Black Hole Binaries

From X-ray binaries to black hole binaries

X-ray binaries (XRBs) are referred to as the binary, X-ray emitting systems.

The compact, X-ray emitting source may be a WD, a NS or a black hole; in this last case, the system is called black hole binary (BHB).

Accretion is still the main driver of X-ray emission.

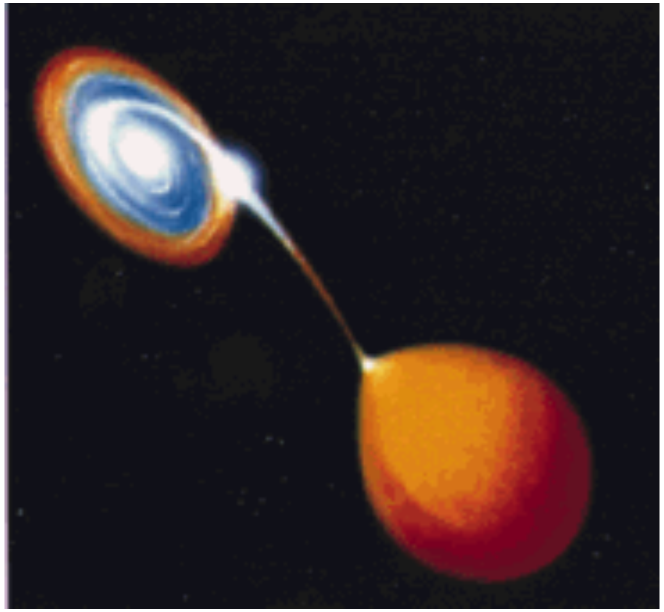
↓

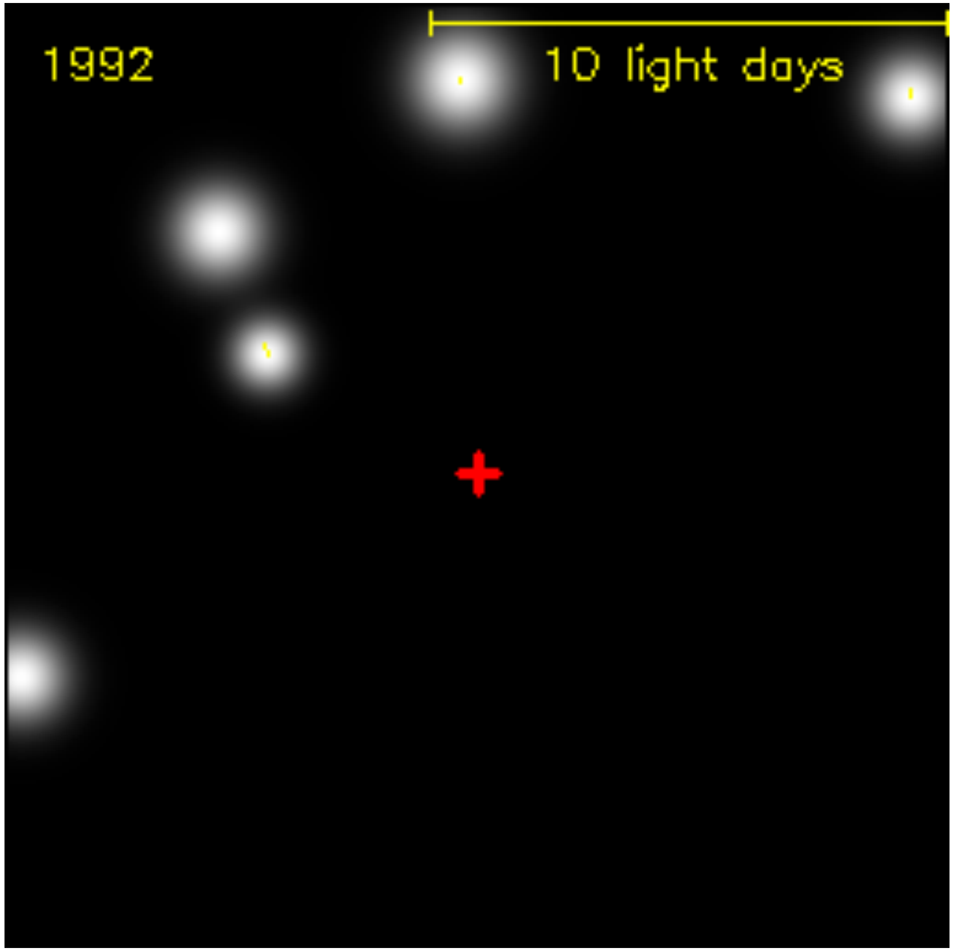
Most X-ray properties in common with systems with an accreting NS
Black Hole Candidates (BHCs) among soft X-ray transients

How can we infer the presence of a Black Hole (it is *black*!)

One way is via gravitational effects on nearby stars (but only for the Galactic Center, SgrA*).

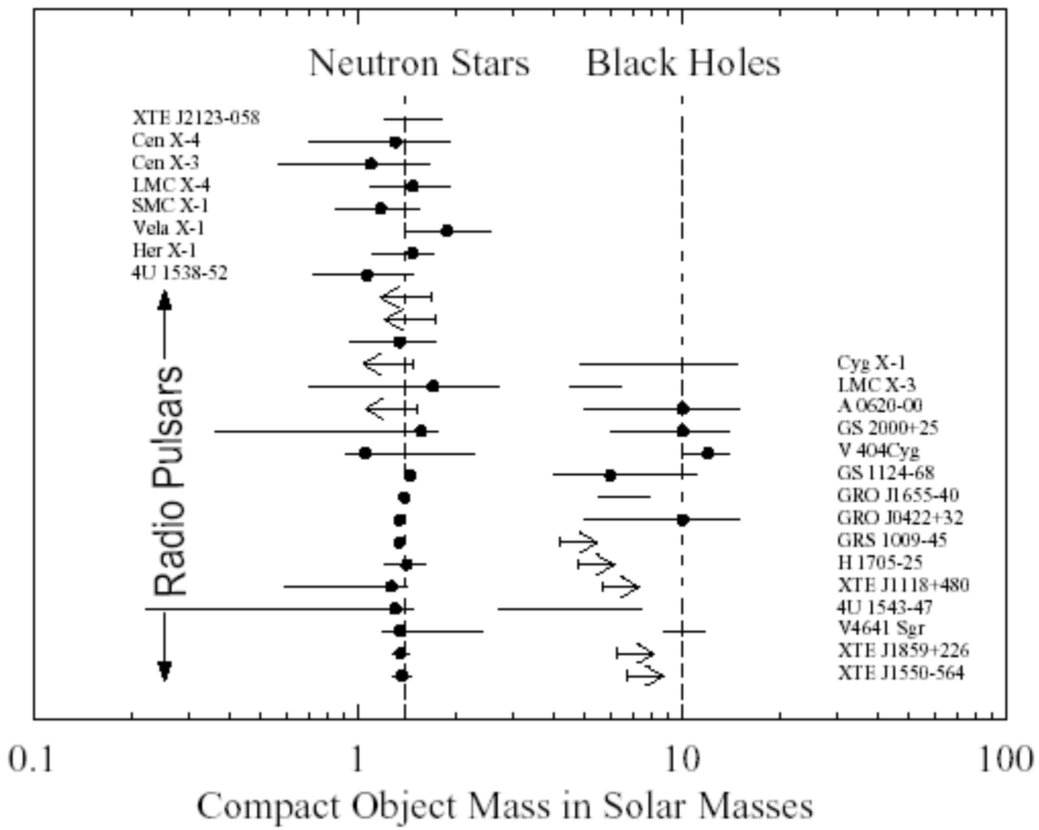
Another way via its “hot” accretion disc and consequent high-energy emission





Confirmed BHBs: statistics (I)

• About 20 BHBs + further 20 BHCs (candidates), where the presence of the black hole is suggested by e.g. the mass function and the X-ray spectral properties (soft thermal + hard power-law emission)



Confirmed BHBs: statistics (II)

Only 3 persistently X-ray bright BHBs, all being HMXBs

All BHBs have recurrent outbursts

Likely, many more to be found ($\geq 10^8$ BH remnants in the Galaxy, van den Heuvel 1992)

If transients (e.g., due to disc instabilities, then bursts)

BHBs are easier to be found

Table 16.1 Black-hole binaries confirmed with the mass function

Source name		Year[a]	Type[b]	$f(M)$ ($M_\odot$)	M_x ($M_\odot$)	X-ray spectrum[c]
Cyg X-1			H, P	0.244 ± 0.005	6.9–13.2	S + PL
LMC X-3			H, P	2.3 ± 0.3	5.9–9.2	S + PL
LMC X-1			H, P	0.14 ± 0.05	4.0–10.0	S + PL
J0422 + 32	V518 Per	'92	L, T	1.19 ± 0.02	3.2–13.2	PL
0620–003	V616 Mon	'17, '75	L, T	2.72 ± 0.06	3.3–12.9	S + PL
1009–45	MM Vel	'93	L, T	3.17 ± 0.12	6.3–8.0	S + PL
J1118 + 480	KV Uma	2000	L, T	6.1 ± 0.3	6.5–7.2	PL
1124–684	GU Mus	'91	L, T	3.01 ± 0.15	6.5–8.2	S + PL
1354–645	BW Cir	'71,'87,'97	L, T	5.75 ± 0.3	>7.8	S + PL
1543–475	IL Lup	'71,'83,'92	L, T	0.25 ± 0.01	7.4–11.4	S + PL
J1550–564	V381 Nor	'98	L, T	6.86 ± 0.71	8.4–10.8	S + PL
J1650–500		2001	L, T	2.73 ± 0.56	4–7.3	PL
J1655–40	V1033 Sco	'94	L, T	2.73 ± 0.09	6.0–6.6	S + PL
1659–487	GX 339–4	($P \sim 460$d)	L, T	5.8 ± 0.5	>5.8	S + PL
1705–250	V2107 Oph	'77	L, T	4.86 ± 0.13	5.6–8.3	S + PL
J1819.3–2525	V4641 Sgr	'99	L, T	3.13 ± 0.13	6.8–7.4	S + PL
J1859 + 226	V406 Vul	'99	L, T	7.4 ± 1.1	7.6–12	S + PL
1915 + 105	V1487 Aql	'92—	L, T	9.5 ± 3.0	10–18	S + PL
2000 + 251	QZ Vul	'88	L, T	5.01 ± 0.12	7.1–7.8	S + PL
2023 + 338	V404 Cyg	'38,'56,'89	L, T	6.08 ± 0.06	10.1–13.4	PL

[a] The year of outburst, including earlier records as optical novae.
[b] H: high-mass binary. L: low-mass binary: P: persistent. T: transient.
[c] X-ray spectral type near the maximum luminosity. S: soft thermal. PL: power-law.

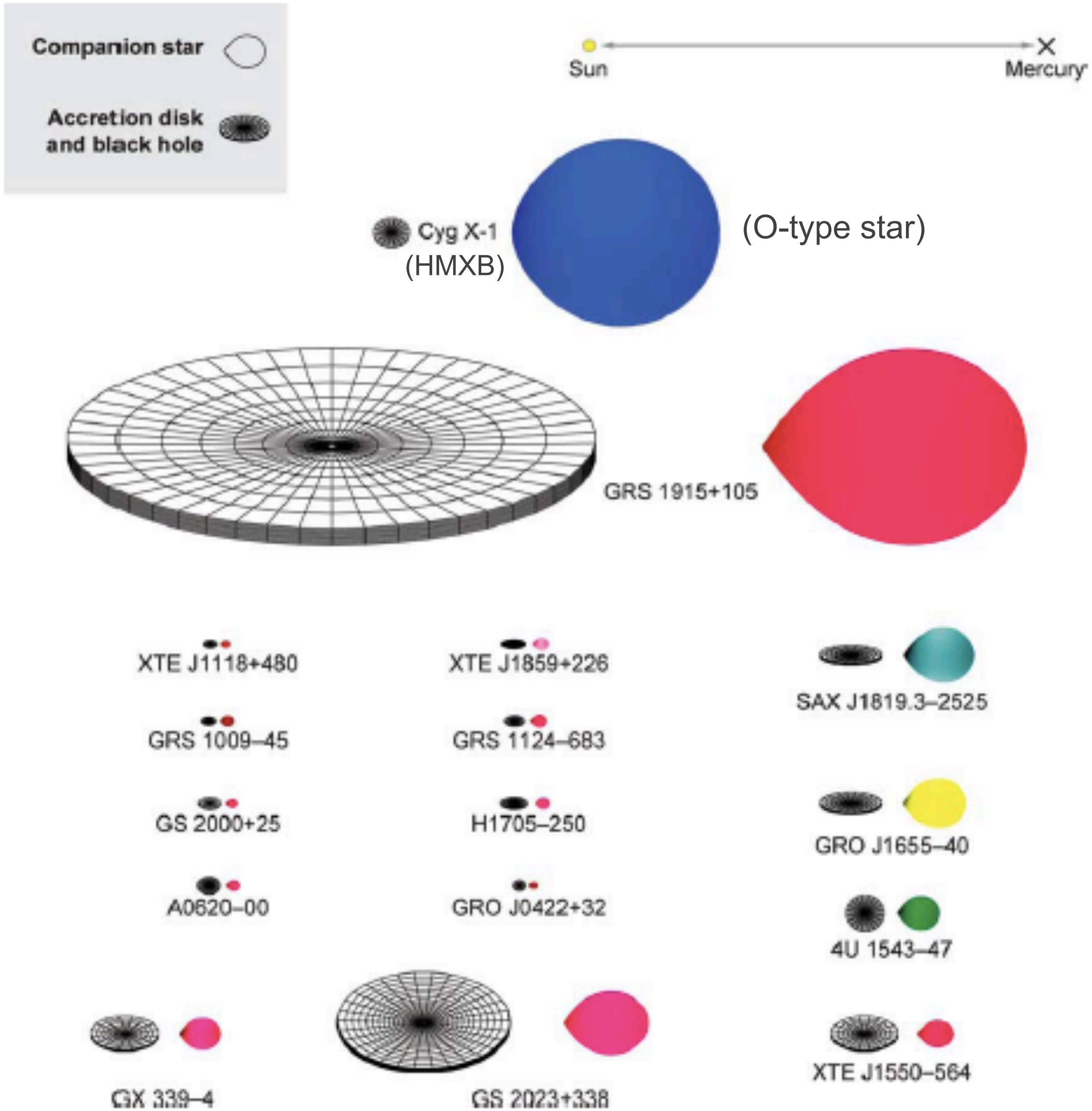
Situation in 2007 (2008)

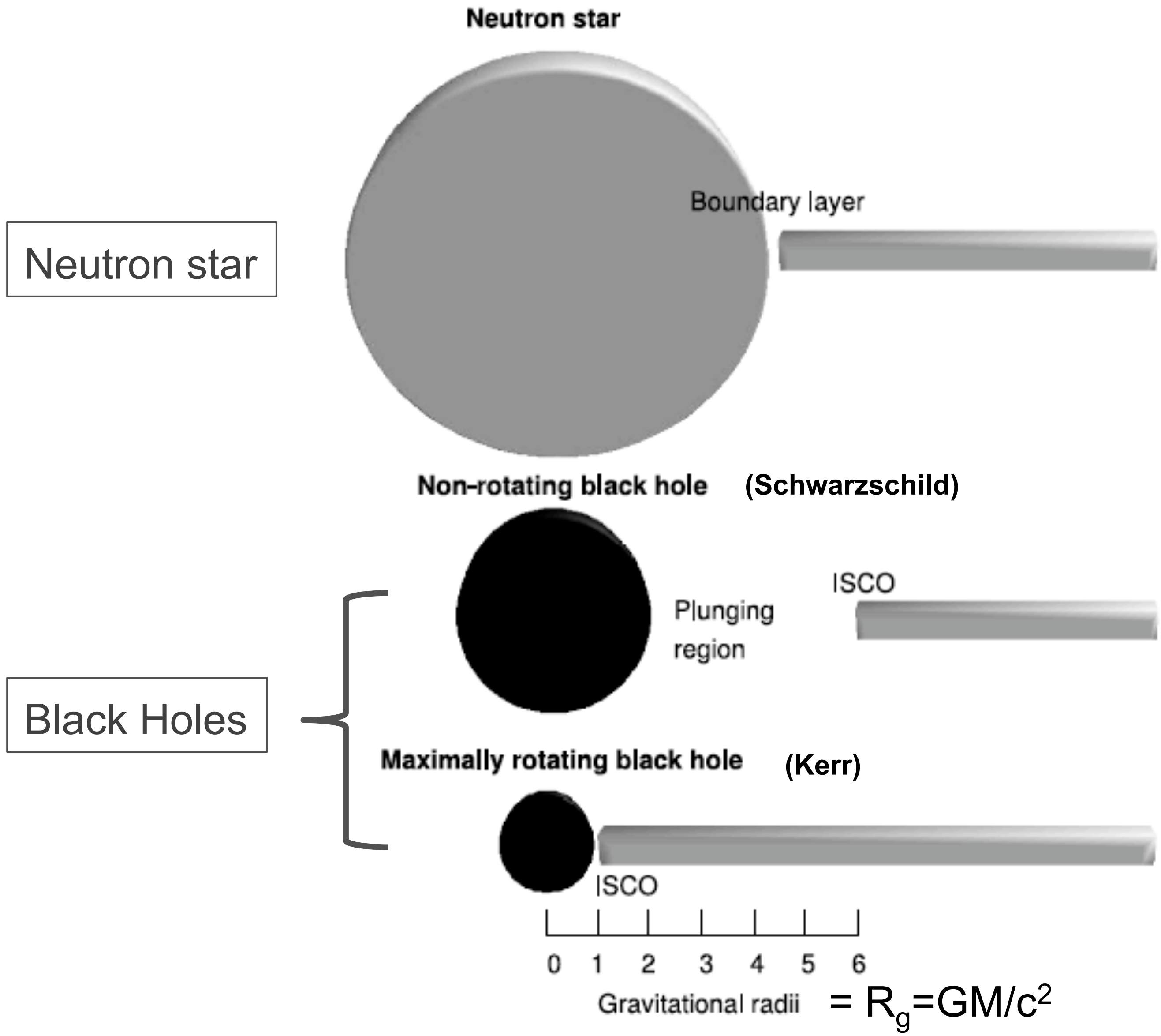
Table 2 Black hole masses in X-ray binaries[a]

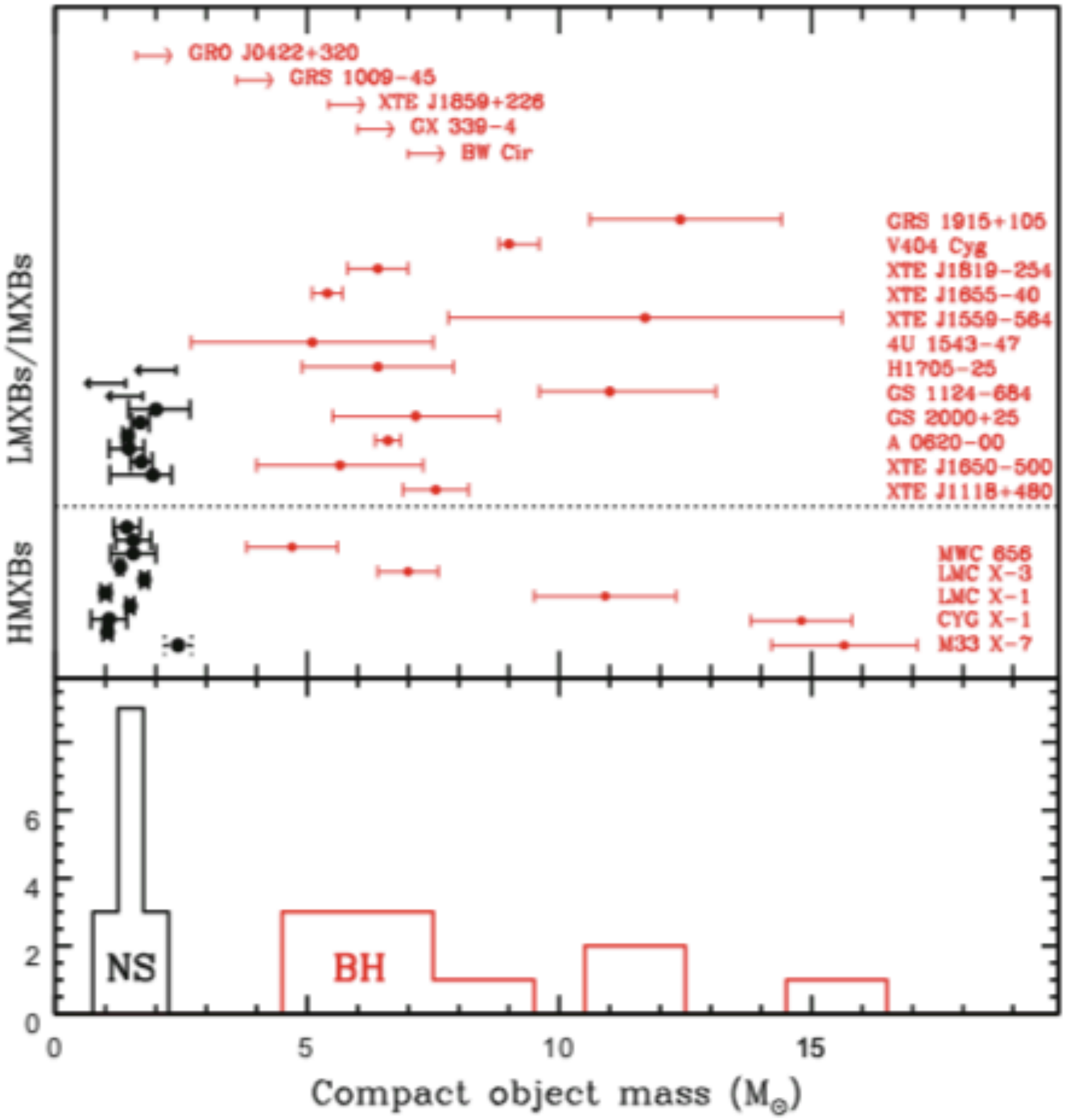
Object	X-ray binary class	Mass ($M_\odot$)	References
GRS 1915+105	LMXB/transient	$12.4^{+2.0}_{-1.8}$	Reid et al. (2014)
V404 Cyg	"	$9.0^{+0.2}_{-0.6}$	
BW Cir	"	>7.0	
GX 339-4	"	>6.0	
XTE J1550-564	"	7.8−15.6	
H1705-250	"	4.9−7.9	
GS 1124-684	"	$11.0^{+2.1}_{-1.4}$	Wu et al. (2016)
GS 2000+250	"	5.5−8.8	
A0620-00	"	6.6±0.3	
XTE J1650-500	"	4.0−7.3	
GRS 1009-45	"	>3.6	
XTE J1859+226	"	>5.42	
GRO J0422+32	"	>1.6	
XTE J1118+480	"	6.9−8.2	
XTE J1819.3-2525	IMXB/transient	6.4±0.6	Macdonald et al. (2014)
GRO J1655-40	"	5.4±0.3	
4U 1543-475	"	2.7−7.5	
Cyg X-1	HMXB/persistent	14.8±1.0	
LMC X-1	"	10.9±1.4	
LMC X-3	"	7.0±0.6	Orosz et al. (2014)
M33 X-7	"	15.7±1.5	
MWC 656	HMXB/transient (?)	3.8−5.6	Casares et al. (2014)

