

Gravitational instabilities in protostellar discs and the formation of planetesimals

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Gravitational instabilities in protostellar discs

- Conditions for instability
- Dynamics of self-gravitating discs:
 - Conditions for fragmentation/self-regulation
 - Local vs global behaviour
- Planetesimal formation and evolution in spiral arms
- Self-regulated disc models and their application to planetesimal formation
- Self-gravitating discs with ALMA

Linear stability criterion

Well known axisymmetric instability criterion:

$$Q = \frac{c_{\rm s}\kappa}{\pi G\Sigma} < \bar{Q} \approx 1$$

- * Equivalent form of the instability criterion $\frac{M_{\rm disc}(R)}{M_{\star}} \gtrsim \frac{H}{R}$
- Need the disc to be cold and/or massive
- * What are the masses and aspect ratio in actual protostellar discs?

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 Midplane temperature for irradiated discs (Chiang & Goldreich 1997, Chiang & Youdin 2009) gives:

$$\frac{H}{R} \simeq 0.02 \left(\frac{R}{\mathrm{AU}}\right)^{2/7}$$

- * Therefore *H*/*R* varies from **0.02** at 1AU to **0.06** at 100 AU
- * Need disc masses of order 5% of the stellar mass to be unstable
- Protostellar disc masses difficult to measure (see Hartmann et al 2006)

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Clear trend to have smaller masses at later stages of evolution
- A substantial fraction of Class I (and even some Class II) objects expected to be unstable
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
- If density profile steep, most of the mass might be hidden in optically thick inner parts (Hartmann 2009)



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Non linear evolution of GI

- Investigated numerically in the last decade by several authors (Laughlin & Bodenheimer 1994, Laughlin et al 1998, Pickett et al 2000, Boss 2000, Gammie 2001, Mayer et al 2002, Lodato & Rice 2004, 2005, Mejia et al 2005, Boley et al 2006)
- Early simulations used an isothermal or polytropic equation of state (Laughlin & Bodenheimer 1994, Mayer et al 2002)
- Starting from Gammie (2001) it has become clear that the evolution is strongly dependent on the cooling time t_{cool}
- Introduce a cooling parameter as the ratio of cooling to dynamical timescale

$$\beta = t_{\rm cool} \Omega$$

Thermal self-regulation of GI

- * Role of cooling time clear if one thinks at the form of the stability parameter Q $Q = \frac{c_{\rm s}\kappa}{\pi G\Sigma} \propto T^{1/2}$
- Development of the instability feeds energy back onto the equilibrium and stabilizes the disc
- * Works as an effective thermostat for the disc
- * Expect the disc to stay close to marginal stability $Q \sim 1$
- In order for thermostat to work, cooling time must not be faster than instability growth time
- * Expect a change in behaviour for $\beta \sim$ a few

Short cooling time: fragmentation

* Fragmentation occurs for cooling times shorter than $\beta \sim 3-5$ (depending on numerical setup, ratio of specific heats, etc... Gammie 2001, Rice, Lodato & Armitage 2005, Clarke, Harper-Clark & Lodato 2007)

Simulation by Peter Cossins $\beta = 4$

If cooling strongly temperature dependent (e.g. close to opacity gap) fragmentation might be easier (Johnson & Gammie 2003, Cossins, Lodato & Clarke 2009b)





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Thermal saturation of GI

Cossins, Lodato & Clarke 2009

- Self-regulation is established through thermal saturation of the spiral waves.
- Amplitude of density perturbation must be related to cooling rate
- We find that:



 Natural if consider that energy content of waves is proportional to the square of the perturbed fields



Spectrum of excited modes

* Many modes excited at the various radii (roughly on the local *H* scale)



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Sonic condition for spiral waves Cossins, Lodato & Clarke 2009

- We have computed the pattern speed of the underlying spiral structure and its Mach number
- The Doppler-shifted Mach number is very close to unity, independently on radius, cooling rate, and disc mass.
- Density jump for almost sonic shocks also directly leads to

$$\frac{\Delta \Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}}$$



Local vs global behaviour Lodato & Rice 2004

- Can the evolution of self-gravitating discs be described within the standard, local,
 α-like prescription?
- Can compute gravitational + Reynolds stresses directly from simulations and compare with expectations from standard *α*-theory (LR04, see also Boley et al. 2006)



