Introduction to Dark Energy

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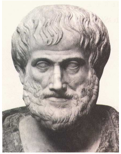
Outline

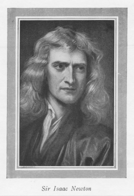
- Gravity, expansion and acceleration
- Cosmological observations
- Dark energy: linear and non-linear properties

Historical perspective, circa 350 b.c.e.



- Gravity is always attractive: how to avoid that the **sky** falls on our head?
- **Aristotle**'s answer: quintessence

Historical perspective, circa 1700 c.e.



- Gravity is always attractive: how to avoid that the **stars** fall on our head?
- **Newton**'s answer: God's initial conditions

Historical perspective, circa 1900 c.e.

- Gravity is always attractive: how to avoid that the **Universe** fall on our head?
- **Einstein**'s answer: to avoid collapse (to make the universe stable) it is necessary to introduce a form of repulsive gravity, by modifying the equations of General Relativity.

There is a simple Newtonian solution

$$\Phi = -G\frac{M}{r} + \lambda r^2$$

Newton himself noted the possibility of the r^2 repulsive behavior but did not comment further!

Original GR equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

geometry *matter*

$$T_\mu^\nu = \left\{ \begin{array}{cccc} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{array} \right\}$$

- These are the most general equations that are

1. covariant
2. covariantly conserved
3. second order in the metric
4. reducing to Newton at low energy

Box on $T_{\mu\nu}$

Basic hydrodynamic equations for a non-relativistic fluid at rest:

$$\begin{aligned} \dot{\rho} &= 0 \\ \nabla p &= 0 \end{aligned} \tag{1}$$

where the energy density $\rho = nmc^2$ and the pressure is $p_i = nmv_i^2$. If we define the matrix

$$T_{\mu\nu} = diag(\rho, p, p, p)$$

then, more simply

$$\frac{\partial T^{\mu\nu}}{\partial x^{\mu}} \equiv T^{\mu\nu}_{,\mu} = 0$$

The relativistic version is the only tensor that depends on $\rho, p, u^{\mu} = dx^{\mu}/ds, g_{\mu\nu}$ and reduces to this limit in the Minkowski space

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} - pg^{\mu\nu} \tag{2}$$

Einstein's equations are complete only when a relation between p and ρ is given: the equation of state:

$$p = w\rho$$

Back to

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- For instance, neglecting the third condition (second order in the metric), we could add whatever rank-2 tensor covariantly conserved, e.g. any E_μ^ν such that

$$E^{\nu}_{\mu;\nu} = 0$$

- like e.g. a linear combination of

$$R R_{\mu\nu},\ R_{;\mu\nu},\ R^2 g_{\mu\nu}, \dots$$

as for instance [$F = F(R)$]

$$E_{\mu\nu} \equiv F' R_{\mu\nu} - \frac{1}{2} F g_{\mu\nu} + g_{\mu\nu} \Box F' - F'_{;\mu;\nu}$$

obtaining a higher-order gravity.

- Neglecting instead the fourth condition, we can add a (*small*) term $\Lambda g_{\mu\nu}$ and rewrite the equations as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- The new term is the **cosmological constant**.

- The big idea of recent years has been to move the new term from right to left

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

thereby introducing a new form of matter

$$T_{\mu\nu(\Lambda)} = \left(\frac{\Lambda}{8\pi}\right) g_{\mu\nu}$$

- This matter has a fundamental property. Writing

$$T^{\nu}_{\mu(\Lambda)} = \left(\frac{\Lambda}{8\pi}\right) \delta^{\nu}_{\mu}$$

or

$$\left\{\begin{array}{cccc} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{array}\right\} = \left\{\begin{array}{cccc} \frac{\Lambda}{8\pi} & 0 & 0 & 0 \\ 0 & \frac{\Lambda}{8\pi} & 0 & 0 \\ 0 & 0 & \frac{\Lambda}{8\pi} & 0 \\ 0 & 0 & 0 & \frac{\Lambda}{8\pi} \end{array}\right\}$$

one gets immediately

$$p_\Lambda = -\frac{\Lambda}{8\pi}, \quad \rho_\Lambda = \frac{\Lambda}{8\pi}$$

that is, the cosmological constant has **negative pressure** (if $\Lambda > 0$).