[a]Adopted from Casares and Jonker (2014) unless otherwise stated in the reference column. Lower limits for BW Cir, GRS 1009-45, XTE J1859 + 226, and GRO J0422 + 32 are based on the absence of eclipses, combined with updated determinations of the mass function and q (when available). The lower limit on GX 339-4 is based on the lack of X-ray eclipses plus constraints provided by the *K-correction*

Casares+2017


Companion star
Accretion disk and black hole
Sun
Mercury
Cyg X-1 (HMXB)
(O-type star)
GRS 1915+105
XTE J1118+480
XTE J1859+226
SAX J1819.3–2525
GRS 1009–45
GRS 1124–683
GS 2000+25
H1705–250
GRO J1655–40
A0620–00
GRO J0422+32
4U 1543–47
GX 339–4
GS 2023+338
XTE J1550–564


Neutron star
Neutron star
Boundary layer
Non-rotating black hole
(Schwarzschild)
ISCO
Plunging
region
Black Holes
Maximally rotating black hole
(Kerr)
ISCO
0 1 2 3 4 5 6
Gravitational radii
= R_g=GM/c^2

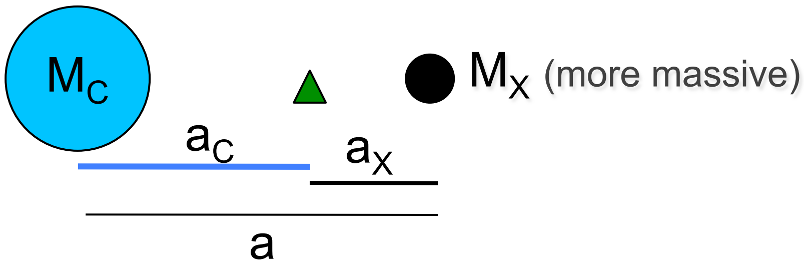


Casares+2017

Mass Function (I)

$$\boxed{\begin{array}{l} a = a_C + a_X \\ M_C a_C = M_X a_X \end{array}}$$

Center of mass



$$\boxed{\mathrm{v}_{\mathrm{C,proj}} = \mathrm{v}_{\mathrm{C,true}} \sin(i) = \left(\frac{2\pi}{P_{\mathrm{orb}}}\right) a_{\mathrm{C}} \sin(i)}$$

(i)

v=projected orbital velocity
i=orbital inclination, i.e., inclination angle between the orbital plane and the plane of the sky (i=0° means face-on)

$$\boxed{G\frac{(M_C + M_X)}{a^3} = \left(\frac{2\pi}{P_{\mathrm{orb}}}\right)^2}$$

III Kepler's law

(ii)

Mass Function (II)

$$\boxed{a = a_C + a_X = a_C + \frac{M_C}{M_X} a_C = a_C \frac{M_C + M_X}{M_X}}$$ (iii)

(ii) + (iii) ⇨

$$\boxed{G \frac{(M_C + M_X) M_X^3}{a_C^3 (M_C + M_X)^3} = G \frac{M_X^3}{a_C^3 (M_C + M_X)^2} = \left(\frac{2\pi}{P_{\text{orb}}} \right)^2}$$

$$\boxed{G \frac{M_X^3 \ \sin(i)^3}{a_C^3 (M_C + M_X)^2} = \frac{4\pi^2}{P_{\text{orb}}^2} \sin(i)^3 \rightarrow \frac{(M_X \sin(i))^3}{(M_C + M_X)^2} = \frac{4\pi^2}{G P_{\text{orb}}^2} (a_C \sin(i))^3}$$

(iv)

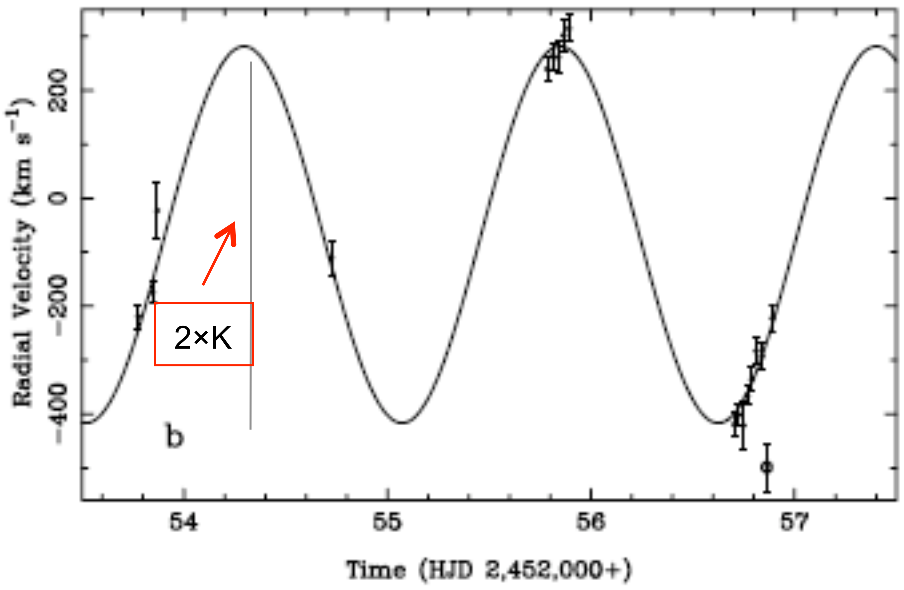
(i) + (iv) ⇨

$$\boxed{f(M) = \frac{(M_X \sin(i))^3}{(M_C + M_X)^2} = \mathrm{v}_C^3 \frac{P_{\text{orb}}}{2\pi G}}$$ Mass function=f(M)

Mass Function (III)

$$f(M) = \frac{M_X^3 (\sin i)^3}{(M_X + M_c)^2} = \frac{PK^3}{2\pi G}$$

X: compact source (accretor)
C: companion source
i: inclination of the binary orbit
P: orbital period
K: amplitude of the Doppler curve which provides the line-of-sight component of the radial velocity=K_C=$V_{C,proj}$



P and **K** are optical measurable quantities from the radial velocity curve (using the Doppler frequency shift of the spectral lines as a function of time/orbital phase)

Application of the III Kepler's law

The mass function sets a lower limit to the real mass of the compact object

Mass Function (IV)

$$f(M) = \frac{M_X^3 (\sin i)^3}{(M_X + M_c)^2} = \frac{PK^3}{2\pi G}$$

$$q = \frac{M_C}{M_X} \text{ mass ratio} \rightarrow f = \frac{M_X^3 (\sin i)^3}{(qM_X + M_X)^2} = \frac{M_X (\sin i)^3}{(q+1)^2}$$

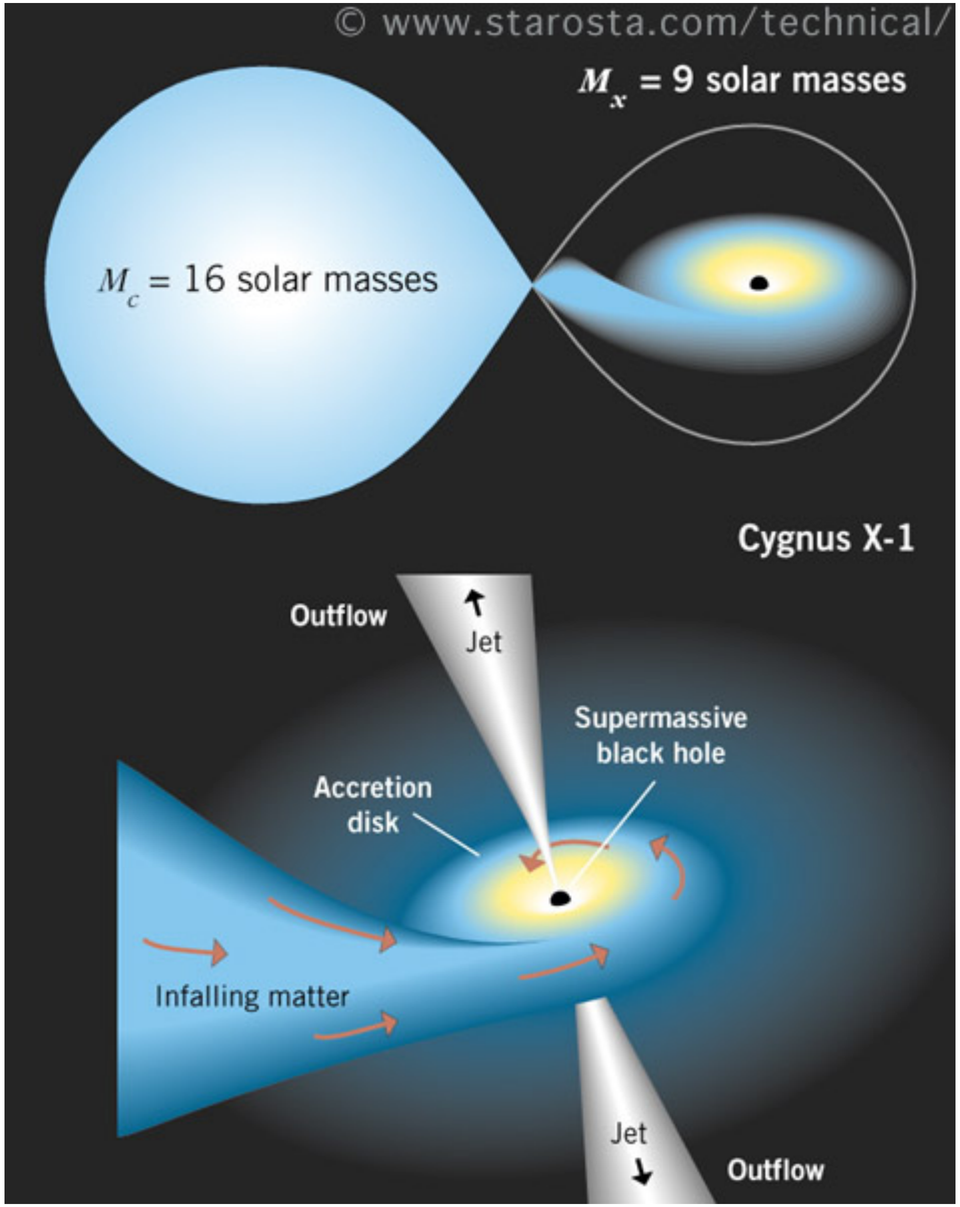
$$\Rightarrow M_X = f \boxed{\frac{(q+1)^2}{(\sin i)^3}} \rightarrow M_X > f(M)$$

>1

➔ If f(M)>3 $M_\odot$, then the compact object should reasonably be a black hole

LMXBs: weak companions
HMXBs: optical light from the companion dominates

Easier to observe the companions once bursts are over



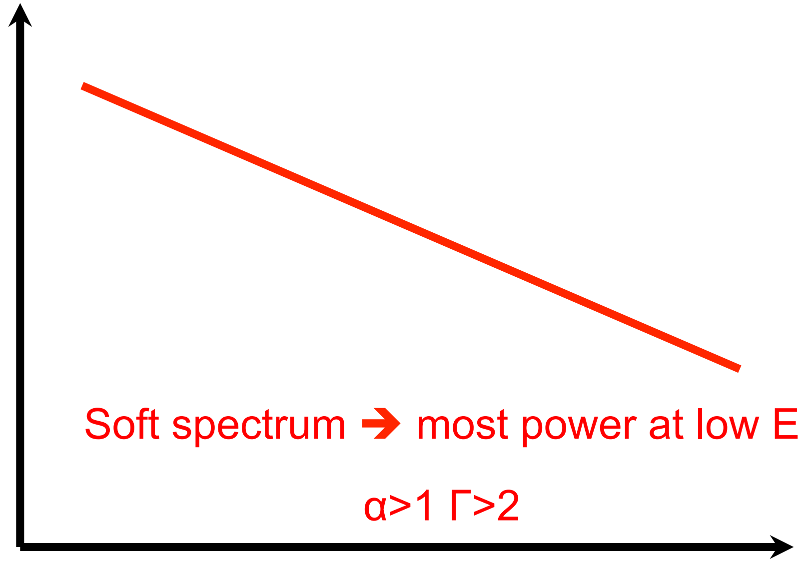
The binary system Cyg X-1

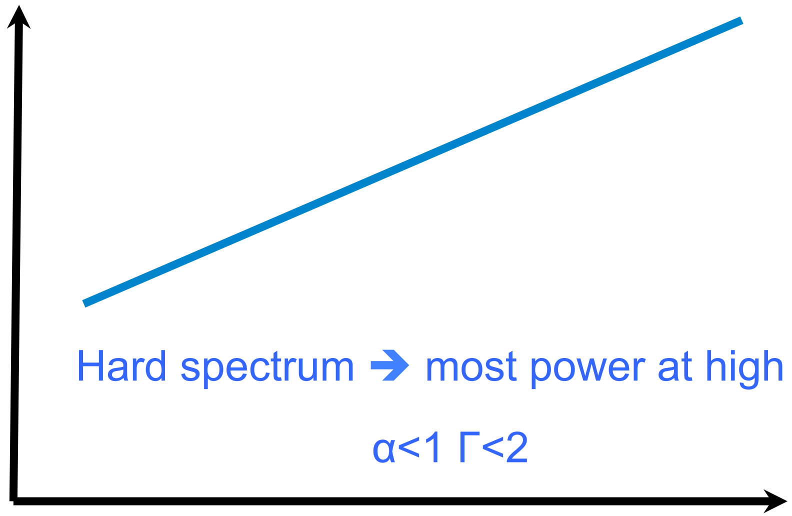
“Internal” structure of BHBs

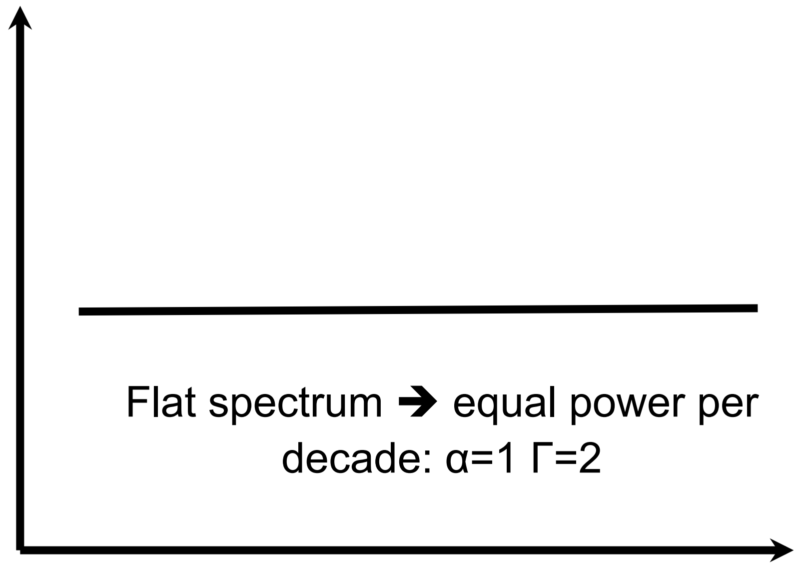
Spectra (basics)

- Plot $\nu f(\nu)$ as this peaks at energy where power output of source peaks.
- $N(E)=AE^{-\Gamma}$
- $F(E)=EN(E)= AE^{-\Gamma+1}=AE^{-\alpha}$

  $\alpha=\Gamma-1$



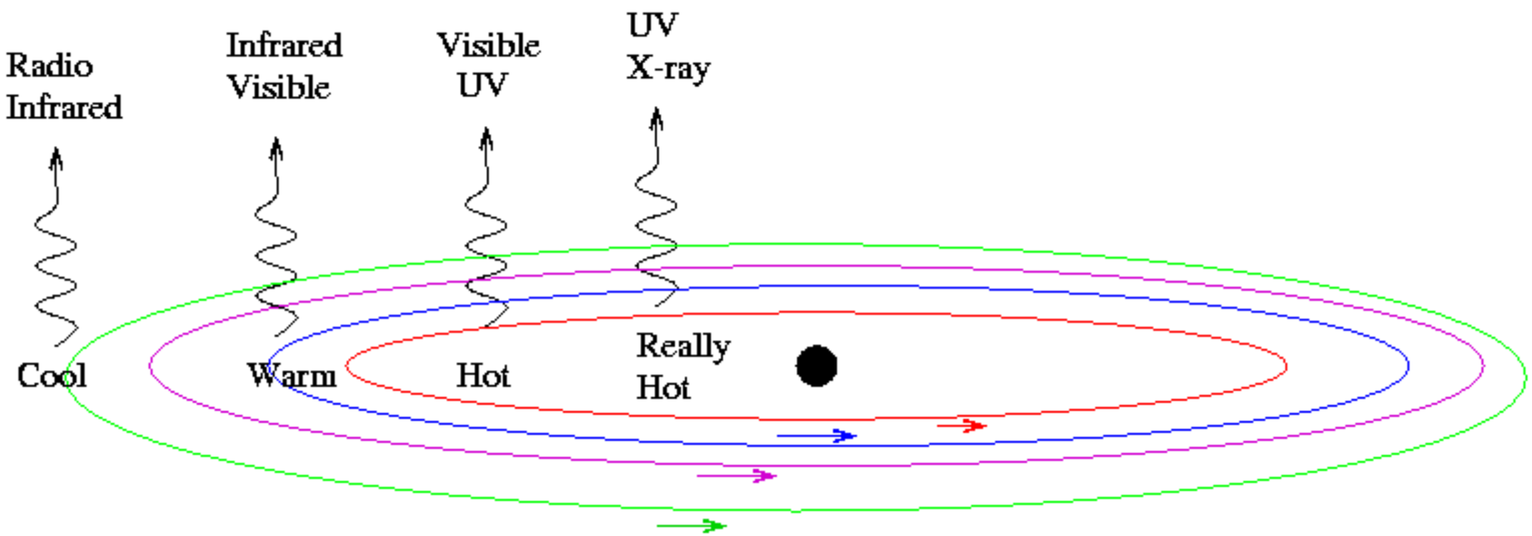




$dL= F(E)\, dE = EF(E)\, dE/E = EF(E)\, d\log E$

(Credits: C. Done)

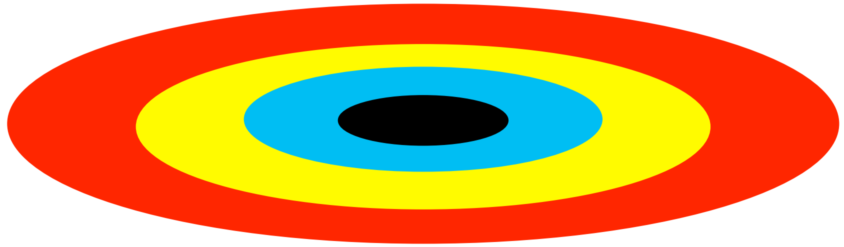
The accretion disc

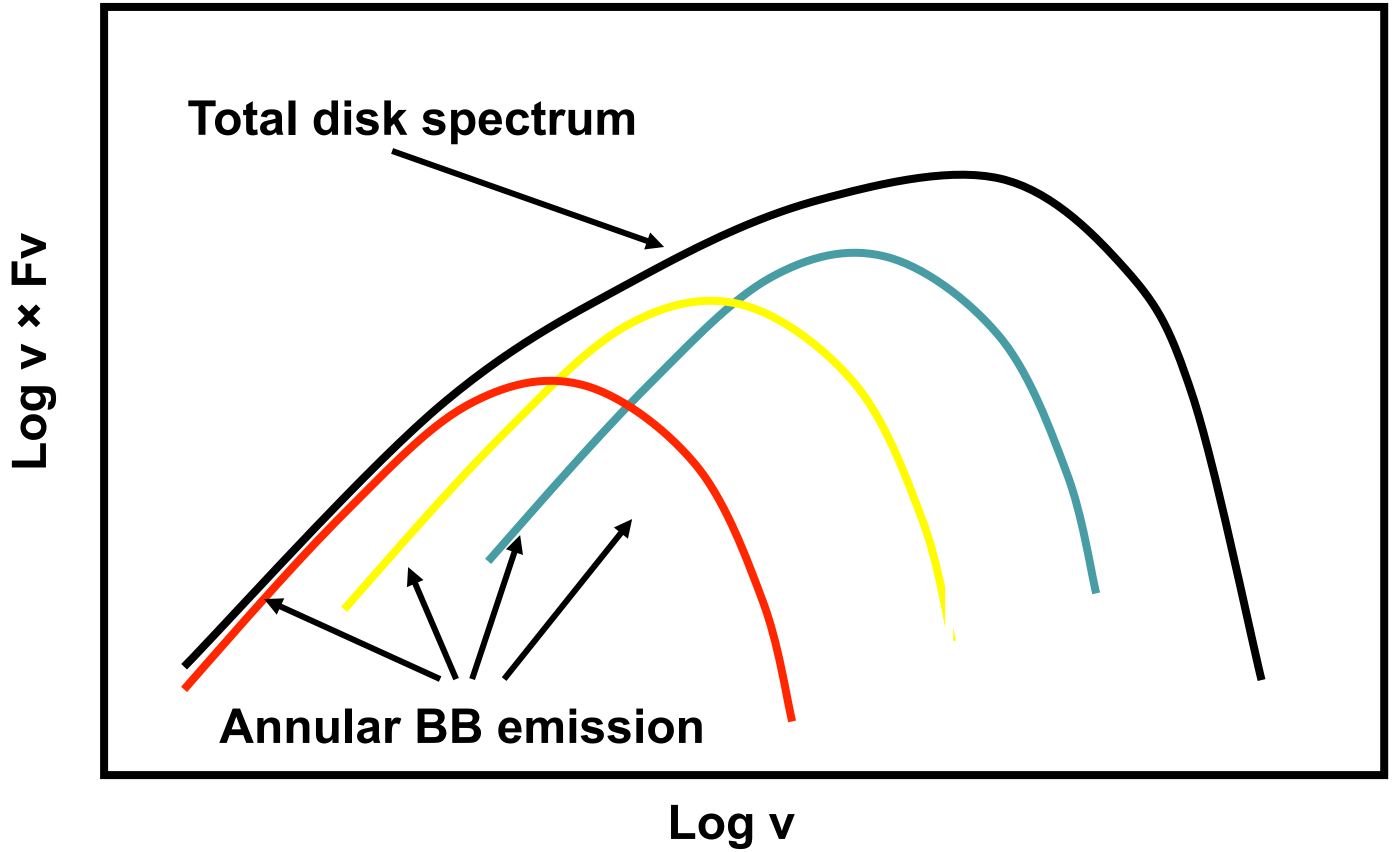


• Friction between adjacent layers which converts gravitational potential energy of the accreting matter into radiation
• Gas in differential rotation, viscosity transports angular momentum outward, while matter is driven inward
• Multi-color blackbody (MCD) emission=BB from layer at different temperatures

