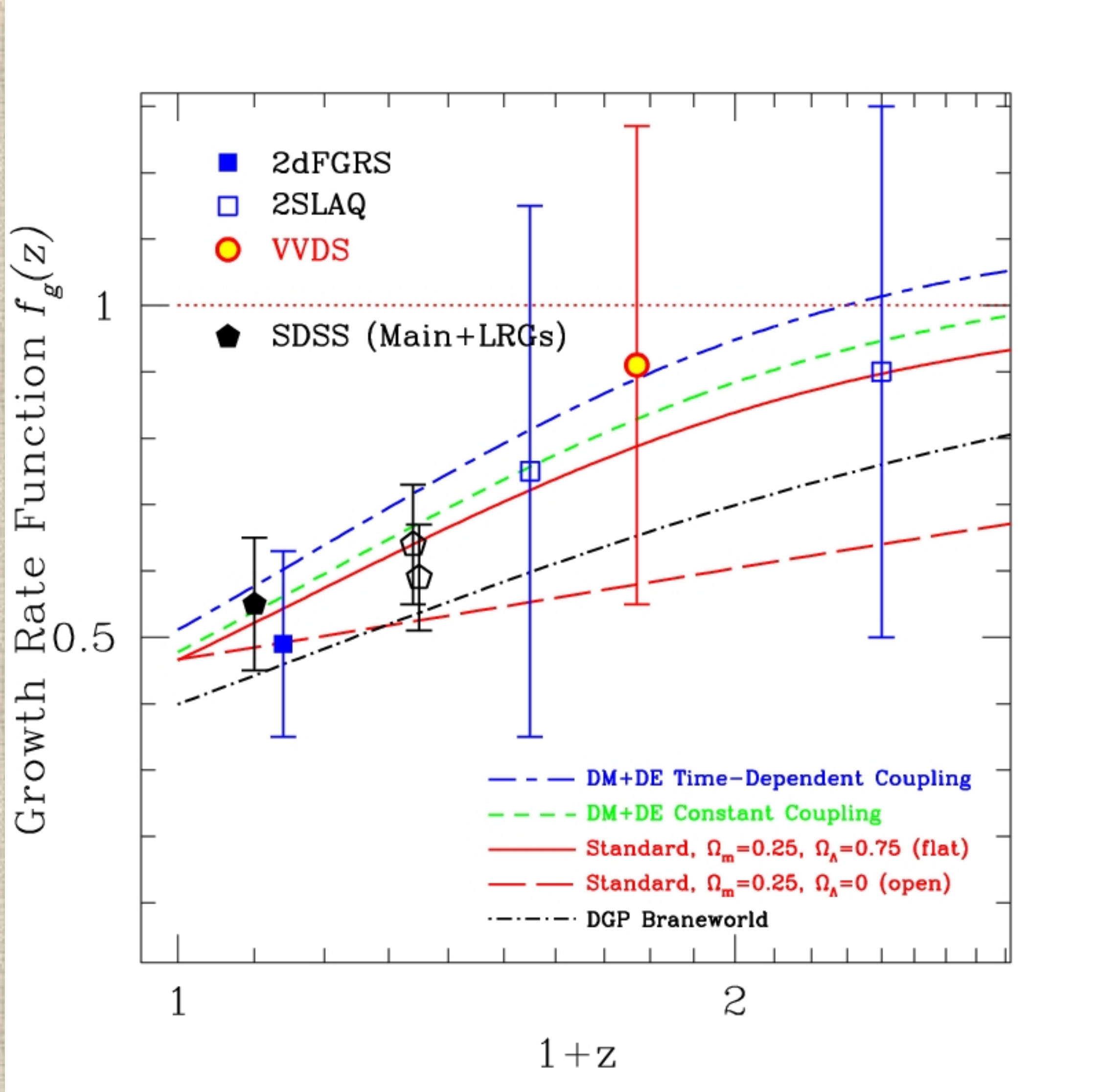
Need to reconstruct the real-space $\xi(r)$: get it from the projected (distortion-free) function

$$w_p(r_p) \equiv 2\int_0^{\infty} \xi(r_p,\pi)d\pi$$

as

$$\xi(r) = \frac{1}{\pi}\int_0^{\infty} \frac{dw_p}{dr_p}\frac{dr_p}{\left(r_p^2 - r^2\right)}$$