- Introducing the equation of state

$$p = w\rho$$

one has that the cosmological constant has a negative eq. of state

$$w = -1$$

As a comparison, the eq. of state of matter (dust or cold dark matter) is

$$p = mv^2 \approx 0 \rightarrow w = 0$$

while for radiation

$$p = \rho/3 \rightarrow w = 1/3$$

A repulsive gravity

- What a **negative pressure** has to do with a **repulsive gravity**?
- Homogeneous and isotropic Friedmann metric

$$ds^2 = dt^2 - a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \sin\theta d\phi^2 + r^2 d\theta^2 \right)$$

For a single perfect fluid, the ten Einstein equations reduce to two equations for the scale factor and the energy density (here we put for simplicity $k = 0$ and always assume $a_0 = 1$)

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p) = -\frac{4\pi}{3}\rho(1 + 3w) \quad (4)$$

From the second one it appears that if

$$w < -1/3$$

then we get accelerated expansion. Therefore the cosmological constant (or any fluid with $w < -1/3$) **accelerates** the expansion $\rightarrow$ "**repulsive gravity**". We call this hypothetical fluid ***Dark Energy.***

- Consider now only the cosm. constant

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho_\Lambda = \frac{\Lambda}{3}$$

from which

$$a = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$$

This accelerated expansion is a prototype of **primordial inflation** **(de Sitter metric).**

- Generally speaking, there are at least three components (plus curvature) so that dynamics is more complicate:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}(\rho_\gamma + \rho_M + \rho_\Lambda) - \frac{k}{a^2}$$

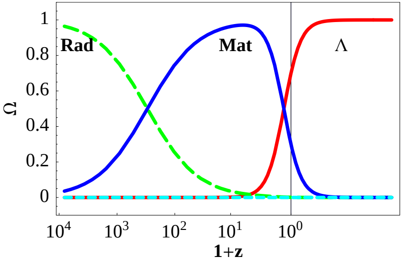
$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0$$

Ordinary matter (baryons plus dark matter) conserves energy during expansion, so that we have **four** different behaviors

$$\begin{aligned} \rho_\gamma &\sim a^{-4} \\ \rho_M &\sim a^{-3} \\ \rho_k \equiv \frac{k}{a^2} &\sim a^{-2} \\ \rho_\Lambda &\sim a^0 \end{aligned}$$

- In general, therefore, we have

$$rad. \to matter \to curvature \to cosm.const.$$



$$\Omega_i \equiv \frac{\rho_i}{\rho_t}$$

Quantistic interpretation

- Think of a field, eg a scalar field, as a series of classical oscillators. Then, every oscillator contributes an energy due to the sum of its potential and kinetic energy.
- When at rest, every oscillator has only its potential energy of the lowest level, that we can always put to zero.
- Quantistically, however, the state of minimum is not at zero energy but rather

$$E_0 = \frac{1}{2}\hbar\omega$$

- Therefore, for a field, the total zero-point energy is

$$E_0 = \sum_i \frac{1}{2}\hbar\omega_i$$

summing over all possible modes. Summing over $k_i = 2\pi/\lambda_i$ where $\lambda_i = L/n_i$ are all the wavelengths of the modes contained in a box of size L, we obtain $dn_i = dk_i L/2\pi$ modes in the range dk_i, so that

$$E_0 = \frac{1}{2}\hbar L^3 \int \frac{d^3k}{(2\pi)^3}\omega_k$$

- where the oscillation frequency is in relation to the particle's mass:

$$\omega^2 = k^2 + m^2/\hbar^2$$

The total energy density integrating up to a cut-off frequency k_{max} is then

$$\rho_{vacuum} = lim \frac{E}{L^3} = \hbar \frac{k^4_{max}}{16\pi^2}$$

- The energy diverges at the high frequencies (ultraviolet divergence). We must suppose then that there is k_{max} beyond which a new interaction modifies the system.
- The problem is, which k_{max} ?. If we assume as limit the Planck energy

$$E_{Planck} = 10^{19} GeV$$

we get

$$\rho_{vacuum} = 10^{92} g/cm^3$$

Now, the experimental limit is

$$\rho = 3H^2/8\pi G \simeq 10^{-29} g/cm^3$$

then, the theoretical estimate is off by 120 orders of magnitude!
- This fundamental theoretical problem is still **open**.

Astronomy of Dark Energy

- We have seen that the Friedmann equation has the form

$$H^2 = \frac{8\pi}{3}(\rho_M + \rho_\Lambda + \rho_k + ...?)$$

- Suppose we know nothing of the matter content of the universe. How to study the structure of the cosmos ?

- *Three levels*:

1. **background** effects (age, distances)
2. **linear** perturbations (large scale clustering, CMB)
3. **non-linear** perturbations (formation of collapsed objects, halo profiles).

Notice that a smooth component, as the Λ, has no **local** observational effects: The Poisson equation remains invaried, because in linear GR the Poisson equation

$$\triangle\Psi = -4\pi G\rho$$

becomes

$$\triangle\Psi = -4\pi G\delta\rho$$

That's why we need cosmology !

From observations to theory

- What we really observe in cosmology is light from **sources** and from **backgrounds**.
- How do we connect these observables to cosmological quantities like $\rho_m, \rho_\gamma, k, a(t), H_0$ etc?
- First, define

$$\Omega_M = \frac{8\pi\rho_0}{3H_0^2}, \quad \Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3H_0^2}, \quad \Omega_k = \frac{8\pi k}{3H_0^2}$$

  and note that

$$1 = \Omega_M + \Omega_\Lambda + \Omega_k$$

  so rewrite Friedman equation as ($a_0 = 1$)

$$H^2 = H_0^2(\Omega_m a^{-3} + \Omega_\Lambda a^0 + \Omega_k a^{-2})$$

- Then, generalize it to several components:

$$\begin{aligned} H^2 &= H_0^2(\Omega_m a^{-3(1+w_m)} + \Omega_\Lambda a^{-3(1+w_\Lambda)} + \ldots) \\ &= H_0^2 \sum_i \Omega_i a^{-3(1+w_i)} = H_0^2 E(a)^2 \end{aligned}$$

- For instance...

name	density Ω_i	state w_i
baryons	0.05	≈ 0
CDM	0.2	0
radiation	0.0001	1/3
massive neutr.	<0.05	≈ 0
cosm. const.	0.75	-1
curvature	<0.03	-1/3
other ?	?	?

- Unknown quantities: H_0, Ω_i, w_i to be determined using:

1. Angular positions of sources, e.g. galaxies: θ_i, φ_i
2. Redshifts: z_i
3. Apparent magnitudes: m_i
4. galaxy ellipticities
5. Age of Universe/age of stars
6. Background radiation/polarization e.g. CMB: $\Delta T / T$

- BASIC RELATION redshift/scale factor:

$$a = \frac{1}{1+z}$$

First basic observable: Age of the Universe

- The age of the universe can be deduced from the Friedmann equation:

$$\left(\frac{da}{dt}\right)^2 = H_0^2 a^2 E(a)^2$$

we get

$$\left(\frac{dz}{H_0 dt}\right) = \quad (1+z)E(z \quad)$$

and finally

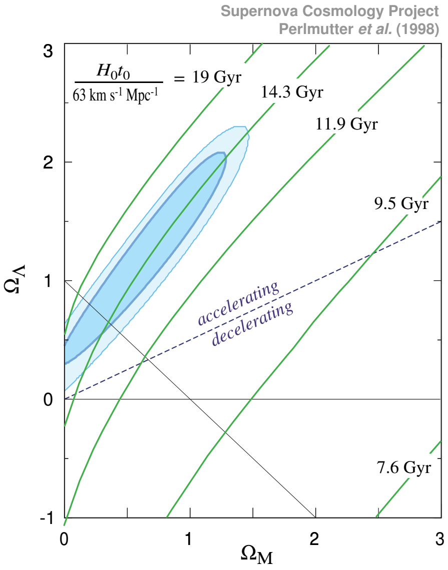
$$t_0 - t_1 = H_0^{-1} \int_0^{z_1} \frac{dz}{(1+z)E(z)}$$