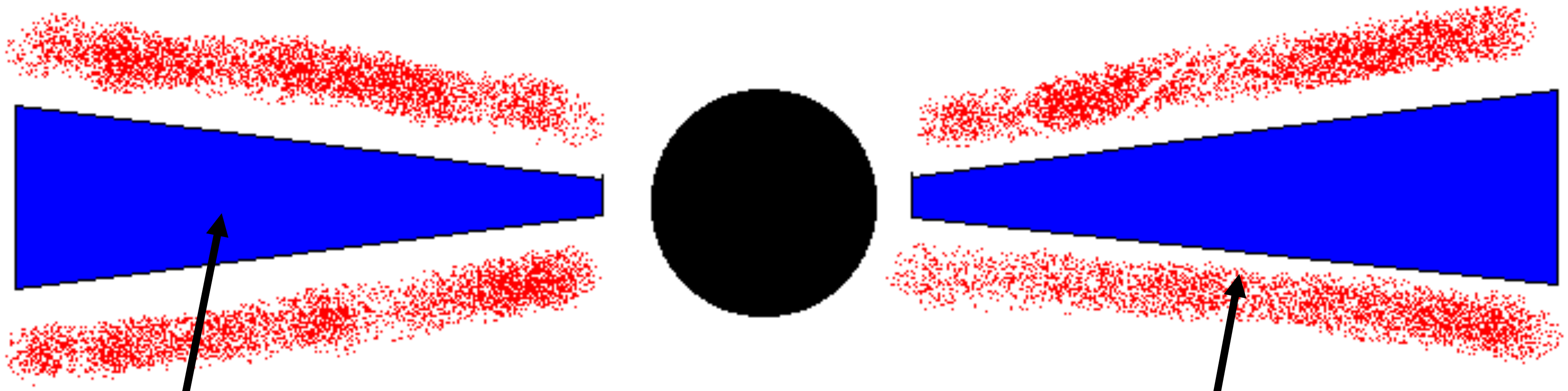
The accretion disc spectrum: multi-color blackbody

M_{BH}=10 $M_\odot$: 1 keV, 10^7 K
M_{BH}=10^8 $M_\odot$: 10 eV, 10^5 K





Accretion disc + corona modeling



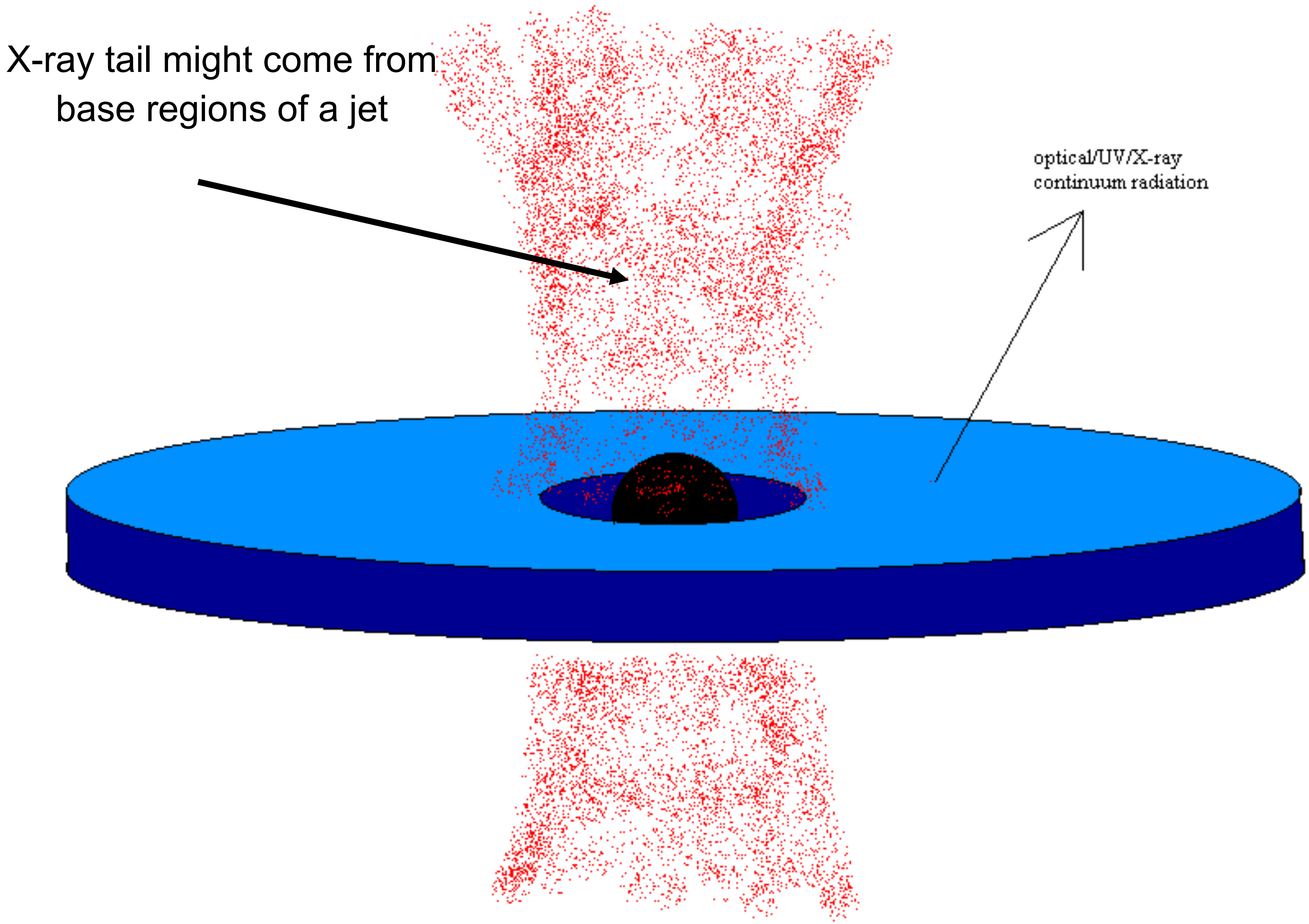
Optically-thick part of the accretion disk emits thermal spectrum… black body radiation with

$$T = \left(\frac{3GM\dot{M}}{8\pi r^3 \sigma_{\rm SB}}\right)^{1/4}$$

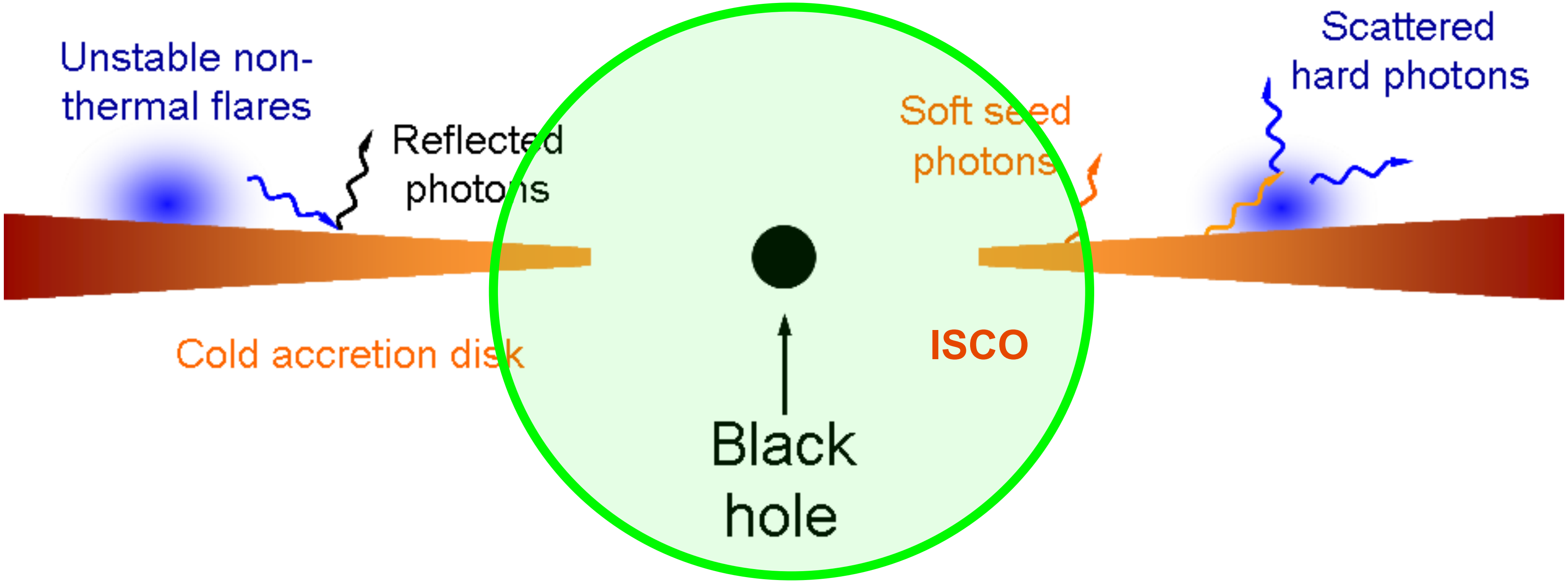
⇨ T(r) prop. $M^{-1/4}$ (see calculations later in this presentation)

X-ray “tail” probably comes from a hot corona that sandwiches the disk… inverse Compton scattering of thermal disk emission by electrons with $T \sim 10^9$K

Similarities with accretion onto super-massive black holes in AGN


X-ray tail might come from
base regions of a jet
optical/UV/X-ray
continuum radiation

Overview of the central region of GBHCs/AGN

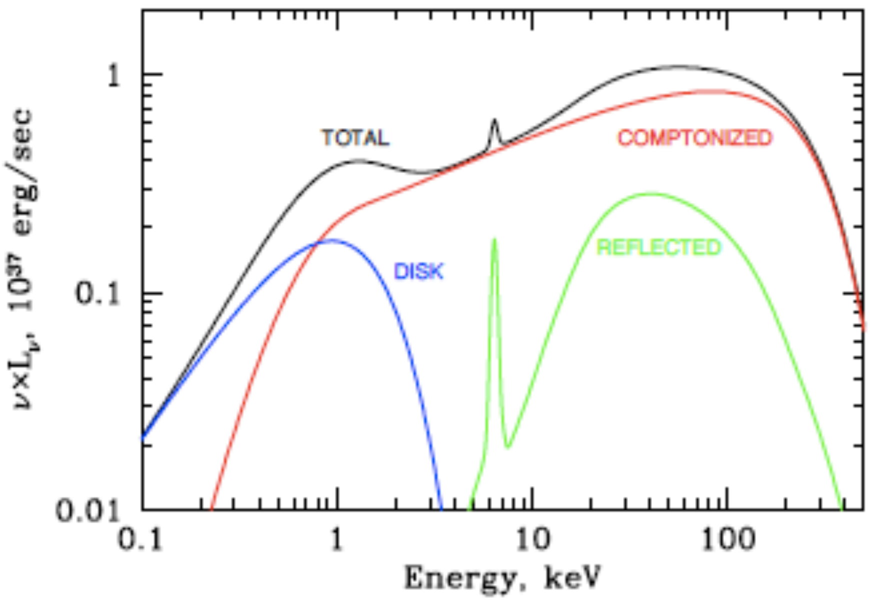


Spectrum:

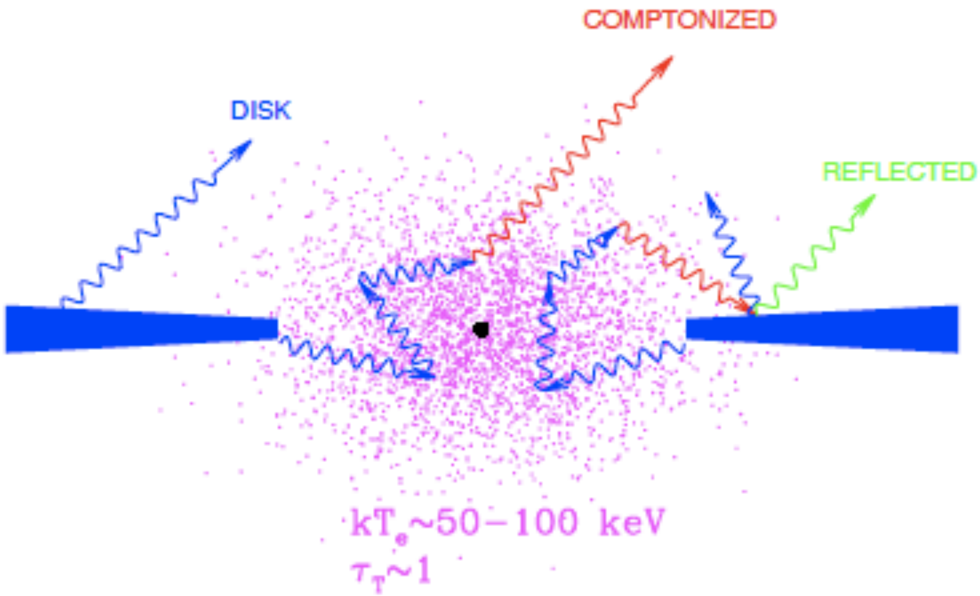
- Direct Continuum (PL + MCD)
- Reflection (Comptonized PL)
- Reprocessed (via photoionization)
  (emission lines; neutral Fe Ka @ 6.4 keV or ionized)

Timing:

- Broad-band/Intrinsic Variability
- Local Variability (QPOs)



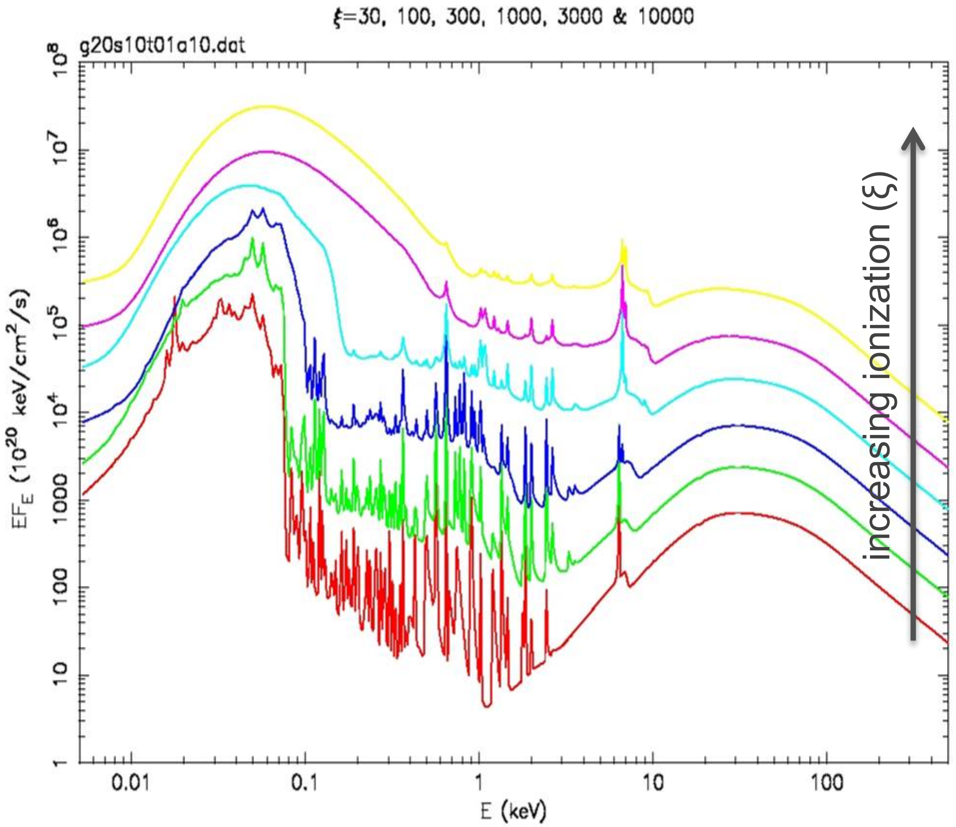
TOTAL
COMPTONIZED
DISK
REFLECTED
$\nu \times L_\nu$, 10^{37} erg/sec
Energy, keV
1
0.1
0.01
0.1
1
10
100

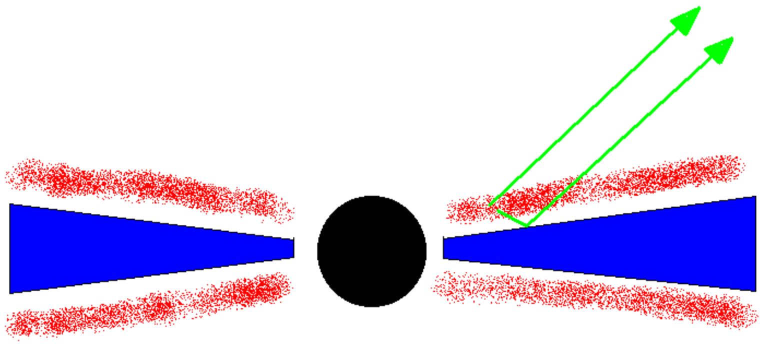


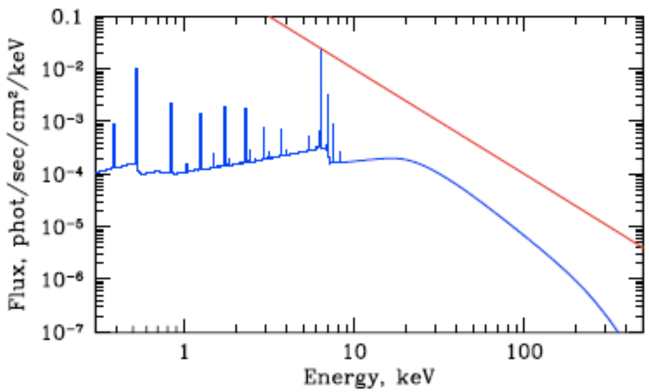
COMPTONIZED
DISK
REFLECTED
$kT_e \sim 50-100$ keV
$\tau_T \sim 1$

X-ray reflection

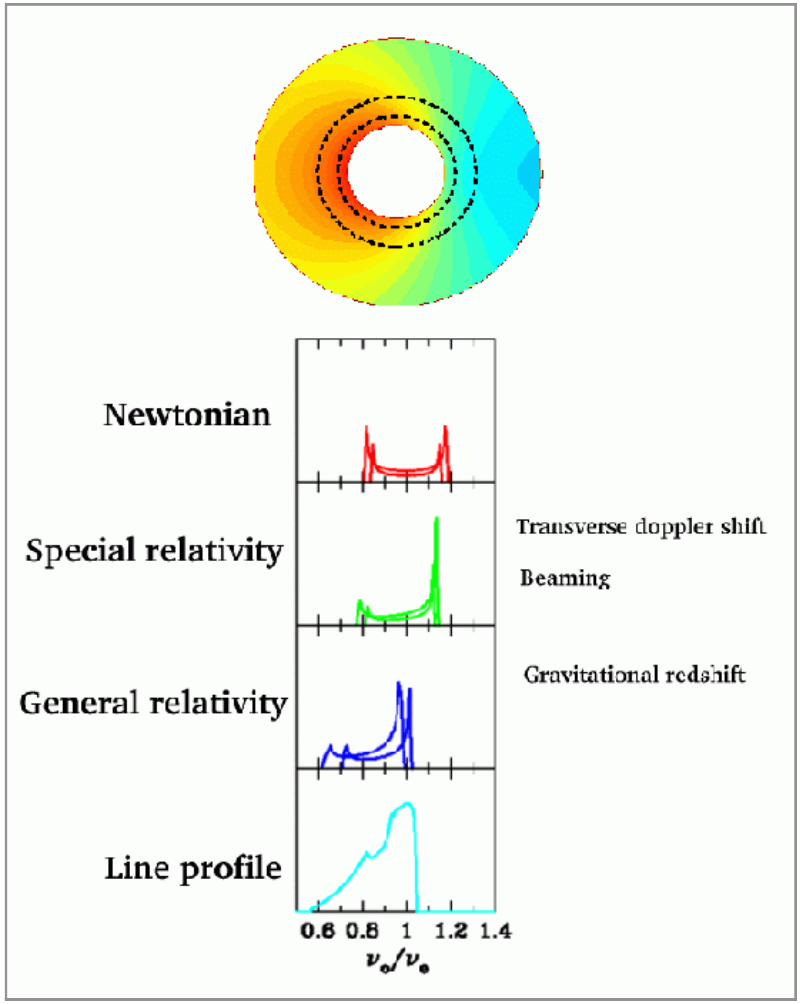
Important consequence of corona: underlying disk is irradiated by intense X-ray source… results in a characteristic spectrum being "reflected" from the disk surface layers