Growth rate of structure from redshift distortions at z~1



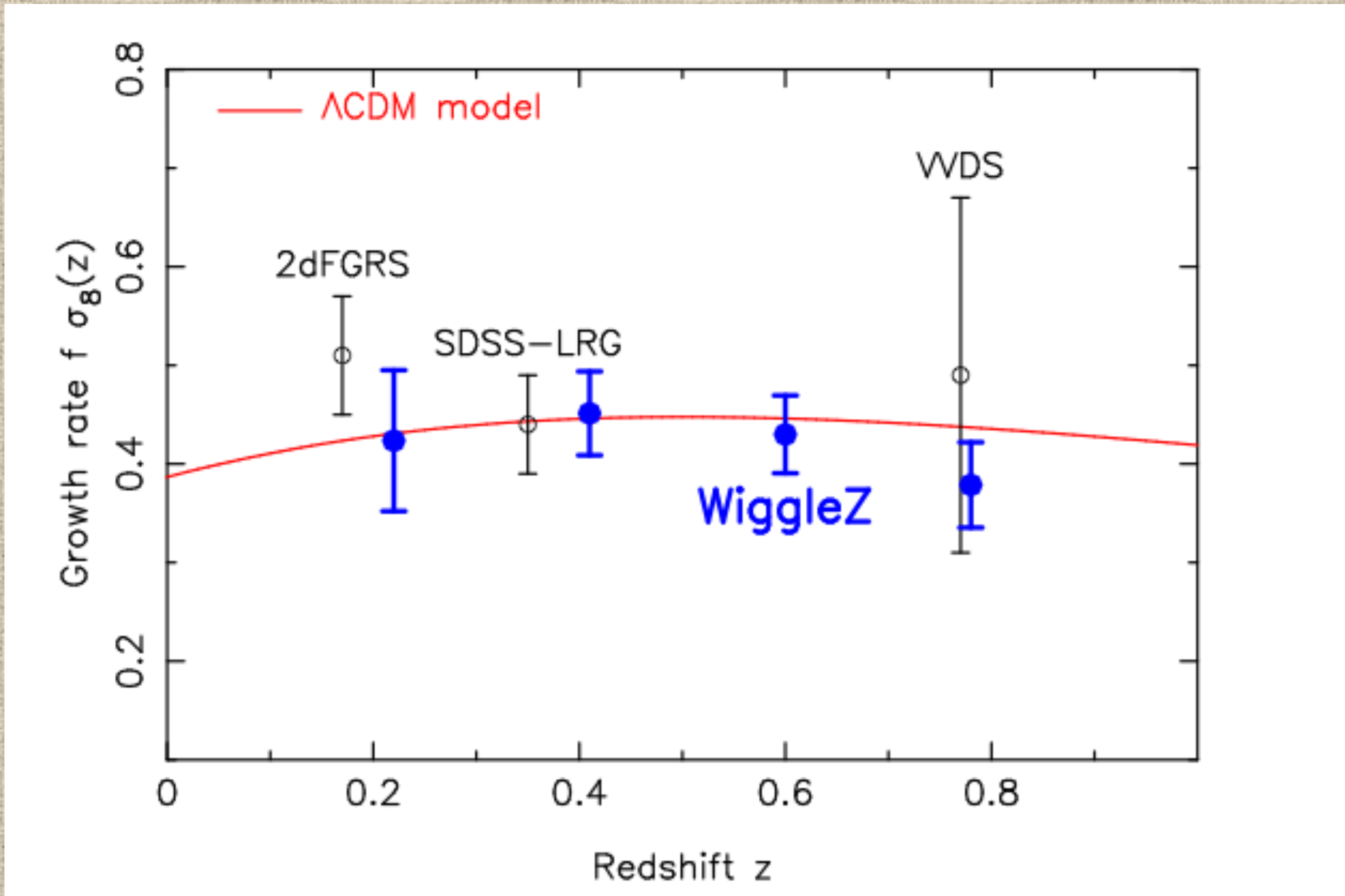
$$f = b_L \beta$$

- 2dFGRS: Hawkins+ 2003
- SDSS main: computed from Tegmark+ 2005
- SDSS-LRG: Tegmark+ 2007, Cabre & Gaztanaga 2008 (see also Yamamoto+ 2008)
- 2SLAQ: Ross+ 2007 (gal), da Angela+ 2007 (QSO)
- VVDS: Guzzo+ 2008

DGP: Lue et al. 2004; DM+DE models: Di Porto & Amendola 2007

f(z) from redshift distortions, recent developments

Blake et al. 2011, arXiv:1104.2948



- WiggleZ survey, 152,000 redshifts 0.1<z<0.9 (aim at 200,000 gals)
- Original main goal: BAO
- Redshift distortions from P(k)
- Large volume, but very sparse sample
- UV-selected emission-line galaxies from GALEX: complex selection function

END of Lecture 2