Notice that the Hubble constant is

$$H_0^{-1} = \frac{1}{100hkm/sec/Mpc} = 9.76h^{-1}Gyr$$

For $z_1 \to \infty$ we get then the age of the universe.

- The effect of the cosmological constant , when $\Omega_{tot} = \Omega_M + \Omega_\Lambda$ is fixed, is to **increase the cosmological age.**



Best fit age of universe: $t_o = 14.5 \pm 1 \ (0.63/h)$ Gyr

Second basic observable: Luminosity distance

- From flat Friedmann's metric

$$ds^2 = c^2 dt^2 - a^2 dr^2$$

and integrating along the null geodesics, we get the **proper distance** which is what you would measure with fixed rods

$$r = \int \frac{cdt}{a(t)} = c \int \frac{da}{\dot{a}a} = c \int \frac{dz}{H(z)}$$

$\rightarrow$ **generalized Hubble law:** measuring distances means measuring cosmology.

- If we compare the energy L emitted by a source at proper distance r with flux f arriving at the observer, we define the **luminosity distance** $d(z)$ such that

$$f = \frac{L}{4\pi r^2 (1+z)^2} = \frac{L}{4\pi d^2}$$

- The two extra factors of $1+z$ take into account the **loss** of energy due to redshift and the **spread** of energy due to the relative dilatation of the emission time versus observer's time. We get

$$d(z) = r(1+z) = cH_0^{-1}(1+z) \int_0^{z_1} \frac{dz}{E(z)}$$

where $cH_0^{-1} = \frac{300.000 km/sec}{100 hkm/sec/Mpc} = 3000 h^{-1} Mpc.$

- Remember our “reference” cosmology

$$E^2(z) = \Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_K(1+z)^2$$

- The luminosity distance therefore depends upon the cosmological constant and, like for the age, increases for Ω_Λ increasing. Therefore, a larger cosm. const. induces a **smaller luminosity of the standard candles**.

- Suppose we have a source of **known** absolute luminosity $M = -2.5 \log L + const$. Then one defines instead of the flux f an **apparent magnitude** $m = -2.5 \log f + const$ as

$$m - M = 25 + 5 \log d(z; \Omega_M, \Omega_\Lambda)$$

If M is the same for every object, then the apparent magnitude gives directly $d(z)$ and is then possible to test for the presence of a cosmological constant.

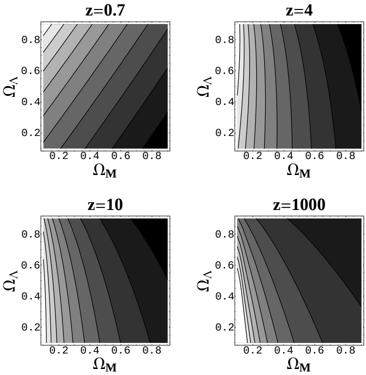
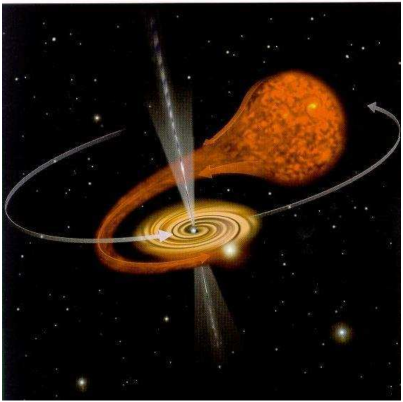