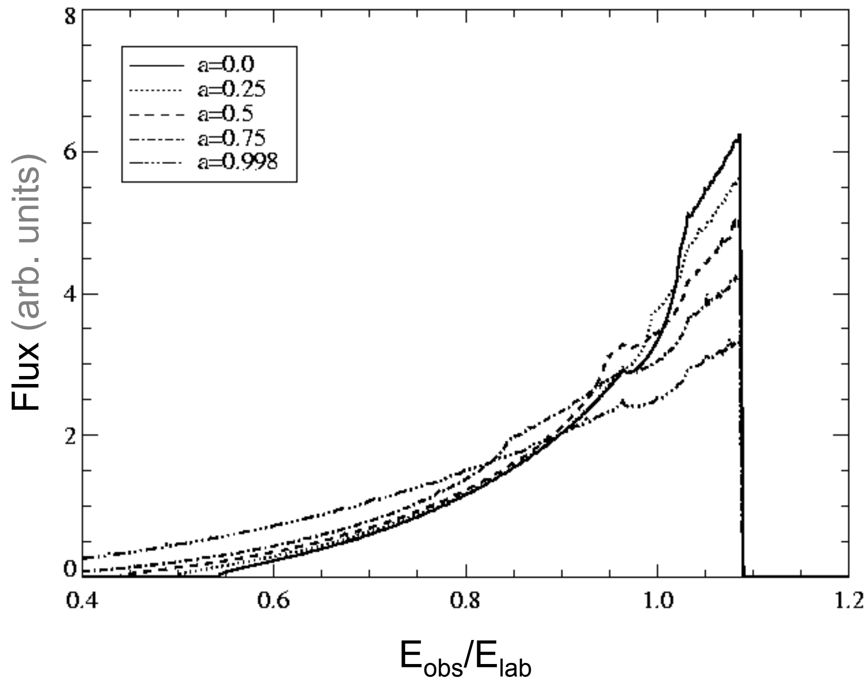






Relativistic effects





X-ray spectral states

X-ray spectral states of BHBs

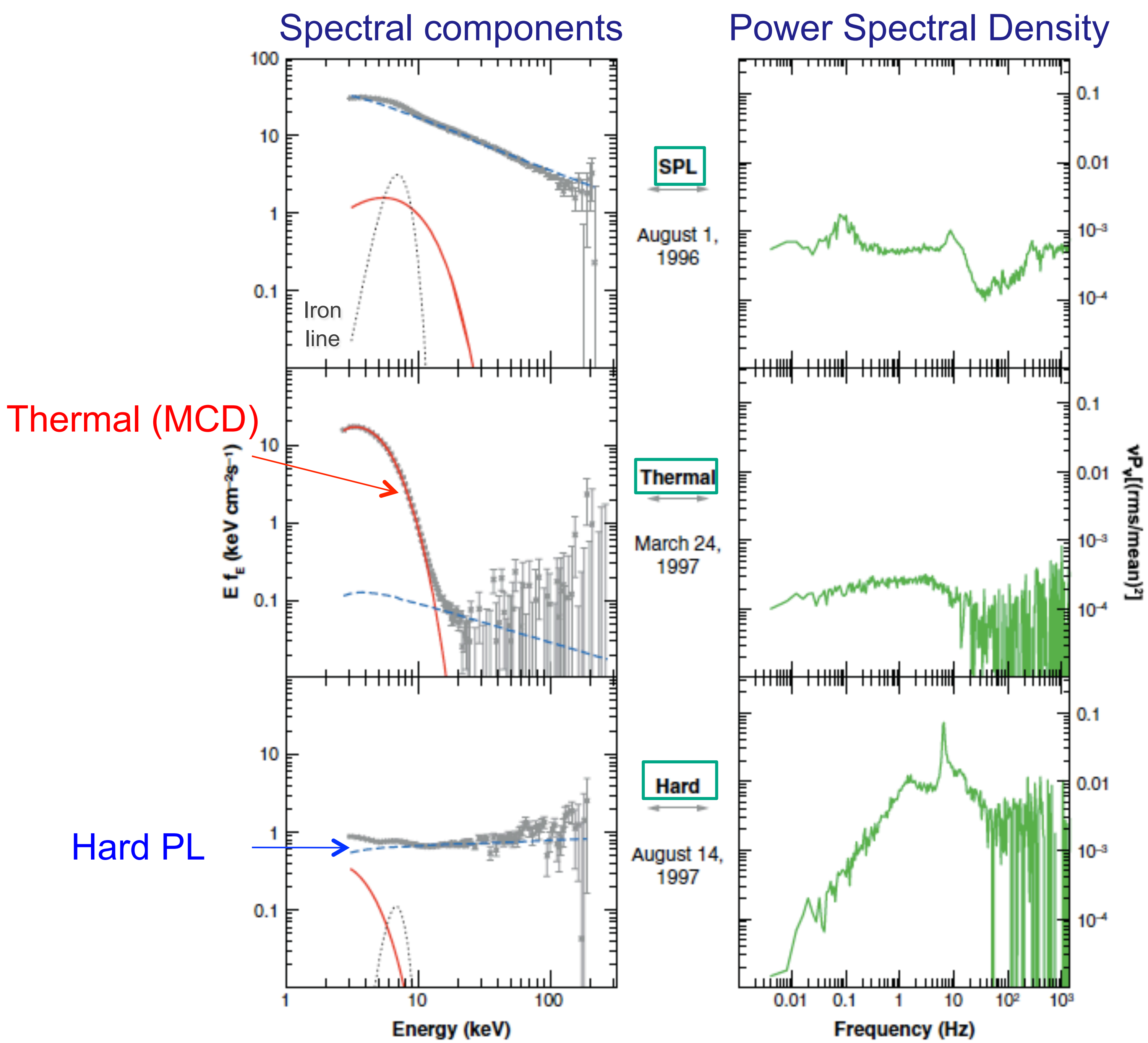
Very High State (VHS) Steep PL State	High State (HS)	Low State (LS)
• **PL emission dominant** ($\Gamma\approx2.5$) from thermal Comptonization, up to high energie (≈MeV) • High-frequency QPOs • At high Mdot, from thin to thick (**slim**) **disc** (Mdot>>L_{Edd}/c^2)	• **Mostly thermal emission from inner accretion disc** (MCD) • Faint $\Gamma\approx2.5$ PL emission due to non-thermal Comptonization • Absent/very weak QPOs	• **Hard PL dominant** ($\Gamma\approx1.4-2.1$, *thermal Comptonization*) + high-E cut-off • $T_e\approx100$ keV, $\tau_e\approx1$ • Presence of a *steady jet* • QPOs present/absent

INTERPLAY DISC-CORONA

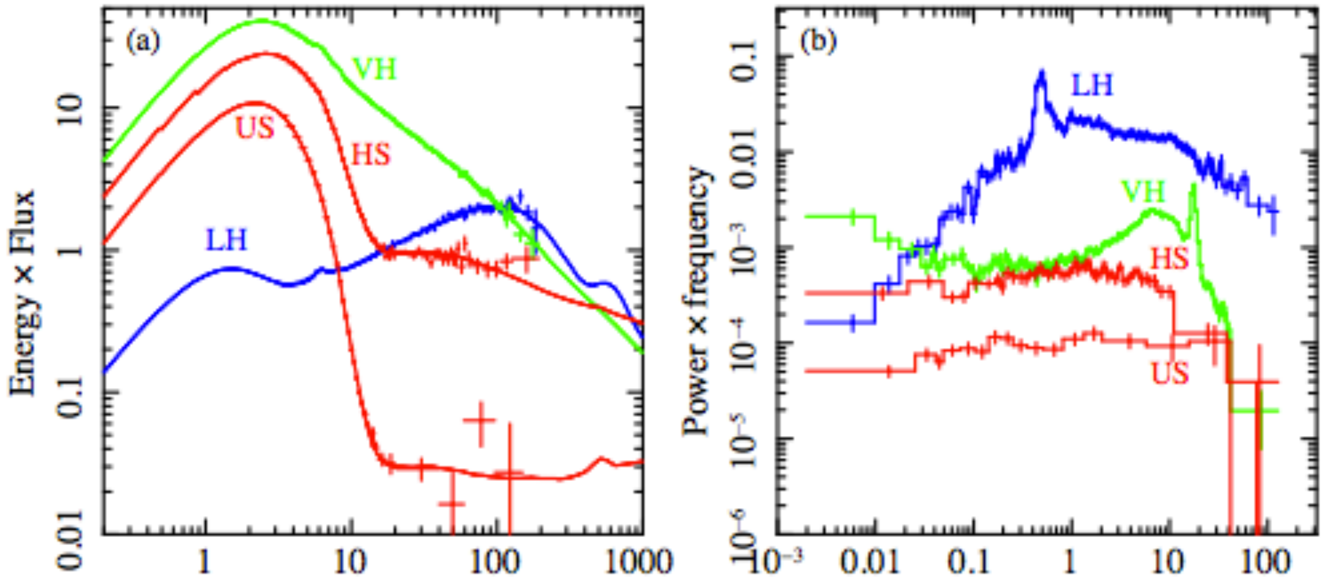
• In the soft/high state, the disc is very close to the last marginally stable Keplerian orbit ➔ no "room" for the corona to Comptonize soft X-ray photons

• In the hard/low state, the accretion disc is probably truncated at larger radii

• Thermal emission is less variable than PL emission



Spectral components
Power Spectral Density
Thermal (MCD)
Hard PL
Iron line
SPL
August 1, 1996
Thermal
March 24, 1997
Hard
August 14, 1997
E f_E (keV cm^-2 s^-1)
Energy (keV)
Frequency (Hz)
νP_ν[(rms/mean)^2]



Done 2010; see also Gierlinski & Done 2003

- ❑ HS (high/soft): disc dominated − looks like a disc but small tail to high energies (L prop. T^4)
- ❑ Very high/intermediate states − at least know something about a disc
- ❑ LH (low/hard) state − looks really different, not at all like a disc!

Summary of the main spectral states

Table 2 Outburst states of black holes: nomenclature and definitions

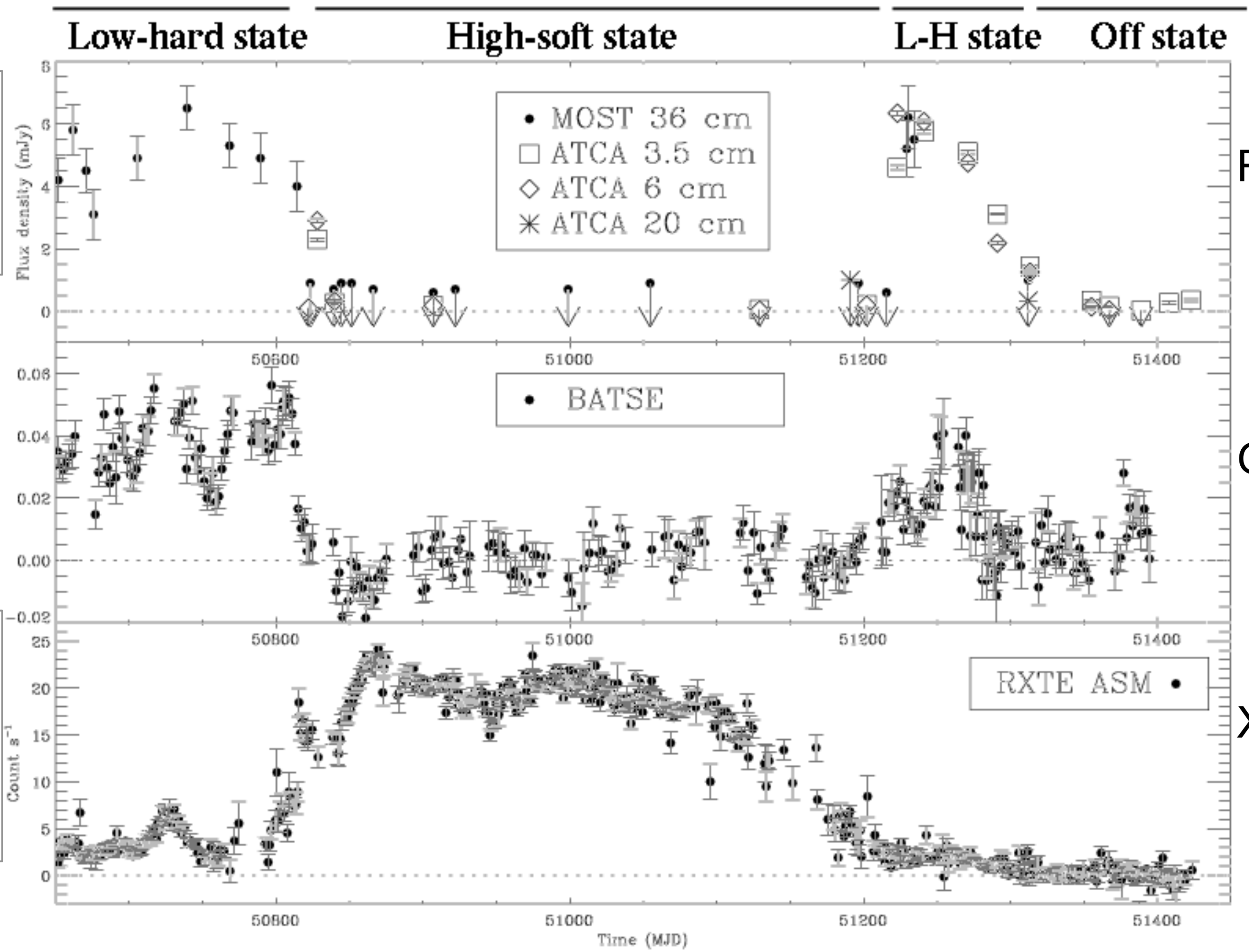
New State Name (Old State Name)		Definition of X-Ray State[a]
Thermal (High/Soft)	HS	Disk fraction $f^b > 75\%$ QPOs absent or very weak: $a^c_{max} < 0.005$ Power continuum level $r^d < 0.075^e$
Hard (Low/Hard)	LH	Disk fraction $f^b < 20\%$ (i.e., Power-law fraction > 80%) $1.4^f < \Gamma < 2.1$ Power continuum level $r^d > 0.1$
Steep Power Law (SPL) (Very high)	VH	Presence of power-law component with $\Gamma > 2.4$ Power continuum level $r^d < 0.15$ Either $f^b < 0.8$ and 0.1–30 Hz QPOs present with $a^c > 0.01$ or disk fraction $f^b < 50\%$ with no QPOs

- HS = thermal, high disk fraction
- LH = hard emission, low disk fraction
- VHS = steep power law: sort of intermediate state with both components

LH

HS

Strong radio in the LH state (jet)



Radio/mm

Gamma-ray

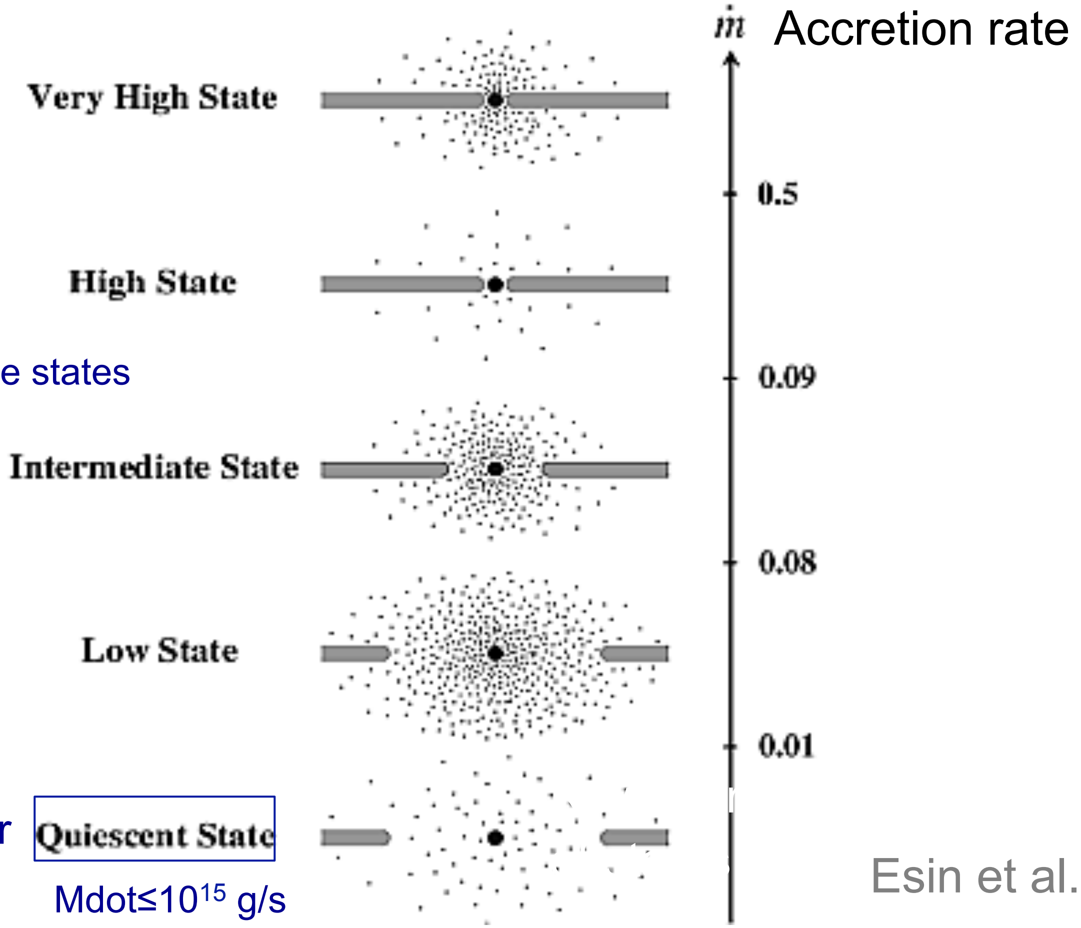
X-ray

Strong X-ray emission in the HS state (disc)

GX339-4

Significant radio-X-ray anti-correlation in the two ‘main’ spectral states

Spectral states



Esin et al. 1997

Presence of intermediate states

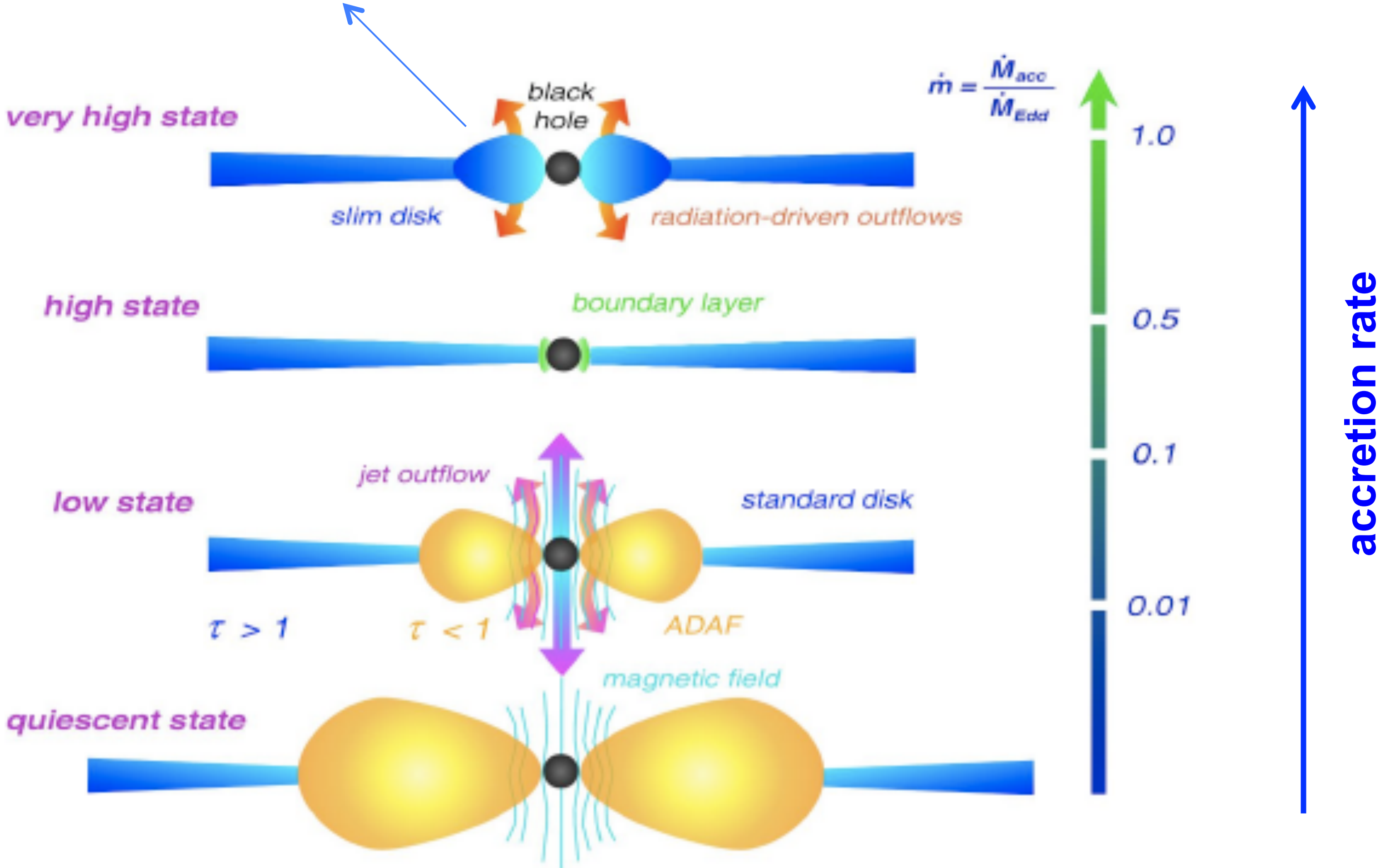
Quiescent state:
$\Gamma \approx 1.5\text{-}2.1$ (+BB in NS)
$L_x \approx 10^{30.5\text{-}33.5}$ erg/s
Adequate conditions for studies of the secondary star (limited heating)