Local vs global behaviour

Cossins, Lodato & Clarke 2009

- Can the evolution of self-gravitating discs be described within the standard, local, α-like prescription?
- Described in detail by Balbus & Papaloizou (1999), recently discussed extensively by Cossins et al (2009)
- * Relation between energy and angular momentum densities in a density wave

$$\mathcal{E} = \Omega_{\rm p} \mathcal{L} \longrightarrow \dot{\mathcal{E}} = \Omega_{\rm p} \dot{\mathcal{L}}$$

* Relation between power and stress due to local (viscous) processes

$$\dot{\mathcal{E}}_{\nu} = \Omega \dot{\mathcal{L}}_{\nu}$$

* If density waves dissipate far from co-rotation, behaviour is non-local

Local vs global behaviour

Cossins, Lodato & Clarke 2009

Degree of non-locality can be measured by

$$\xi = \left| \frac{\Omega - \Omega_{\rm p}}{\Omega} \right|$$

 Sonic condition for wave dissipation also tells us something about this:

 $\xi \approx \frac{c_{\rm s}}{v_{\phi}} = \frac{H}{R}$

To the extent that the disc is thin (*H*<<*R*), global behaviour should be negligible

 Possible to construct local, viscous models of disc evolution (Clarke 2009, Rafikov 2009)



- * Recent work has cast doubts on convergence of critical cooling time for fragmentation as obtained by SPH (Meru & Bate 2010, 2011)
- Two different results:
 - 1. Meru & Bate (2010): location of fragments depends on surface density profile! (Very difficult to understand....)
 - * In particular, critical value of β depends on $\Sigma(R)R^2$

$$m(R) = \frac{\Delta(R)R}{M_{\star}}$$

2. Meru & Bate (2011): critical value of β keeps increasing with increasing number of particles

- * Lodato & Clarke (2011): both effects are due to resolution!
- Indeed,

$$\frac{h}{H} = \frac{\eta}{m(R)} \left(\frac{2q}{\pi^2 Q^2 N}\right)^{1/3}$$

- * Lodato & Clarke (2011): both effects are due to resolution!
- * Indeed, $\frac{h}{H} = \frac{\eta}{m(R)} \left(\frac{2q}{\pi^2 Q^2 N}\right)^{1/3}$ Smoothing length/ Thickness







$$\frac{h}{H} = \frac{\eta}{m(R)} \left(\frac{2q}{\pi^2 Q^2 N}\right)$$

- Simple interpretation if
 - Frag. inhibited for insufficient resolution
 - * Resolution depends on β

$$\beta_{\rm res} \approx 2 \frac{H}{h} \propto m(R) N^{1/3}$$

More details in Lodato & Clarke (2011)







- * Origin of cooling rate dependent fragmentation:
 - Artificial heating? Resolution criterion corresponds to artificial viscosity providing a fixed fraction (5%) of the cooling rate.
 - Smearing of the density peak?
- Astrophysical consequences:
 - slight adjustment in location of fragmentation in protostellar discs
 - No major effects on local/global behaviour

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Planetesimal formation

- * The first step of the core accretion model for planet formation
- * How to form km-sized planetesimals?
- Combined effect of local pressure maxima, drag force, and dust selfgravity
- The so-called streaming instability (Youdin & Johansen) works only at rather large metallicities
- * Alternatives?

- Effects of gas drag on solid particles is to induce fast migration towards pressure maxima.
- In a laminar disc this produces a fast inward migration of meter-sized particles (Weidenschilling 1977)

$$v_r = \frac{\Delta v}{\Omega t_{\rm s} + 1/\Omega t_{\rm s}}$$

$$\Delta v \approx \frac{c_{\rm s}^2}{v_{\rm K}} \frac{\partial \ln \rho}{\partial \ln R}$$

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- Pressure maxima in spiral structure efficient trap for meter sized objects (see also Haghighipour & Boss 2003, Durisen et al 2005).
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Solid agglomeration in pressure maxima (Rice, Lodato et al 2004, 2006)

- Density of meter sized objects enhanced by up to two orders of magnitude
- Density becomes high enough to become comparable to Roche density
- Gravitational collapse of solids is possible
- Confirmed through additional simulations including the solids self-gravity (Rice et al. 2005)
- Resulting planetesimals mass expected to be high (but difficult to measure from simulations)



Evolution of large bodies in a selfgravitating disc (Britsch, Lodato, Clarke 2008)