Figure: **Curves of constant** $d(z; \Omega_M, \Omega_\Lambda)$**.**

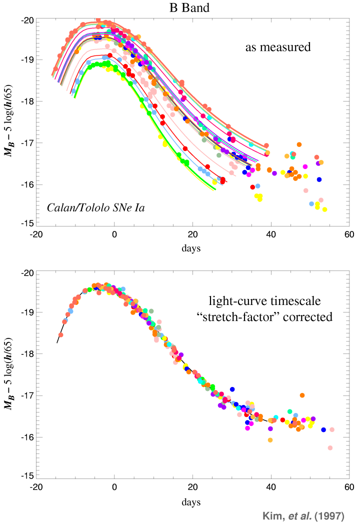
▶

Standard candles

- There exist **standard candles** in nature ?
- The best such thing so far are **supernovae Ia**.



- This hypothesis can be tested and calibrated through a local sample whose distance we know by other means.



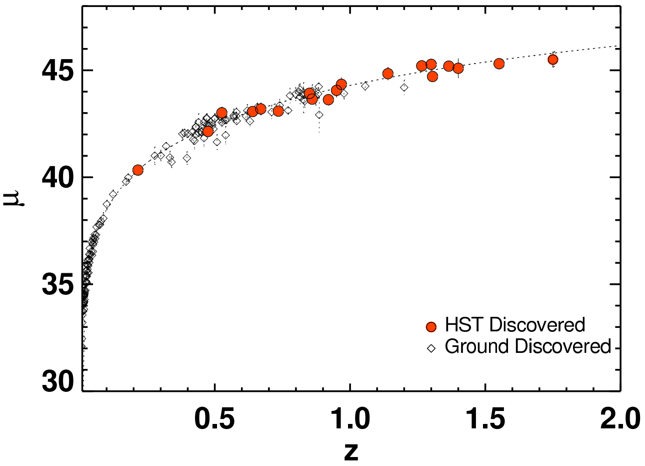
- Then, we compare $m_{obs}(z)$ with

$$m_{theor}(z) = M + 25 + \log d(z; \Omega_M, \Omega_\Lambda, ..)$$

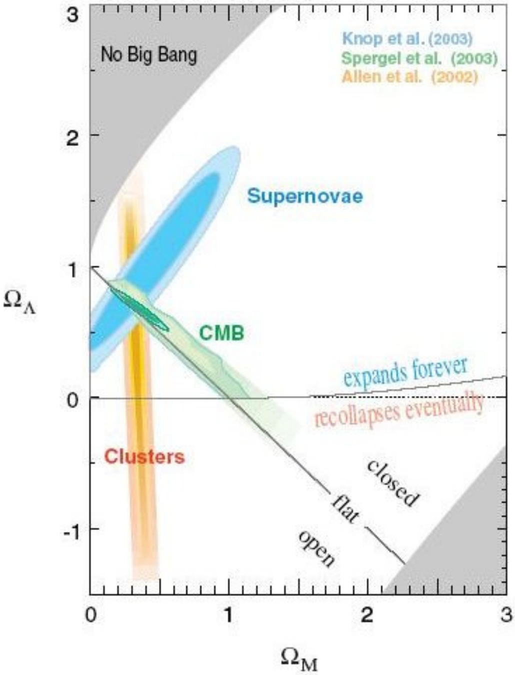
- For instance

$z = 1, \quad M = -19.5$	
$\Omega_M = 0, \quad \Omega_\Lambda = 1$	$\Omega_M = 1, \quad \Omega_\Lambda = 0,$
$m_{theor} = 24.4$	$m_{theor} = 23.2$

More than twice as bright !