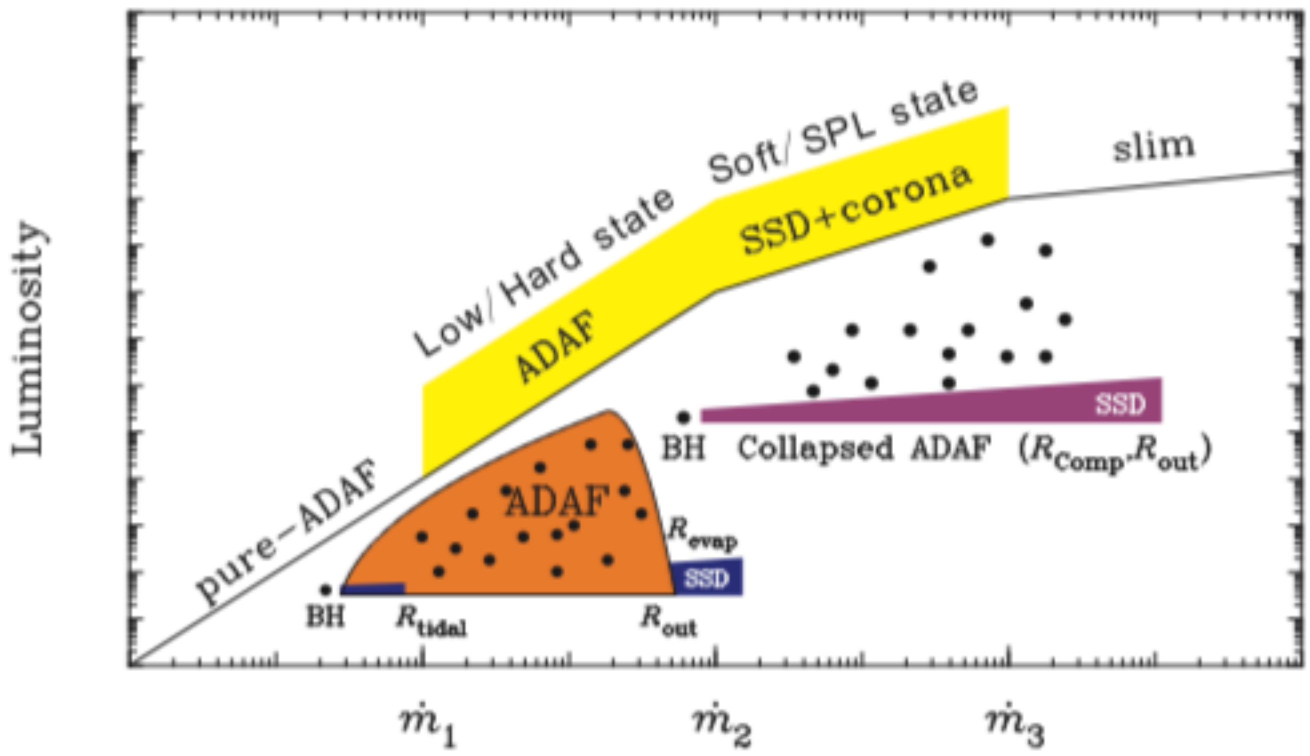
Mdot$\leq 10^{15}$ g/s

Spectral states mainly due to different disc/corona geometry and their interplay

Inefficient radiation (ADAF?) in quiescent state (outer disc as merely a store of matter)

Slim disk: the disk is radiation pressure-dominated and becomes 'thick' (geometrically+optically, advecting the accretion energy with the flow. Spectrally: broader, less-peaked disc spectrum (as in ULXs)





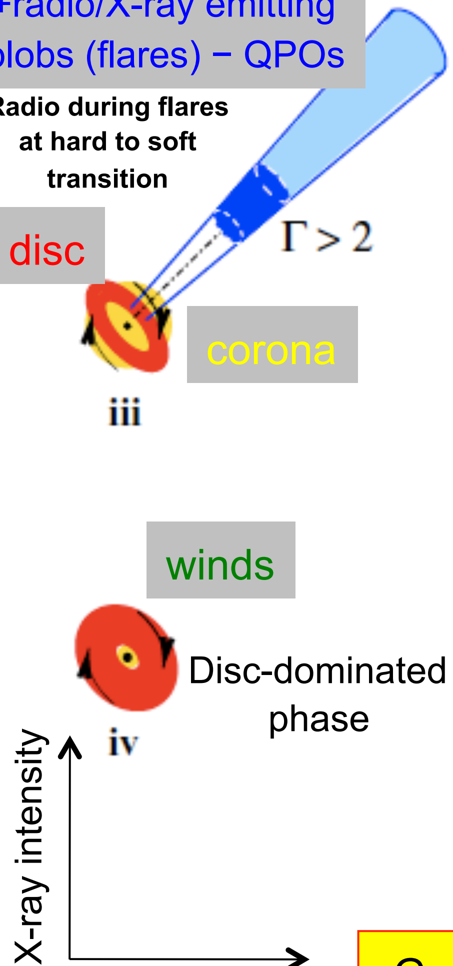
Luminosity
pure−ADAF
Low/ Hard state
ADAF
Soft/ SPL state
SSD+corona
slim
ADAF
BH
$R_{\rm tidal}$
$R_{\rm out}$
$R_{\rm evap}$
SSD
BH
Collapsed ADAF
($R_{\rm Comp}$,$R_{\rm out}$)
SSD
$\dot{m}_1$
$\dot{m}_2$
$\dot{m}_3$

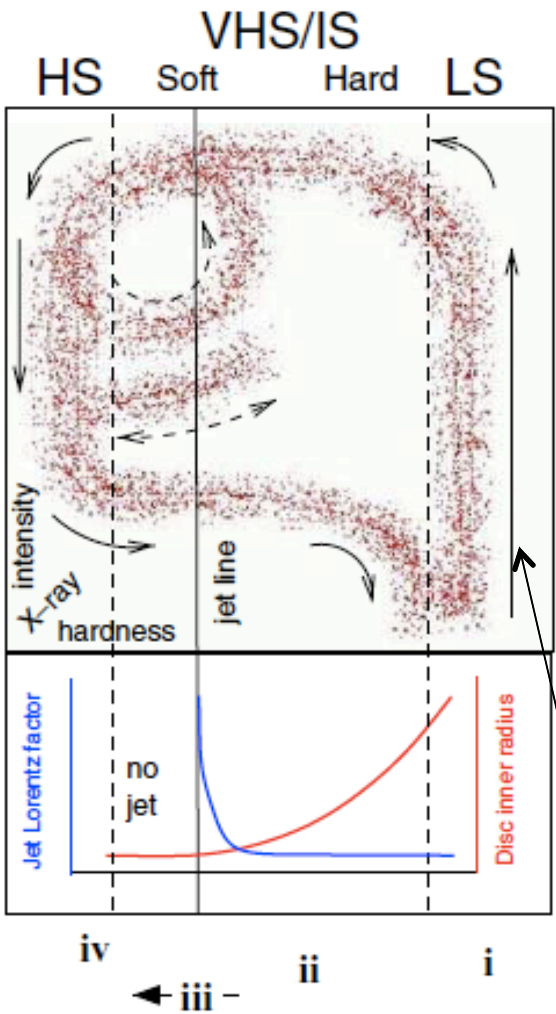
Wang+12

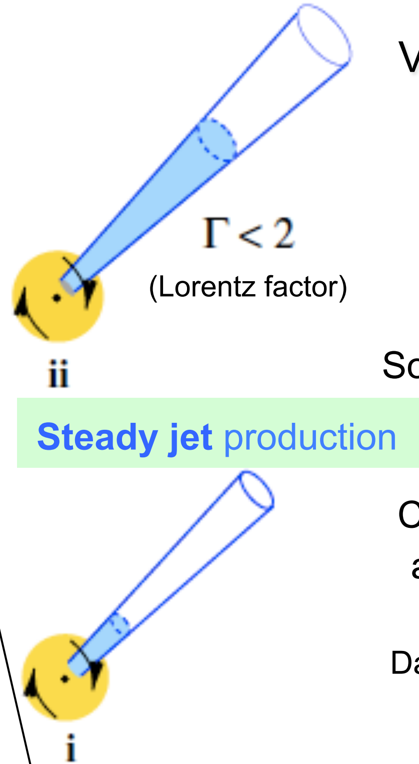
Discs and jets (disc-jet coupling)

Jet+shocks +radio/X-ray emitting blobs (flares) – QPOs

Radio during flares at hard to soft transition







LS – low/hard state
HS – high/soft state
VHS/IS – very high and intermediate states

Shocks during jet production

Some jet re-activation also in the soft state

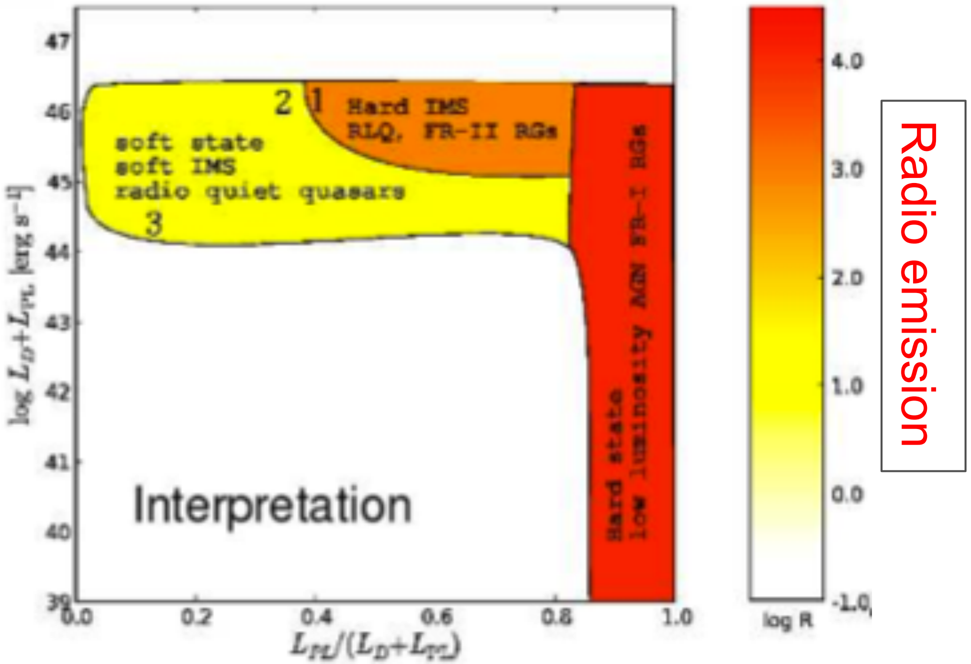
Constant radius (ISCO?) and structure in the HS

Data for GX 339-4, full path in ≈1 year

track of a simple X-ray transient outburst with a single optically thin jet production episode

Gravitational energy into radiative and kinetic (jets/winds) outputs

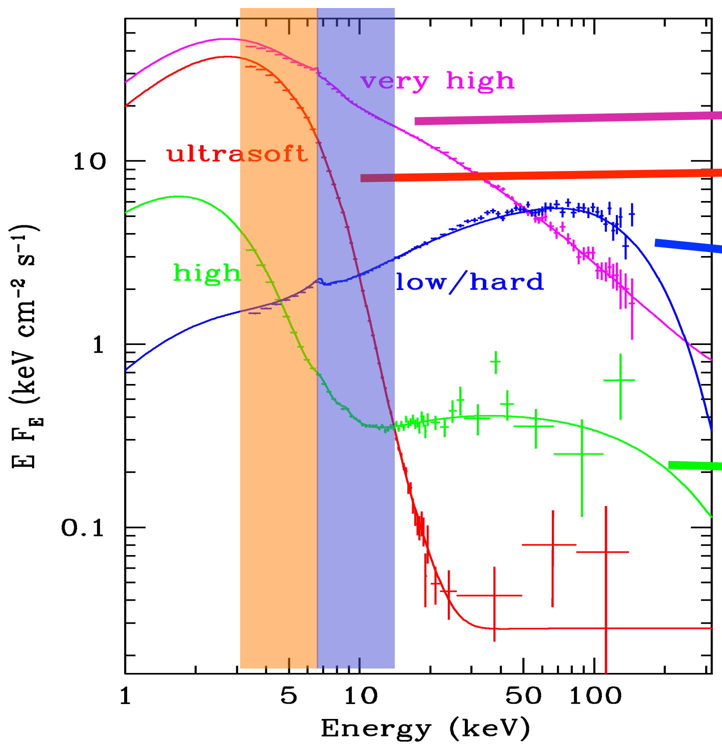
(from Fender et al. 2004; Remillard and McClintock 2007)

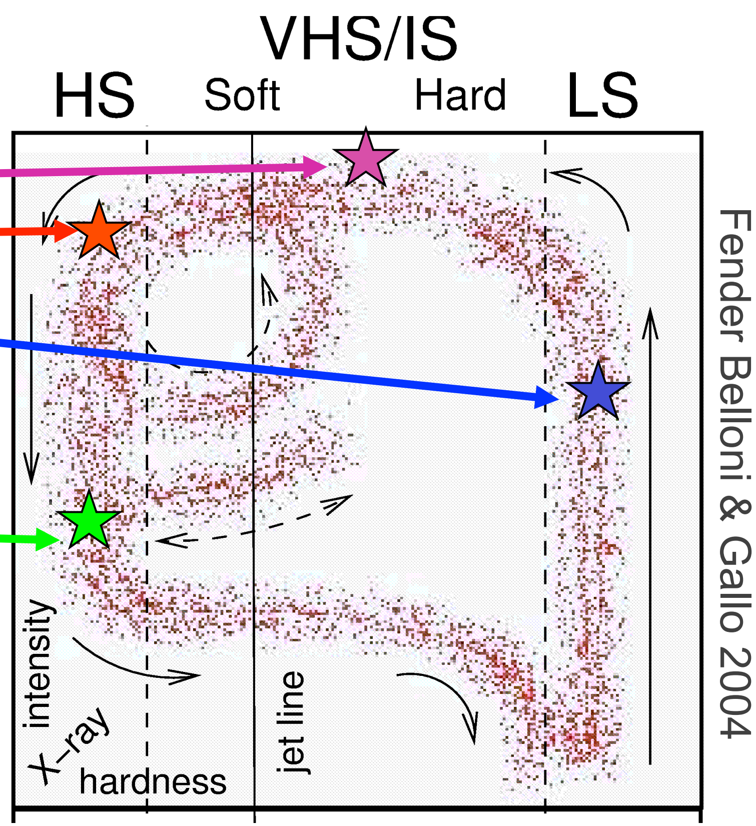


Radio (steady) jets during the low/hard state (no disc accretion)
Radio ≈100 times lower in the high/soft state (disc-dominated phase)
In some cases, faster radio jets in the hard-to-soft transition due to blob emission and likely shoks

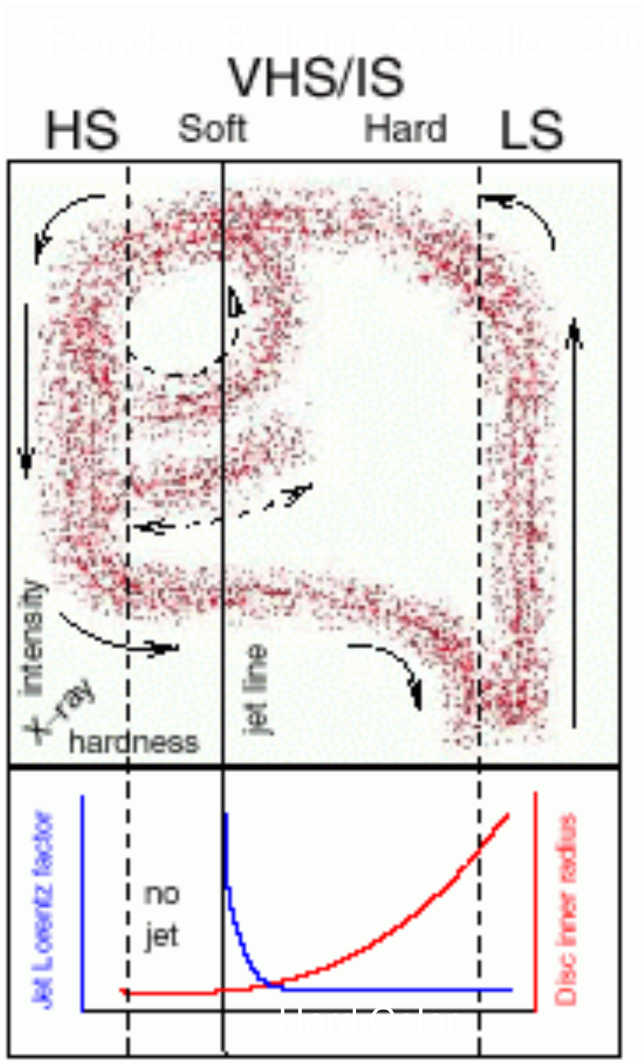
Hardness – intensity diagram

- Outburst starts hard, source stays hard as source brightens
- Then source softens to intermediate/very high state/steep power law state ➔ major hard-soft state transition
- Then the source is in the disc-dominated state, then hardens to make transition back to low/hard state – hysteresis as generally at lower L



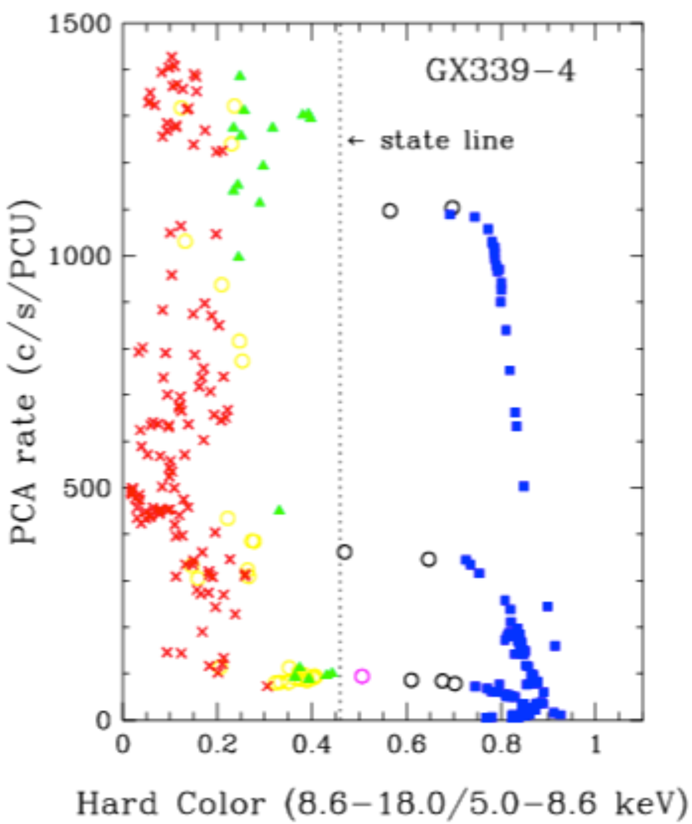


Unified model for jets in BH binaries



Remillard 2005

Thermal x Hard (jet) ■



sort of source spectral slope

Feedback

$$P_{\rm spin} + P_{\rm accretion} = L_{\rm bol} + L_{\rm kin} - L_{\rm advected}$$

'Input' powers (from BH spin and mass accretion)

$$L_{\rm kin} = L_{\rm jet} + L_{\rm wind}$$

$$P_{\rm accretion} = \frac{GM\dot{m}}{R} = \eta\ \dot{m}\ c^2$$

Bolometric radiative power

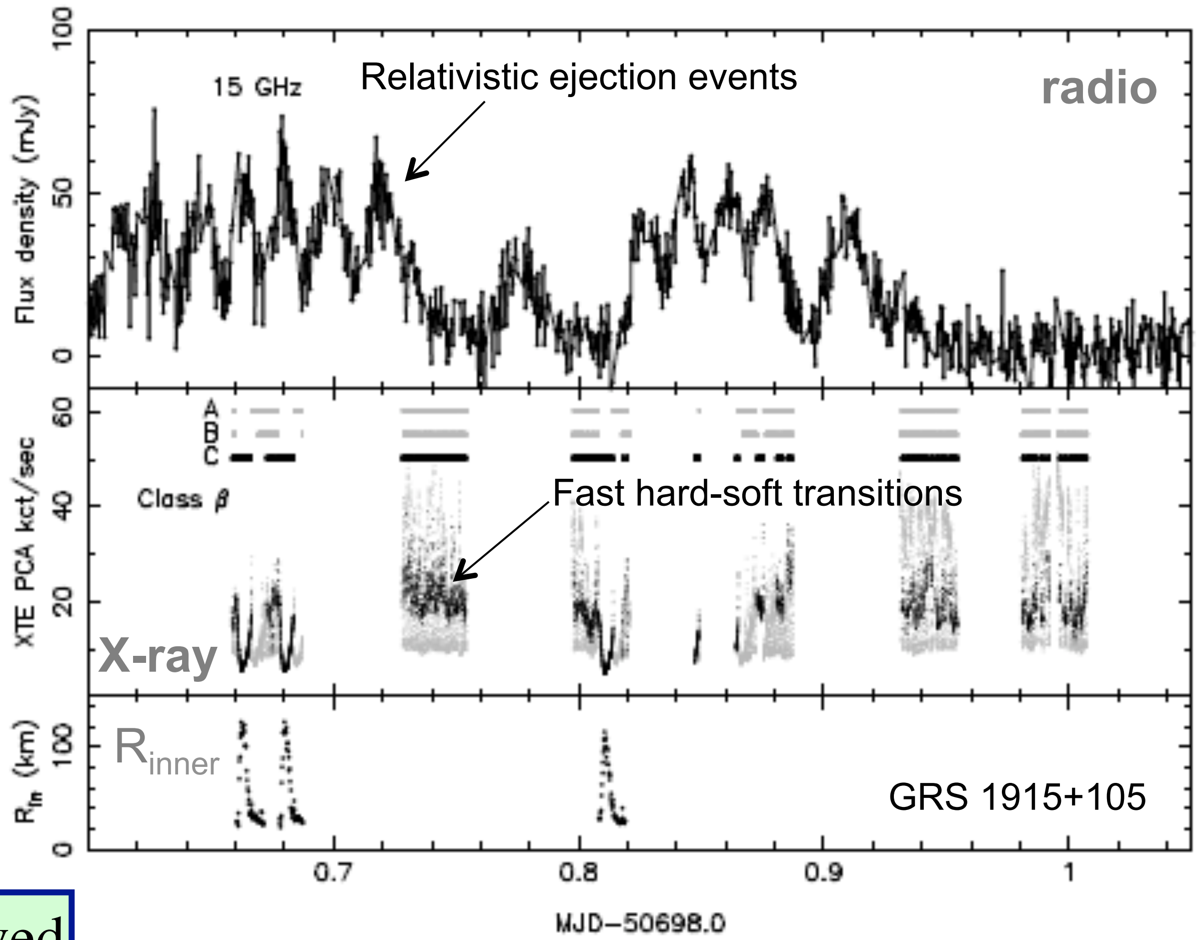
$P_{acc} \geq 0$
P_{spin} can be positive/negative (spin energy, accreting BHs can gain E_{spin}) – refer to spin-up/down in NSs/X-ray PSRs