- What is the dynamics of large (decoupled) bodies in a self-gravitating disc?
- Simulations of km-10³km sized bodies
- Similar to the case of planetesimals in an MRI turbulent disc (Nelson 2005), they undergo strong stochastic migration
- Reach average eccentricities of order e ~ 0.07
- Significantly reduce ability of further growth
- Induce potentially disruptive collisions



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Planetesimals in self-gravitating discs

- Particle traps in spiral arms are an effective way of producing large solid bodies in the disc:
 - Resulting planetesimal mass quite large
 - Dynamically stirred population of planetesimals
 - * Expected to occur in early phases of star formation (<~ 1Myr)
 - * <u>Is this process limited to some specific radial range in the disc?</u>
 - * Note: Rice et al. used an idealized cooling function leading to a rather large amplitude spiral $\Delta\Sigma/\Sigma\approx 0.1$
 - * Need a detailed model of self-gravitating discs with realistic cooling

 If transport is local (cf. Cossins et al 2009), then in thermal equilibrium (and absent other sources of heating, e.g. irradiation):

$$\alpha = \frac{4}{9} \frac{1}{\gamma(\gamma - 1)} \frac{1}{\Omega t_{\text{cool}}}$$

- Possible to construct models of self-regulated discs (Q~1), where viscosity is related to cooling time (Clarke 2009, Rafikov 2009)
- * Identify various possible regimes for self-gravitating protostellar discs

Clarke 2009, Cossins, Lodato & Clarke 2010, Rafikov 2009



 Note: such models generally have a steep density profile in the inner disc, e.g.:

$$\Sigma \propto R^{-9/4}$$

 Most of the mass might be hidden in the optically thick innermost few AU (cf. Hartmann 2009)

Clarke 2009, Cossins, Lodato & Clarke 2010, Rafikov 2009



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Where do planetesimals form? Clarke & Lodato (2009)

- * Check at which radii spiral arms concentration can produce large solid bodies
- * Under what conditions is solid concentration effective?
- * In the presence of a density perturbation on a scale λ :

$$v_r \approx v_\phi - v_{\rm K} \approx \frac{c_{\rm s}^2}{v_{\rm K}} \left(\frac{R}{\lambda}\right) \frac{\Delta \Sigma}{\Sigma}$$

Migration time is:

$$t_{\rm mig} = \frac{\lambda}{v_r} \approx \left(\frac{\lambda}{H}\right)^2 \left(\frac{\Delta\Sigma}{\Sigma}\right)^{-1} \frac{1}{\Omega}$$

* For a self-gravitating spiral structure $\lambda \sim H$, and hence t_{mig} is comparable to structure lifetime for

$$\frac{\Delta\Sigma}{\Sigma} > f \approx 0.1$$

Where do planetesimals form?

Clarke & Lodato (2009)

- Planetesimal formation through this process occurs at 30AU < R < 50 AU
- Roughly coincident with the location of the Kuiper belt
- Some evidence for a large inner hole in debris disc systems (Currie et al. 2008), based on the apparent increase of debris disc brightness at late ages ~ 10-15 Myrs
- Rapid production of large bodies in the outer disc may preserve sub-mm emission in the T Tauri phase (Takeuchi, Clarke & Lin 2005)



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66 antennas with a maximum baseline of 18km

Unprecedented angular resolution (down to ~ 1mas)

First science in 2012

At mm wavelengths disc emission optically thin --> possible to detect non-axisymmetric, spiral structures?

venerdì 18 febbraio 2011

- Start from one of our simulations, e.g. $M_* = 1M_{Sun}$, $M_{disc} = 0.2M_*$
- * "Place" disc at 140pc (in Taurus) or at 50 pc (distance to TW Hya)
- * Assume a "standard" opacity law





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0.2

-0.2

0.2

-0.2

0.2

-0.2

Conclusions

- Class I discs are likely to be gravitationally unstable
- Self-regulated evolution of GI leads to sustained angular momentum transport for ~ 1 Myr, bringing the disc into the T Tauri phase
- Density waves dissipate when they become sonic
- Induced transport is local IF disc is sufficiently thin
- Spirals induce rapid formation of planetesimal in an annular region at large distances (30-50 AU): possibly consistent with observations of debris discs and the Kuiper belt
- * ALMA could be able to detect such discs very soon!