45
40
35
30
μ
0.5
1.0
1.5
2.0
z
HST Discovered
Ground Discovered

Supernova Cosmology Project



Linear perturbations

- General problem

$$\delta(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) = 8\pi\delta(T_{\mu\nu})$$

- This introduces, at linear level, the following quantities:

$$\delta\rho, \delta p, \delta v, \delta g_{\mu\nu}$$

- The general perturbed metric can be written as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu}$$

where in all generality

$$g^{(1)}_{\mu\nu} = a^2 \begin{pmatrix} 2\psi & w_i \\ w_i & 2\phi\delta_{ij} + h_{ij} \end{pmatrix} \quad (5)$$

- However, this is far too general for what we need. First, as any tensor field, the metric tensor can be written as a sum of terms that depend on purely scalar, vector and tensor quantities. For instance

$$w \equiv w^{\parallel} + w^{\perp} = \nabla w_s + w^{\perp} \tag{6}$$

and

$$h_{ij} \equiv h\delta_{ij}/3 + h_{ij}^{\parallel} + h_{ij}^{\perp} + h_{ij}^{T} \tag{7}$$

Only the scalar quantities couple to $\delta\rho$ so we consider only these now.

- Then, we can choose any reference frame, i.e. we can change the coordinates to $y^{\mu} = f(x^{\nu}) \rightarrow 4$ conditions on the metric coefficients. However, we would like to keep the unperturbed part $g_{\mu\nu}^{(0)}$ as it is. Then we can subject the perturbed part to 4 extra conditions: this is called ***gauge choice*.**
- One of the simplest choice is called *longitudinal* or *Newtonian*: we put $w_i = h = 0$ and obtain finally

$$g_{\mu\nu}^{(1)} = a^2 \begin{pmatrix} 2\Psi & 0 \\ 0 & 2\Phi\delta_{ij} \end{pmatrix} \tag{8}$$

- Then we get the general perturbation equation. In particular, we also obtain $\Psi = \Phi$ if the fluid is a perfect fluid (no anisotropic stress).

- **Newtonian regime**: small scales with respect to horizon H^{-1}, small velocities, for a single component

$$\begin{aligned}\dot{\delta} &= -\nabla v \\ \dot{v}_i &= H v_i - \nabla \Phi \\ \triangle \Phi &= -4\pi a^2 \rho \delta\end{aligned}$$

- Deriving the first we get for the density perturbations $\delta = (\rho - \rho_M)/\rho_M$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi\rho_M \delta \equiv \frac{3}{2} H^2 \delta$$

$\rightarrow$ evolution of a density contrast under a gravitational potential proportional to ρ_M.

- To solve the equation is sufficient to know the behavior of $H(t)$ and $\rho(t)$. If there is only matter, $H^2 = H_0^2 a^{-3}$ and the solutions are

$$\delta \sim a, \quad \delta \sim a^{-3/2}$$

- Dark energy introduces **two main effects**: Growth of perturbations and Integrated Sachs Wolfe effect.

Growth of perturbations

- If there is a smooth component as the cosmological constant (for which $\delta_\Lambda = 0$) then all it changes is that $H^2 = H_0^2 \Omega_m a^{-3}$ where $\Omega_m < 1$: weaker gravity forcing. If $\Omega_m \approx const.$ then

$$\begin{aligned} \delta &\sim a^p \\ p &= \frac{1}{4}(-1 \pm \sqrt{1 + 24\Omega_m}) \end{aligned}$$

- If the cosmological constant is the dominating component, the second member vanishes (there is no potential gradient) : $\delta \sim$ const.
- A better way to study perturbation growth is to parametrize the growth in this way

$$\frac{d \log \delta}{d \log a} \approx \Omega_m(a)^\gamma, \quad \gamma \approx 0.55$$

Beyond the cosmological constant

- A cosm. constant, as we have seen, has a negative pressure and does not fluctuate.
- But any fluid with pressure $p = w\rho$ such that $w < -1/3$ has in fact similar properties.
- The simplest case is a scalar field ϕ. Pressure and energy for a potential $V(\phi)$ are

  $$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

  So that the conservation equation

  $$\dot{\rho} + 3H(\rho + p) = 0$$

  is in fact the **Klein Gordon** equation

  $$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

The eq. of state is

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

kin. energy dominates	$w \to 1$	stiff fluid
equipartition	$w \to 0$	dust
pot. energy dominates	$w \to -1$	cosm. const.
negative kin. energy	$w < -1$	phantom