Amount of available power advected across the BH event horizon (EH)

Most of the available accretion energy is 'trapped' in hot ions which cross the EH before they can radiate

Support from observations

A+B: soft states
C: hard states



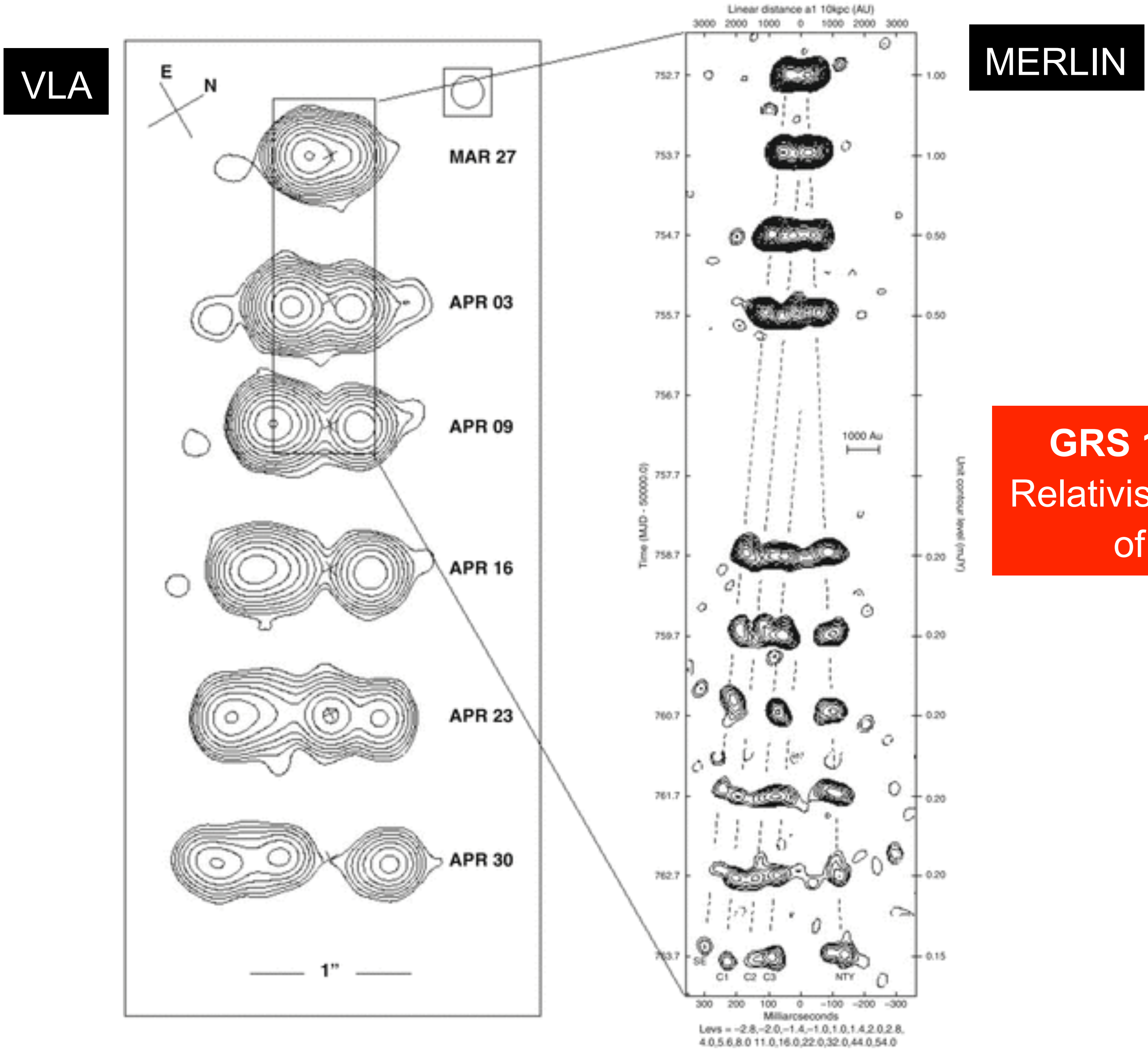
$L_R \propto L_X^{0.7}$ observed

$L_R \propto P_{jet}^{1.4}$ theory

$\rightarrow \mathrm{P_{jet}} \propto L_X^{0.5}$

Major ejection events during state transitions

In BHB low states, jets may dominate

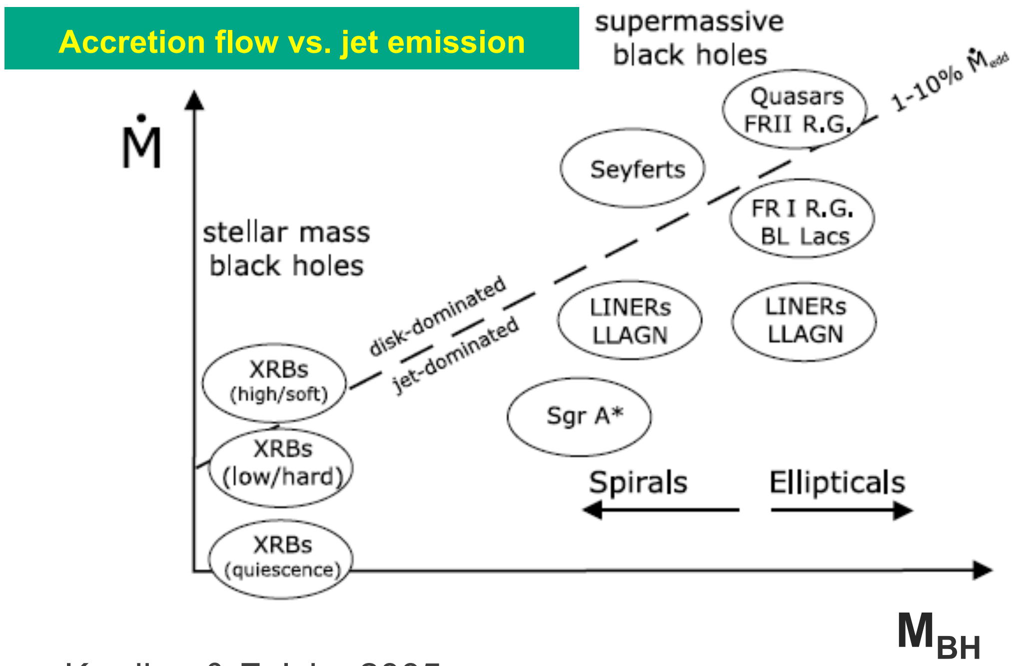


GRS 1915+105
Relativistic ejections of blobs

Mirabel & Rodriguez (1994) and Fender et al. (1999) - adapted

AGN/BHCs “fundamental plane”

Unified model for BHB and low-power AGN



Kording & Falcke 2005

$$\boxed{L_{\rm synch} \approx \left(q_j \frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^2 M}$$

$$\boxed{L_{\rm SSC} \approx \left(q_j \frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^3 M}$$

M=BH mass

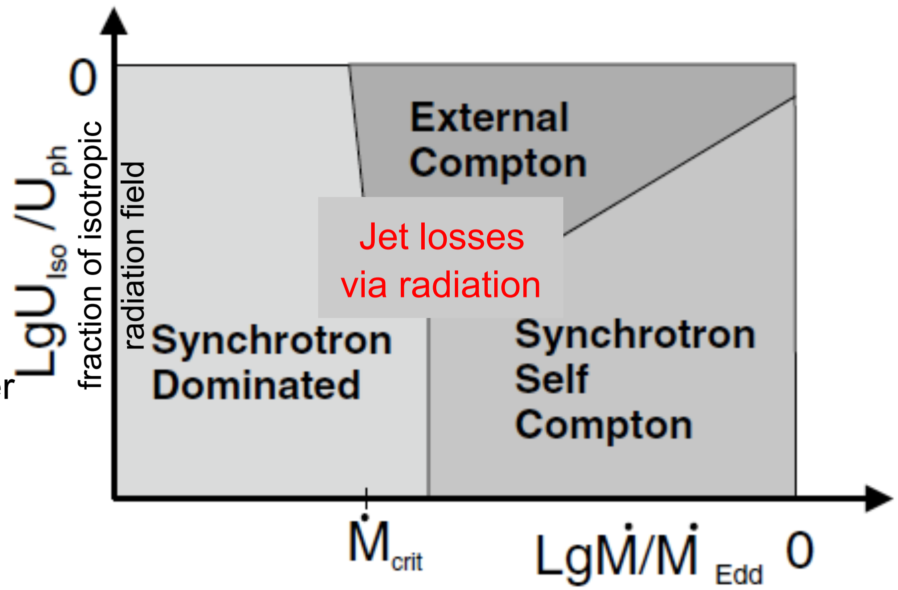
$$\boxed{L_{\rm EC} \approx \left(q_j \frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^2 M U_{\rm ph}}$$

q_j=percentage of accretion power injected into the magnetic field in the jet

U_{ph}=energy density of external photon field

Weakly accreting systems: the jet is in equipartition without loosing a significant fraction of its energy via radiation (synchrotron phase)

At increasing accretion rates, SSC and EC play a role and the jet can be quenched if there is material around the BH that scatters the photons from the accretion disc



A unified scheme for accretion

I) The accretion flow and disc form a coupled jet-disc system, with both components always present

II) Below a certain critical accretion rate (≈0.01 Edd. accretion rate), the inner part of the accretion flow becomes radiatively inefficient (low emission, ADAF)

III) Below this critical Mdot, or for face-on orientation (relativistic beaming), the jet emission (mostly radio/IR) dominates the emission from the accretion flow (X-rays, with maybe some contribution from the jet via synchrotron)

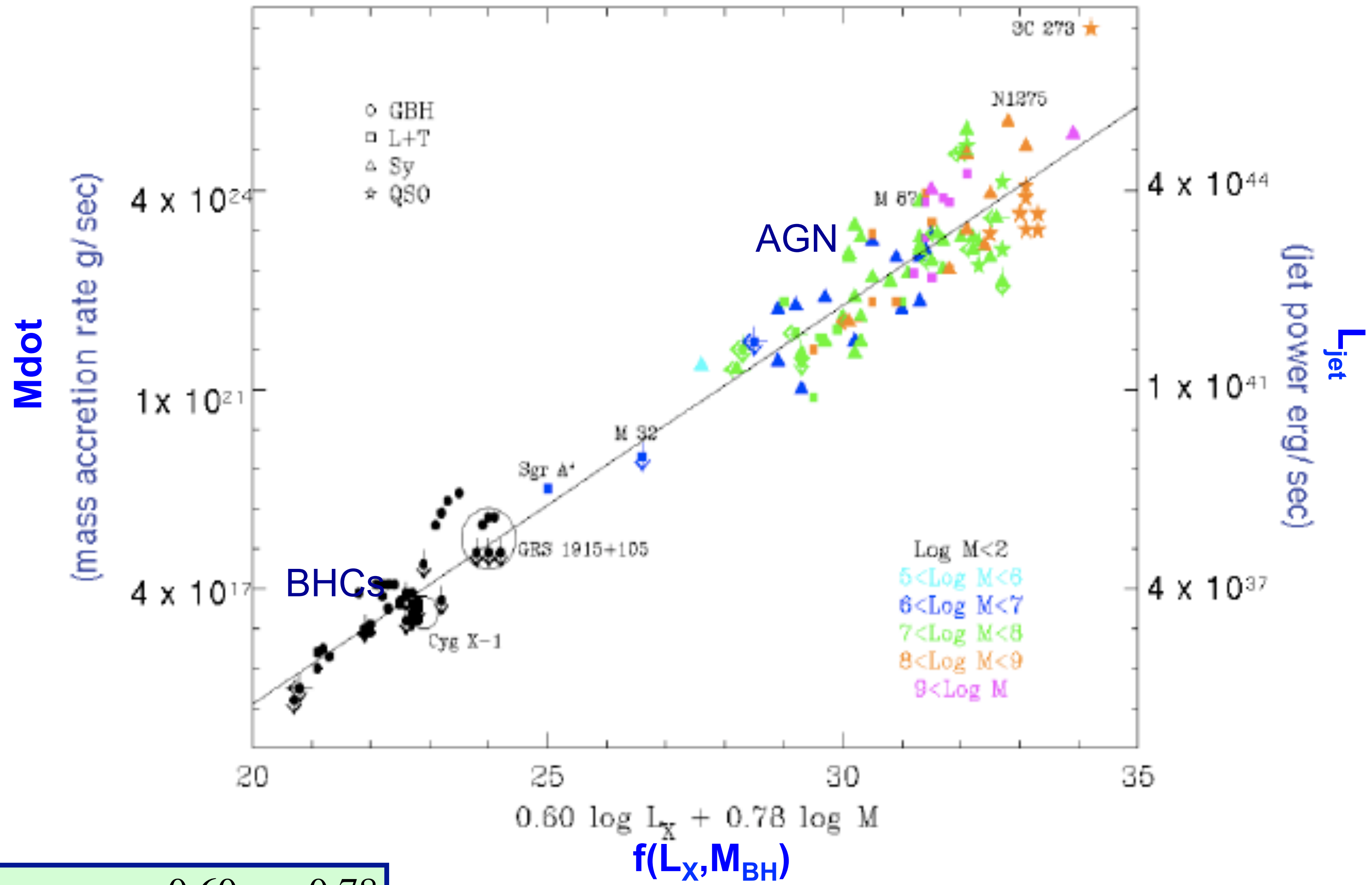
➔ jet-dominated accretion flow

➔ Near-Eddington, black holes are *disc-dominated* (with production of winds), while at sub-Eddington rates, black holes are *jet-dominated* (not thermally dominated)

➔ low-state binaries ≅ BL Lacs, FRIs, LINERS

One prediction is that the region of the onset of particle acceleration in the jet is at $\approx 100-1000\ R_G$

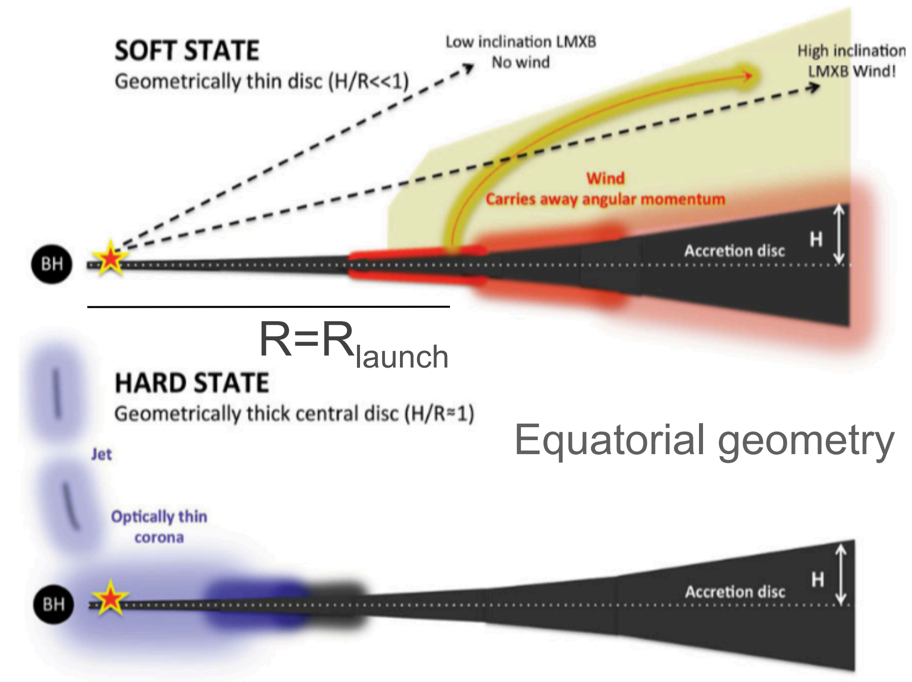
A “fundamental” plane for AGN and BHBs



$$L_{\mathrm{radio}} \propto L_X^{0.60} M^{0.78}$$

from Merloni et al. 2003 – Here L_{jet} instead of L_{radio}
see also Yuan et al. 2009 and Gutelkin et al. 2019

Winds in high/soft state

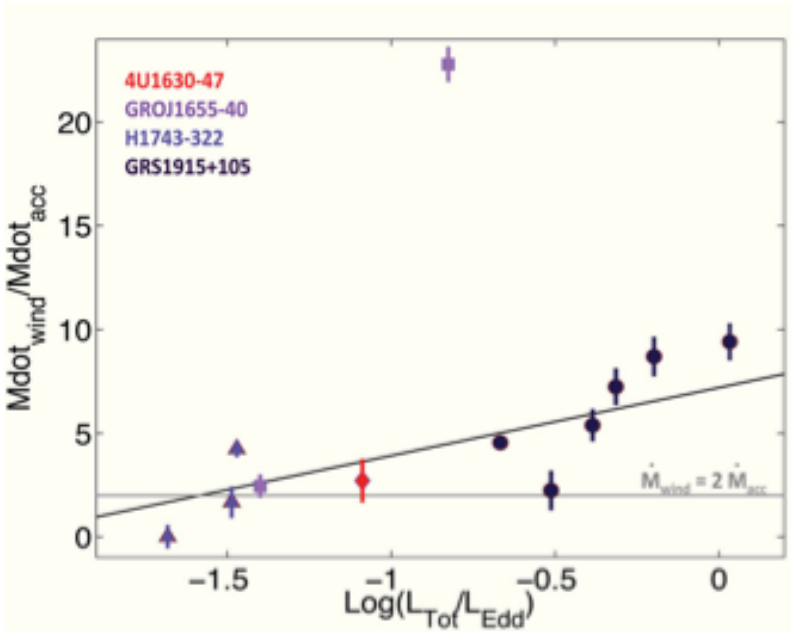


$$\dot{M} = 4\pi R^2 n m_p v_{out} \frac{L_X}{\xi} \frac{\Omega}{4\pi}$$

Mdot~10^{19} g/s

L_W~10^{35} erg/s

V_{out}: wind outflow
ξ: ionization parameter (from abs. lines)
Ω: solid angle subtended by the wind
Thermal pressure to launch, then radiative and magnetic pressure

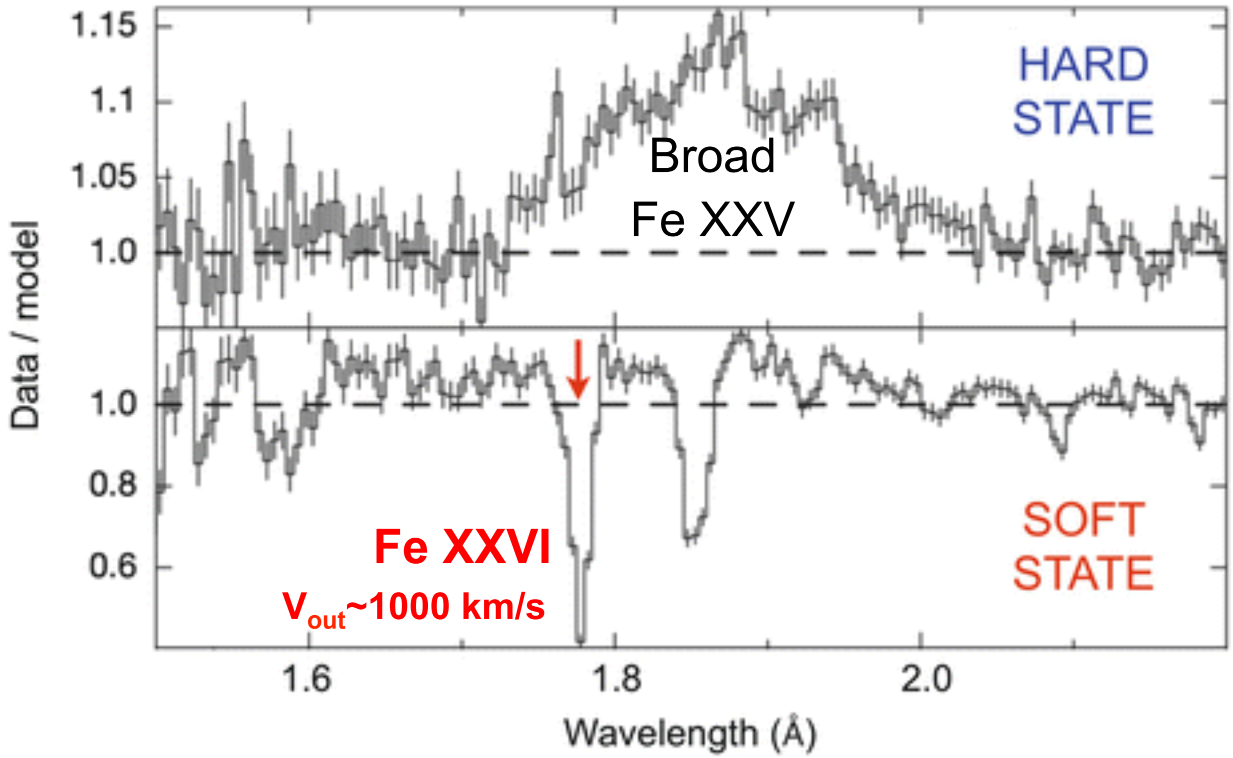


The mass outflow rate carried out by these winds may be higher than the inner accretion rate

➔ Responsible for the quenching of the jet?

Ponti et al. 2012

Winds revealed via blueshifted iron features

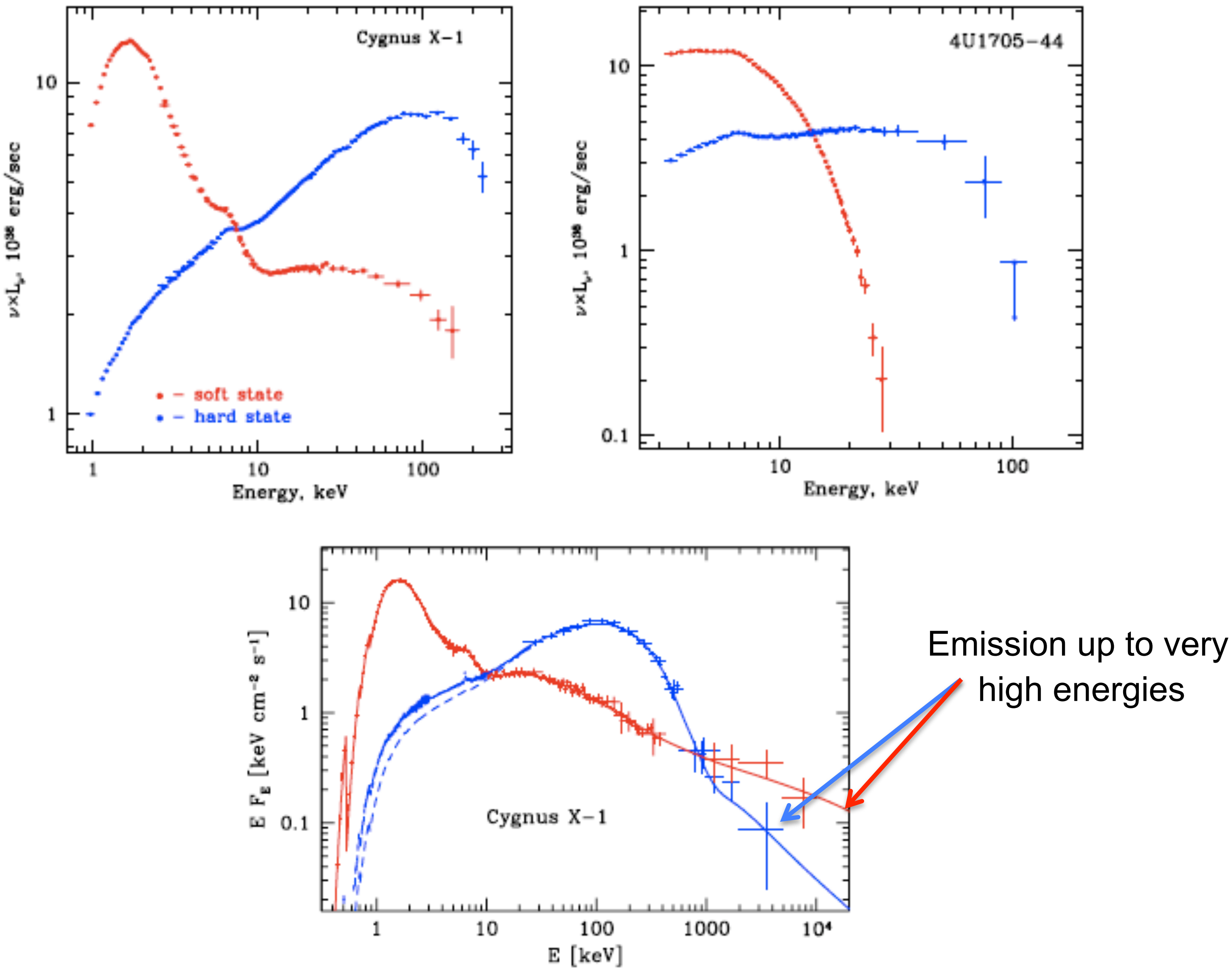


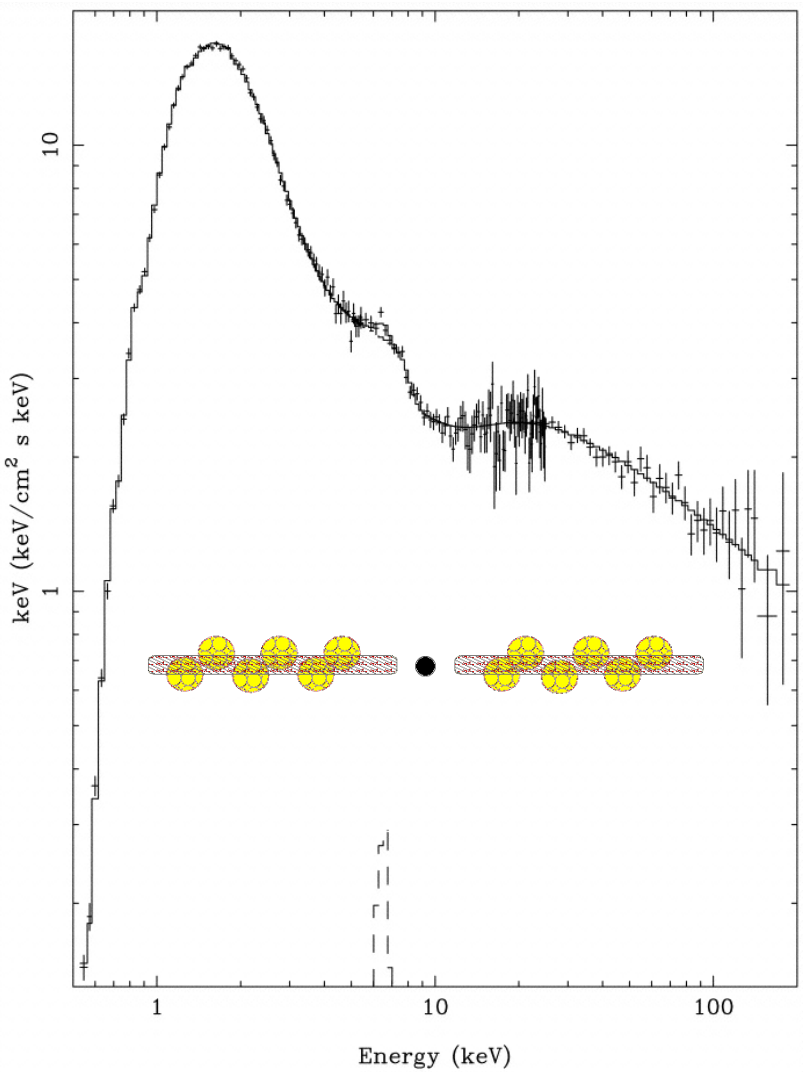
GRS 1915+105

Adapted from Neilsen et al. 2012 – see also Fender & Munoz-Darias 2016

Some “real” X-ray spectra

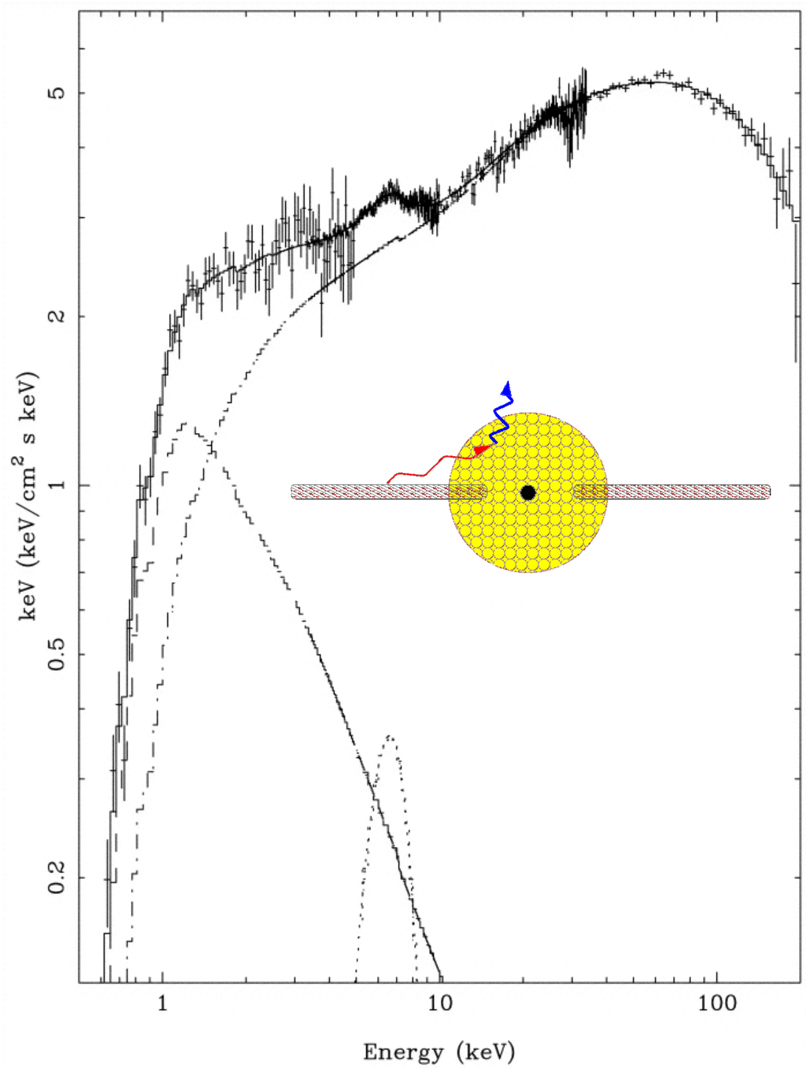
Soft vs. hard state in BHBs





Soft (high) state
thermal disk emission + hard tail

BeppoSAX



Hard (low) state
hard X-ray spectrum, little thermal disk

Quasi periodic oscillations in BHBs

Main properties of QPOs in BHBs

Low-frequency QPOs (LFQPO: 0.1−30 Hz): tied to the flow of matter in the AD

$\nu_{LFQPO} << \nu_{Kepl,disc}$ ➔ $R \approx 100\ R_g$

Typically they are stable and persistent

High-frequency QPOs (HFQPO: 40−450 Hz): tied to $R \approx R_{ISCO}$

stable, do not shift in frequency (vs. NS)

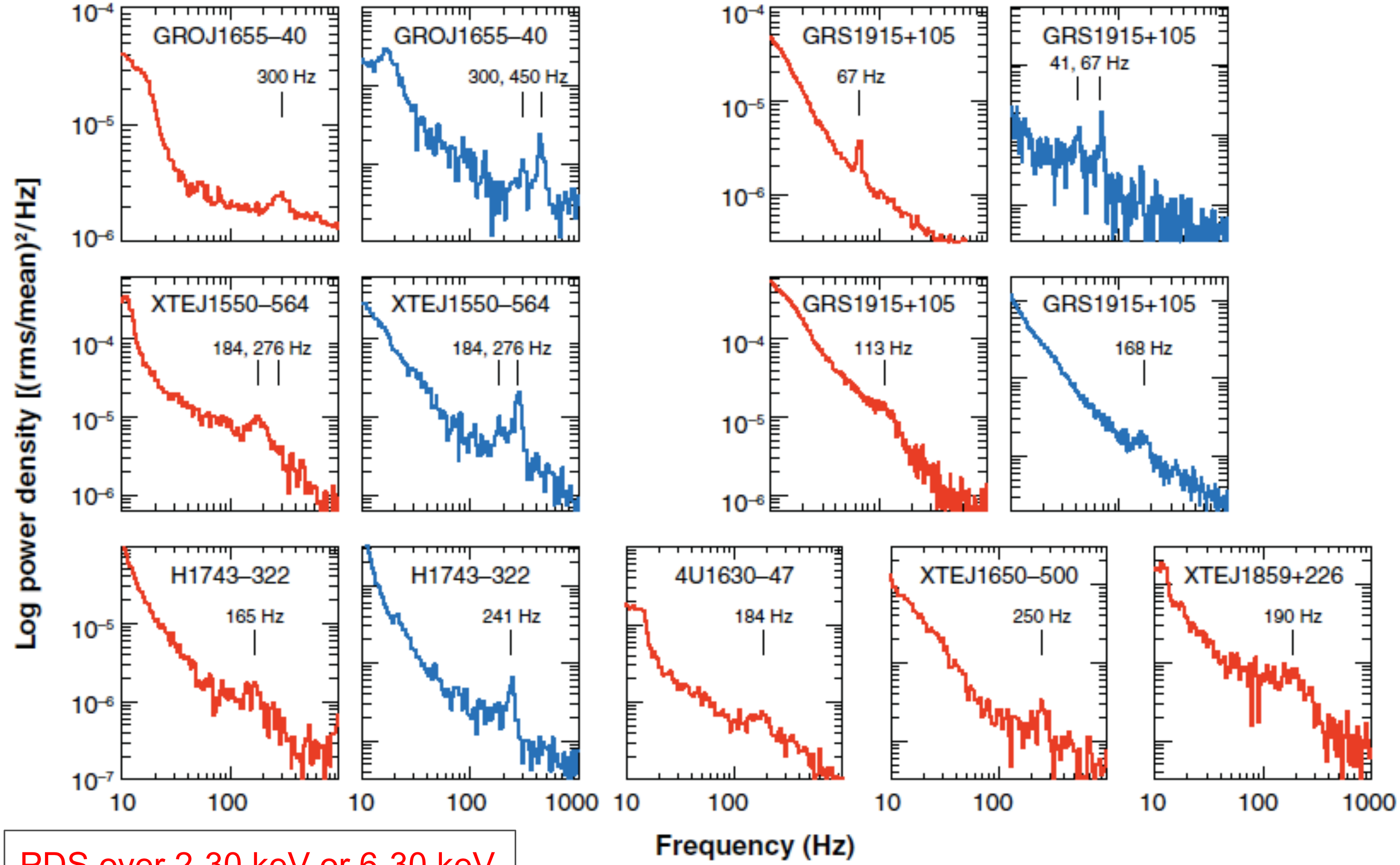
3:2 pairing in frequency (resonance)

➔models explains such QPOs if the BH is rapidly spinning (Kerr BH) ➔ all sources with such QPOs have jets

Transient and subtly variable

Possible models: global disk oscillations, radial oscillations associated with spiral shocks, oscillations in the region separating the cool disk from the hot corona

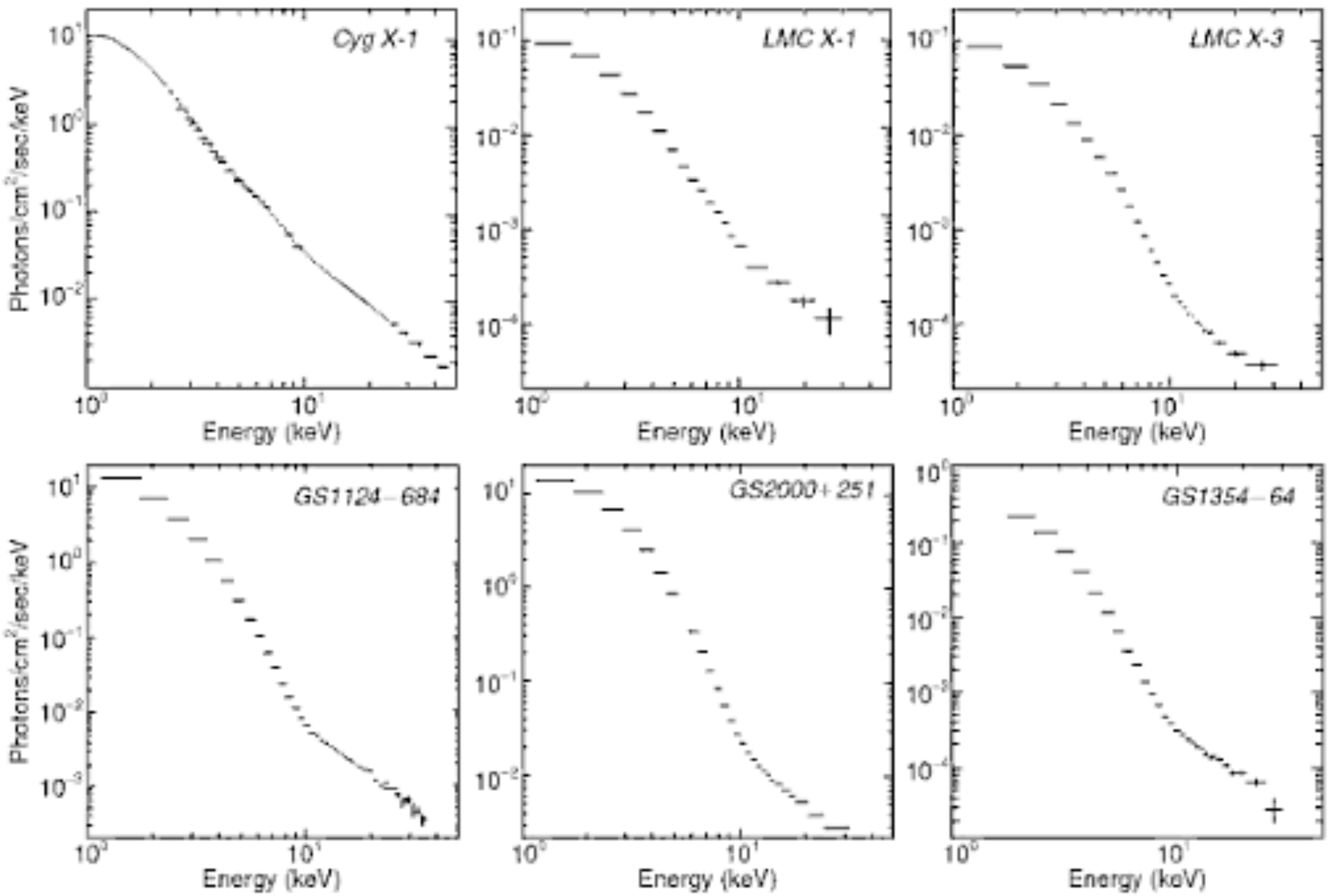
QPOs in BHBs and BHCs



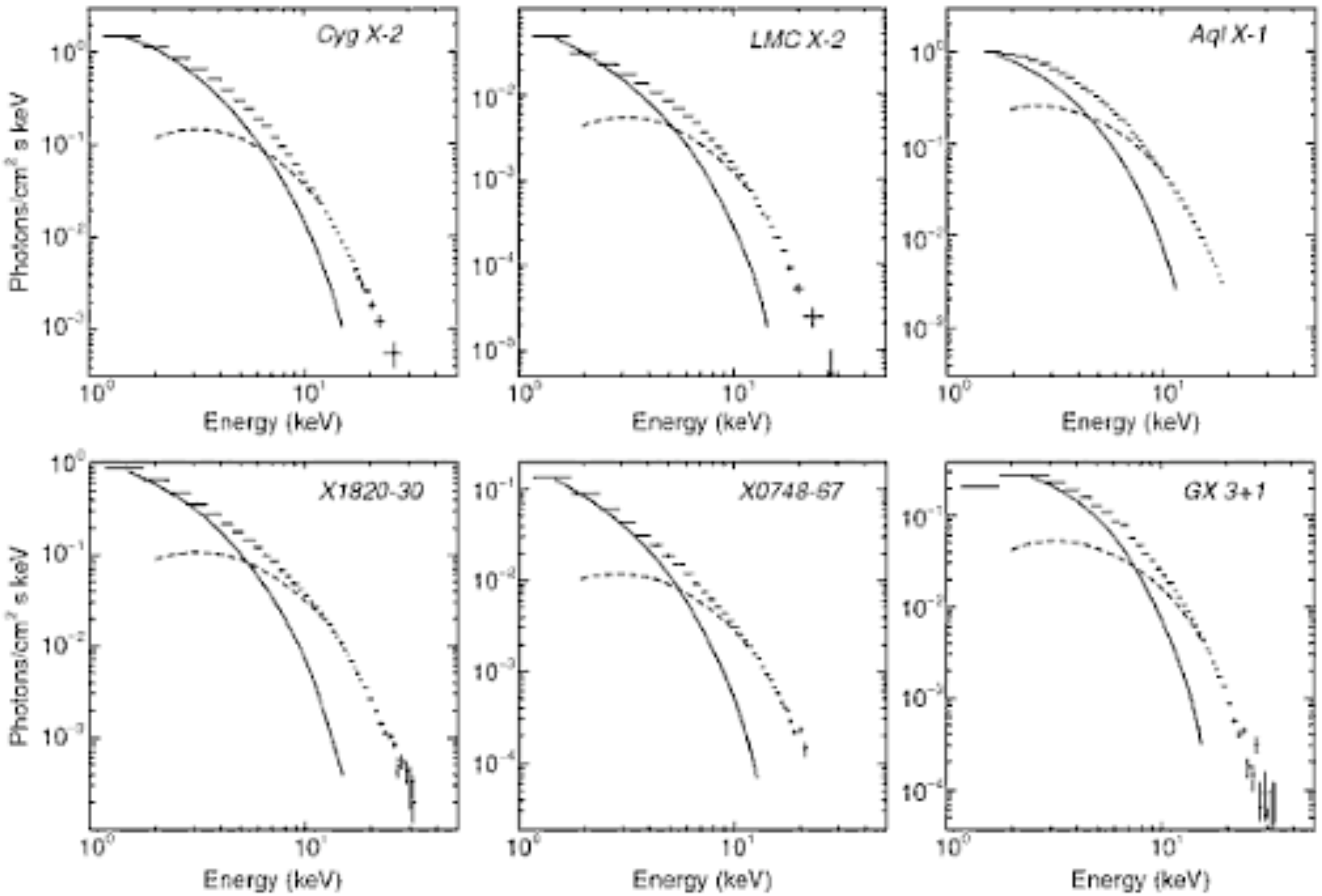
PDS over 2-30 keV or 6-30 keV
PDS over 13-30 keV

BHBs vs. NS in the same state

High-state spectra of BHBs (thermal + PL tail)



High-state spectra of NS (MCD+BB component)



• NS have an additional ≈2 keV blackbody emission probably linked to the cooling of the NS surface (BH have no surface!) If not observed, possible obscuration by the corona