- In general, the equation of state will vary with time.
- As a first approximation

$$w = w_0 + w_1 z$$
$$w = w_0 + w_1 (a - 1)$$

- This matter component, denoted dark energy or quintessence, has properties similar to the cosm. constant but

1. it **fluctuates** at large scale
2. is composed of particles of microscopical **mass**

$$H\hbar = 10^{-32} eV$$

   (the energy of a particle with Compton wavelength is equal to the horizon scale 3000 Mpc !)
3. gives an expansion different from the cosm. constant, and therefore in principle **observable** with the same methods as above

- For instance, if the potential is an exponential

$$V(\phi) = \exp(-\mu\phi)$$

the scale factor grows, when the dark energy is dominating, as a power law

$$a \sim t^{p=\frac{3}{\mu^2}}$$

instead of as an exponential (as for Λ), just as a perfect fluid with **constant** equation of state

$$w = \frac{2}{3p} - 1$$

In general, of course, $w = w(z)$.

- The crucial point is that a scalar field does not cluster because its "**sound speed**"

$$c_s^2 = \frac{\delta p}{\delta \rho}$$

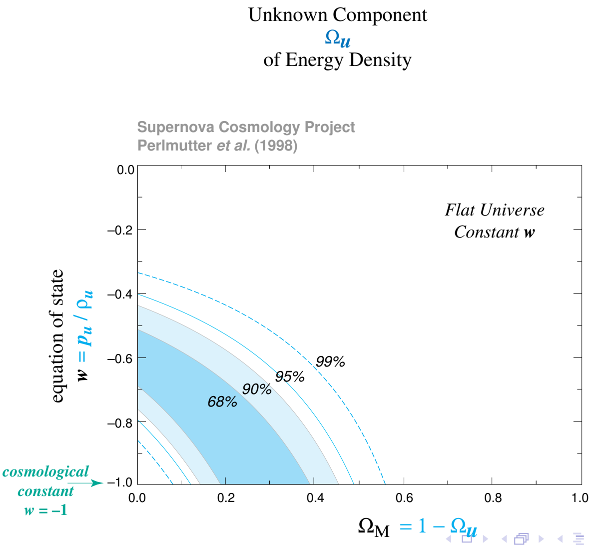
is equal to the speed of light. That is, its own pressure resists gravitational collapse. Perturbing the Klein-Gordon equation in Fourier space:

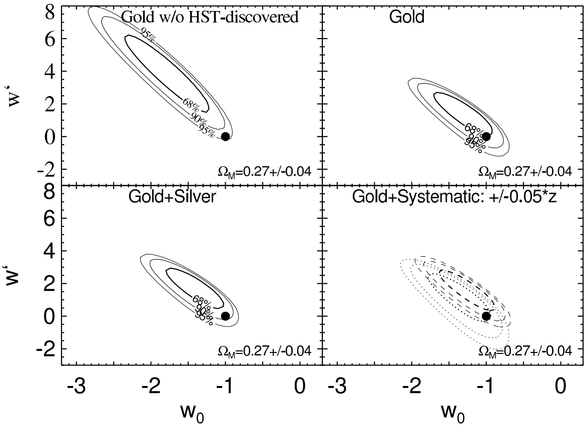
$$\ddot{\varphi} + 2H\dot{\varphi} + c_s^2 k^2 \varphi + a^2 U'' \varphi = 0$$

At small scales, k dominates and the solution for φ **oscillates acoustically** around zero instead of growing. Therefore, a scalar field is a good candidate for dark energy.

- Repeat the SNIa fit with

$$E^2(z) = \Omega_M(1+z)^3 + \Omega_{DE}(1+z)^{3+3w_{DE}}$$




Gold w/o HST-discovered
Gold
Gold+Silver
Gold+Systematic: +/-0.05*z
68%
90%
95%
Ω_M=0.27+/-0.04
w'
w_0
8
6
4
2
0
-2
-3
-2
-1
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