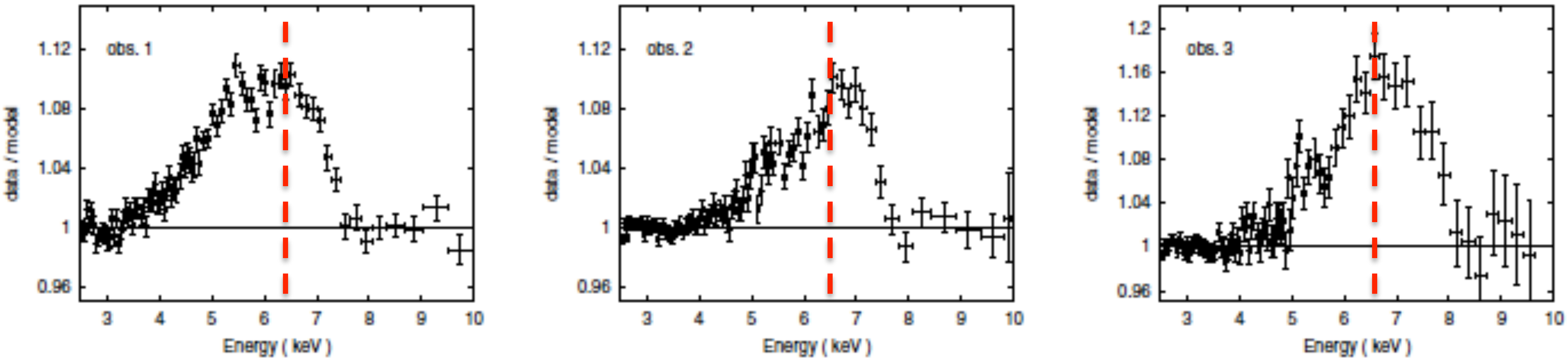
• No Type I bursts in BHBs

• kT_{in} prop $M_X^{-1/4}$ ➔ NS slightly "hotter" than BH (easier to appreciate BHs vs. SMBHs difference)

Broad (relativistic) iron lines

Broad (relativistic) iron Kα lines in BHCs

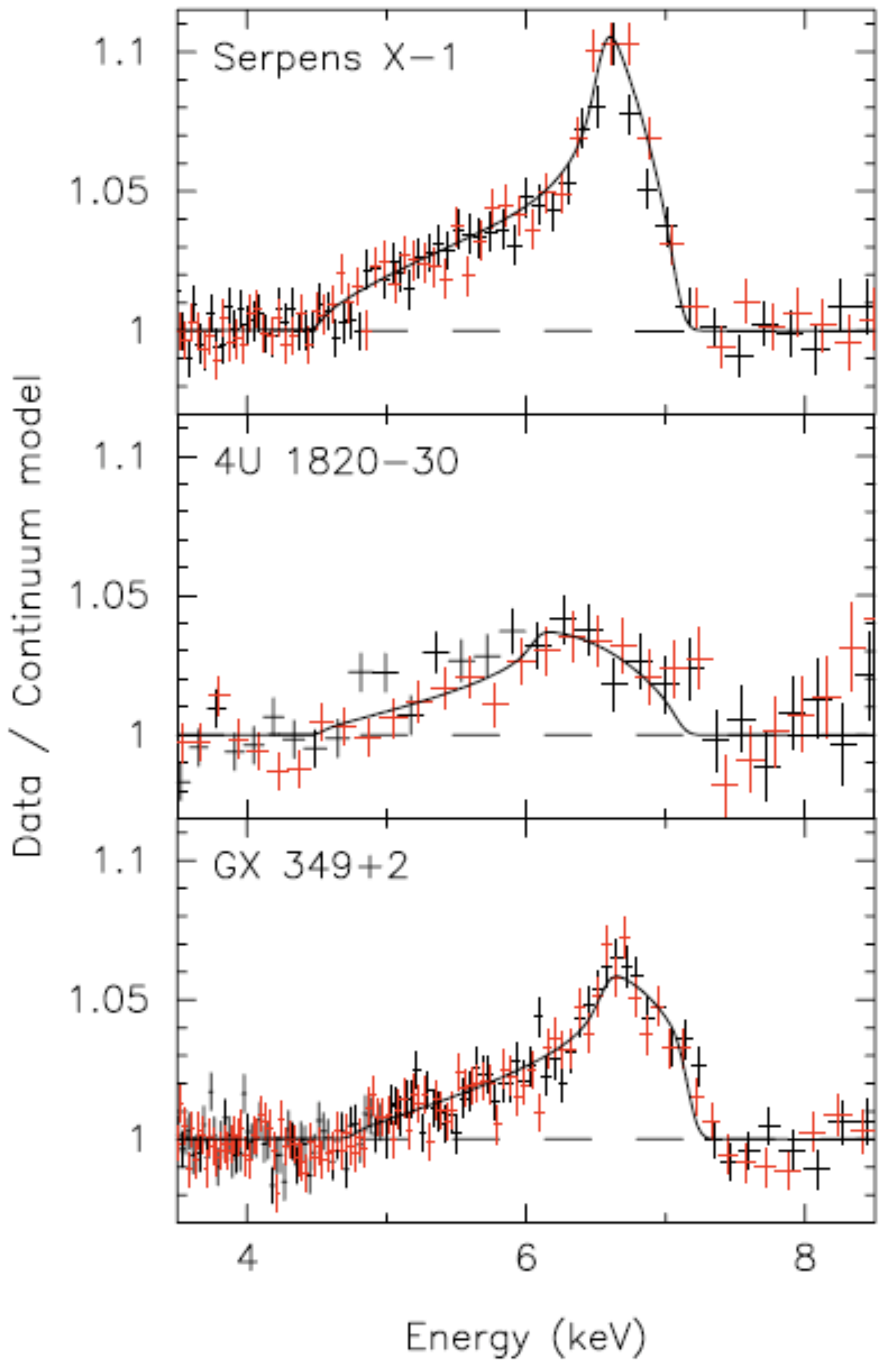
XTE J1650-500 (BH candidate) during outburst



Miniutti et al. (2004)

and in NS LMXBs

LMXBs with *Suzaku* observations

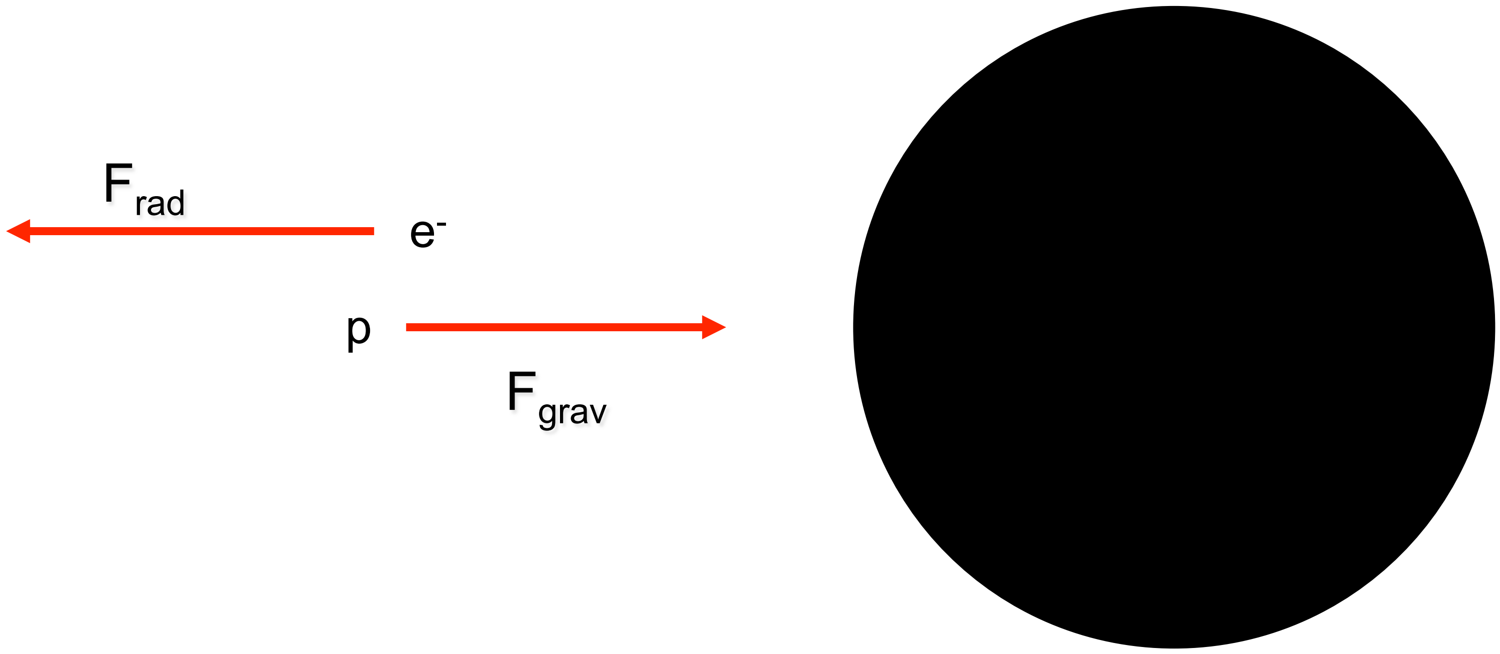


Cackett et al. (2008)

Eddington Luminosity

How luminous can an accreting black hole be?

ASSUMPTION: spherically symmetric steady-state radial infall of ionized H



$$P_{\text{rad}} = \frac{F_{\text{rad}}}{\text{c}} = \frac{L}{4\pi R^2 c}$$

Outward momentum flux = pressure

$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi R^2 c}$$

Outward radiation force on a single electron

$$F_{\text{grav}} = \frac{G(m_p + m_e)M}{\text{R}^2} \approx \frac{Gm_p M}{R^2}$$

Gravitational energy acting on a proton

Eddington luminosity (limit)

max luminosity allowed under the defined assumptions

$$F_{\text{rad}} \le F_{\text{grav}} \rightarrow \frac{L\sigma_T}{4\pi R^2 c} \le \frac{GMm_p}{R^2} \rightarrow L \le \frac{4\pi Gcm_p}{\sigma_T} M$$

$$\rightarrow L_{\text{Edd}} = 1.26 \times 10^{38} \left(\frac{M}{M_{\text{sun}}} \right) \text{erg/s}$$

there are cases where this luminosity is likely exceeded

Accretion luminosity and accretion rate

Accretion luminosity: luminosity released by gravitational energy due to gas falling onto a star/compact object with mass M and radius R

$$L = \frac{GM\dot{M}}{\mathrm{R}} = \eta \dot{M} c^2$$

η=efficiency of the conversion of gravitational energy into radiation

$$L_{\mathrm{Edd}} = \frac{4\pi G c m_p M}{\sigma_{\mathrm{T}}} = \frac{GM\dot{M}_{\mathrm{Edd}}}{\mathrm{R}} \rightarrow \dot{M}_{\mathrm{Edd}} = \frac{4\pi c m_p}{\sigma_{\mathrm{T}}} R$$

$$\rightarrow \dot{M}_{\mathrm{Edd}} \approx 1.5 \times 10^{-8} \left(\frac{R}{10\ \mathrm{km}} \right) M_{\mathrm{sun}}/\mathrm{yr}$$

[cgs system]

$m_p = 1.7 \times 10^{-24}$ g

$M_\odot = 2 \times 10^{33}$ g

$\sigma_T = 6.65 \times 10^{-25}$ cm^2

$G = 6.67 \times 10^{-8}$ cm^3/g/s^2

All calculations

$$\dot{M}_{\mathrm{Edd}} = \frac{4\pi c m_p}{\sigma_{\mathrm{T}}} R = \frac{12.56\ 3\ 10^{10}\,(\mathrm{cm/s})\ 1.7\ 10^{-24}\,(\mathrm{g})}{6.65\ 10^{-25}\,(\mathrm{cm}^2)} R =$$

$$= 9.63\ 10^{11}\ \mathrm{R\ (g/cm\ s)} = 9.63\ 10^{11} \left(\frac{R}{10\ \mathrm{km}}\right) 10^{6}\,(\mathrm{cm})\ \frac{3600 \times 24 \times 365\ (\mathrm{s})}{2\ 10^{33}\ (\mathrm{g})}\ [M_{\mathrm{sun}}/\mathrm{yr}] =$$

$$= \dot{M}_{\mathrm{Edd}} \approx 1.5 \times 10^{-8} \left(\frac{R}{10\ \mathrm{km}}\right) [M_{\mathrm{sun}}/\mathrm{yr}]$$

Accretion rate in a super-massive black hole (I)

$$L_{\rm Edd} = \eta \dot{M} c^2 = 1.3\times10^{38}\left(\frac{M}{M_{\rm sun}}\right) \ \text{erg/s}$$

$$\eta_{0.1} = \eta / 0.1$$

$$\text{erg=g cm}^2/\text{sec}^2$$

$$\frac{\rm g}{\rm s} = \frac{\rm M_{sun}}{2\times10^{33}}\times\frac{365\times24\times3600}{\rm yr} = 1.58\times10^{-26}\ \frac{\rm M_{sun}}{\rm yr}$$

$$\rightarrow \dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\eta c^2} = \frac{1.3\times10^{38}(M/M_{\rm sun})}{0.1\times\eta_{0.1}\times9\times10^{20}}\left[\frac{\rm erg/s}{\rm cm^2/s^2}\right] =$$

$$= \frac{1.44\times10^{18}}{\eta_{0.1}}\left(\frac{\rm M}{\rm M_{sun}}\right)[\rm g/s] = \frac{1.44\times10^{18}}{\eta_{0.1}}\times1.58\times10^{-26}\left[\frac{\rm M_{sun}}{\rm yr}\right] =$$

$$\approx \frac{2.2\times10^{-8}}{\eta_{0.1}}\left[\frac{\rm M_{sun}}{\rm yr}\right] = 2.2\ M_8/\eta_{0.1}\ [\rm M_{sun}/yr]$$

$$\rightarrow \dot{M}_{\rm Edd} = 2.2 M_8/\eta_{0.1} \qquad [\rm M_{sun}/yr]$$

M_8= M in units of $10^8\,M_\odot$
$\eta_{0.1}=\eta/0.1$

Accretion rate in a super-massive black hole (II)

Eddington time

Eddington time: time taken by a body to radiate its entire mass at the Eddington rate

$$\dot{M}_{\mathrm{Edd}} = \frac{M}{t_{\mathrm{Edd}}} \rightarrow t_{\mathrm{Edd}} = \frac{M}{\dot{M}_{\mathrm{Edd}}} = \frac{M}{2.2 M_8 / \eta_{0.1}} = \frac{10^8}{2.2} \eta_{0.1} = 4.5 \times 10^7 \eta_{0.1} \text{ yr}$$

Rate of increase in mass for a black hole

$$\frac{dM}{dt} = \dot{M}_{\mathrm{Edd}} = \frac{M}{t_{\mathrm{Edd}}} \rightarrow \int_{M_0}^{M} \frac{1}{M'} dM = \frac{1}{t_{\mathrm{Edd}}} \int_{t_0}^{t} dt' = \frac{1}{t_{\mathrm{Edd}}} (t - t_0) \overset{t_0=0}{=} \frac{t}{t_{\mathrm{Edd}}}$$

$$\rightarrow \ln(\mathrm{M}) - \ln(\mathrm{M}_0) = \frac{t}{t_{\mathrm{Edd}}} \rightarrow M = M_0 \; e^{(t/t_{\mathrm{Edd}})}$$

M_0 is often referred to as the mass of the **BH seed** (see lesson on the high-redshift AGN at the end of the course and the topic concerning the nature of their progenitors)

T(R) in accretion discs (I)

$$L = \frac{1}{2}\frac{GM\dot{M}}{R}$$

Half of the E_{grav} is radiated, half goes into heating of the gas

$$L = 2\pi R^2 \sigma_{SB} T^4$$

πR^2=area of the disc (×2: both surfaces)
$\sigma_{SB} T^4$: flux passing through the surface

Stefan-Boltzmann law

$$T = \left(\frac{GM\dot{M}}{4\pi\sigma_{SB}R^3}\right)^{1/4} \rightarrow \left[\frac{3GM\dot{M}}{8\pi\sigma_{SB}R^3}\left(1 - \left(\frac{R_{in}}{R}\right)^{1/2}\right)\right]^{1/4}$$

Proper treatment

$$R_{in} \approx R_S = 2GM/c^2$$

$$R >> R_{in} \rightarrow 1-(R_{in}/R)^{1/2} \approx 1$$

$$R^{-3/4} = (R/R_S)^{-3/4} \times (2GM/c^2)^{-3/4}$$

$$T(R) = \left(\frac{3c^6}{64\pi\sigma_{SB}G^2}\right)^{1/4} M^{-1/2}\, \dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4}$$

see also Done 2010, arXiv:1008:2287

All calculations

$$T = \left(\frac{3GM\dot{M}}{8\pi\sigma_{SB}}\right)^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} \left(\frac{2GM}{c^2}\right)^{-3/4} =$$

$$= \left(\frac{3G}{8\pi\sigma_{SB}}\right)^{1/4} M^{1/4} \dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} \left(\frac{c^2}{2G}\right)^{3/4} M^{-3/4} =$$

$$= \left(\frac{3GG^{-3}c^6}{8\pi\sigma_{SB}2^3}\right)^{1/4} M^{-1/2} \dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} =$$

$$= \left(\frac{3G^2c^6}{8\pi\sigma_{SB}2^3}\right)^{1/4} M^{-1/2} \dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} =$$

$$= \left(\frac{3c^6}{64\pi\sigma_{SB}G^2}\right)^{1/4} M^{-1/2} \dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4}$$

T(R) in accretion discs (II)

$$\dot{M}_{\rm Edd} = 2.2 M_8/\eta_{0.1} \quad [M_{\rm sun}/{\rm yr}]$$

$$T = {\rm cons} \times M^{-1/2}\,\dot{M}^{1/4}\left(\frac{R}{R_S}\right)^{-3/4}$$

$\eta=0.1$

$M^{-1/2}$

$M_{dot}^{1/4}=(M_{dot,Edd} \times M_{dot}/M_{dot,Edd})^{1/4}$

$$T(R) = {\rm cost} \times (10^8 M_8 M_{\rm sun})^{-1/2}(2.2 M_8)^{1/4}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{1/4}\left(\frac{R}{R_S}\right)^{-3/4}$$

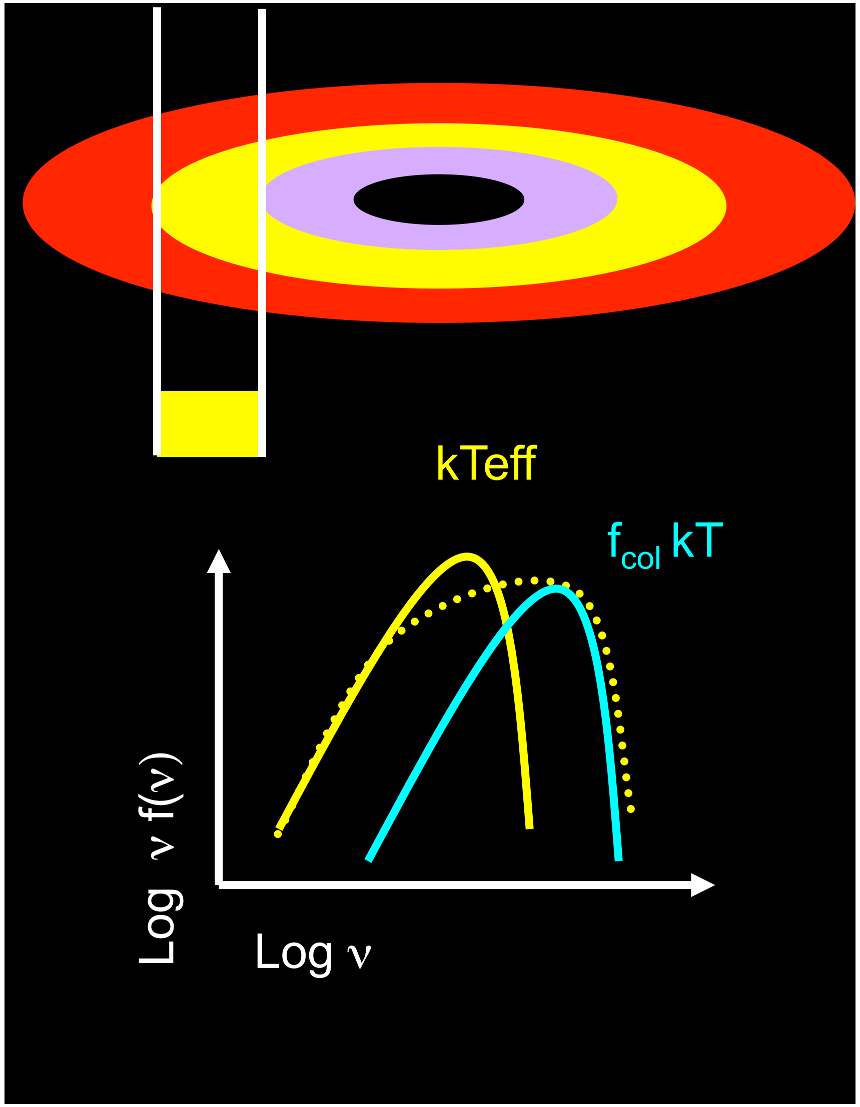
$$T(R) \approx 2.1\times 10^5\ M_8^{-1/4}\ (\dot{M}/\dot{M}_{\rm Edd})^{1/4}\ ({\rm R/R_S})^{-3/4}\ {\rm K}$$

$$\propto M^{-1/4}\ (\dot{M}/\dot{M}_{\rm Edd})^{1/4}$$

$T_{BB}(R) \approx M^{-1/4}$: thermal (disk) emission mostly emitting in UV for AGN (*big blue bump*), in soft X-rays in BHBs

T(R) in accretion discs (III)

- Disc annuli not blackbody – too hot, so little true opacity. Compton scattering important.
- Modified blackbody (Shakura & Sunyaev 1973)
- Describe by colour temperature f_{col}
- Relativistic smearing effects on the spectra at each radius



as already seen in the lesson on accreting NS with discs