Emission mechanisms
Part I

partly derived and adapted from Prof. G. Matt's lesson for the 2008 high-energy school held in Urbino

Some reference textbooks + pubs:

- Bradt, *"Astrophysics Processes", Cambridge University Press*
- Rybicki & Lightman, *"Radiative processes in astrophysics", Wiley*
- Ghisellini, *"Radiative processes in High-Energy Astrophysics"* (arXiv:1202.5949) & Lecture Notes in Physics 873 (Springer)

Outline of part I

- Radiative transfer: basics
- Black body radiation
- Bremsstrahlung emission
- Synchrotron emission
- Compton scattering (Inverse Compton)
- Cherenkov radiation

Radiative transfer: basics. I

absorption

$$dI_\nu = -\alpha_\nu \, I_\nu \, ds$$

Decrement of radiation from a source while passing through an infinitesimal path of length ds

$$\alpha_\nu = \mathrm{n} \, \sigma_\nu$$

Absorption coefficient [length^{-1}]: density of absorbers times the cross section of the absorbing process

$$d\tau_\nu = \alpha_\nu \, ds = n \, \sigma_\nu \, ds$$

Optical depth (opacity)

τ_υ>>1: optically thick case

emission

$$dI_\upsilon = j_\nu \, ds$$

Contribution from the emitters along ds

$$S_\upsilon = j_\nu / \alpha_\nu \rightarrow j_\nu = \alpha_\nu S_\upsilon$$

Source function: emissivity/opacity ratio

Thermal equilibrium: S_ν=Planck function

➔ **Kirchcoff's Law**

Absorption+Emission

$$\frac{dI_\nu}{ds} = -\alpha_\nu \, I_\nu + j_\nu$$

Radiative transfer: basics. II

Only absorption

$$dI_\nu = -\alpha_\nu \, I_\nu \, ds \;\rightarrow\; \int_{I_0}^{I} \frac{1}{I_\nu} dI_\nu = -\int_0^{\tau} d\tau'_\upsilon$$

$$\left[\ln I_\nu\right]_{I_0}^{I} = -\left[\tau'_\upsilon\right]_0^{\tau} \;\Rightarrow\; I_\upsilon = I_{0,\nu}\, e^{-\tau_\upsilon}$$

Absorption coefficient assumed constant across the width of the layer

$I_{0,v}$=intensity of the radiation before passing the absorbing layer

Only emission

$$dI_\upsilon = j_\nu \, ds \;\rightarrow\; I_\nu = I_{0,\nu} + \int_0^{S} j_\nu \, ds$$

S=total emitting path

$I_{0,v}$=intensity of the radiation before passing the emitting layer

Radiative transfer: basics. III

Both terms and thermal equilibrium

$$\Rightarrow \frac{dI_\upsilon}{ds} = -\alpha_\nu\, I_\nu + j_\nu \rightarrow \quad \frac{dI_\upsilon}{\alpha_\nu ds} = \frac{dI_\upsilon}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu} = -I_\nu + S_\nu$$

$$I_\upsilon(\tau_\nu) = I_{\nu,0}\ \mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} \mathrm{e}^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu)\ \mathrm{d}\tau'_\nu = I_{\nu,0}\ \mathrm{e}^{-\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

$$\frac{dI_\upsilon}{d\tau_\nu} = -I_\nu + S_\nu : \text{if TE, LTE: } \frac{dI_\upsilon}{d\tau_\nu} = 0 \rightarrow I_\nu = S_\nu = \text{Planck function}$$

$$\Rightarrow \frac{dI_\upsilon}{ds} = -\alpha_\nu\, I_\nu + j_\nu \rightarrow \quad I_\nu = I_{\nu,0}\ \mathrm{e}^{-\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

Radiative transfer: basics. IV

Detailed integration

$$d\tau_\nu = \alpha_\nu ds \qquad S_\nu = j_\nu / \alpha_\nu$$

$$dI_\upsilon = -\alpha_\nu \, I_\nu ds + j_\nu ds = -I_\nu d\tau_\nu + S_\nu d\tau_\nu$$

$$dI_\upsilon + I_\nu d\tau_\nu = S_\nu d\tau_\nu$$

$$e^{\tau_\nu}(dI_\upsilon + I_\nu d\tau_\nu) = e^{\tau_\nu}(S_\nu d\tau_\nu)$$

$$d(e^{\tau_\nu} I_\nu) = e^{\tau_\nu} dI_\upsilon + I_\nu e^{\tau_\nu} d\tau_\nu$$

$$\rightarrow \; d(e^{\tau_\nu} I_\nu) = e^{\tau_\nu} S_\nu d\tau_\nu$$

starting point for the integration: τ_ν, $I_{\nu,0}$

$$e^{\tau_\nu} I_\nu - I_{\nu,0} = \int_0^{\tau_\nu} e^{\tau_\nu '} S_\nu \; d\tau_\nu '$$

$$e^{-\tau_\nu}(e^{\tau_\nu} I_\nu - I_{\nu,0}) = I_\nu - I_{\nu,0} e^{-\tau_\nu} = \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu \; d\tau'_\nu = S_\nu - S_\nu \; e^{-\tau_\nu} = S_\nu (1 - e^{-\tau_\nu})$$

$$\rightarrow \; I_\upsilon(\tau_\nu) = I_{\nu,0} \; e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu d\tau'_\nu = I_{\nu,0} \; e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu})$$

Radiative transfer: basics. V

$$I_{\nu,0}=0 \rightarrow I_\nu(\tau_\nu)=S_\nu(1-e^{-\tau_\nu})=\frac{j_\nu S_0}{\alpha_\nu S_0}(1-e^{-\tau_\nu})=j_\nu S_0\,\frac{1-e^{-\tau_\nu}}{\tau_\nu}$$

$$\tau_\nu >> 1 \rightarrow I_\nu = \frac{j_\nu S_0}{\tau_\nu}$$

$$\tau_\nu << 1 \rightarrow I_\nu = j_\nu S_0$$

Optically thick regime
typically at low ν

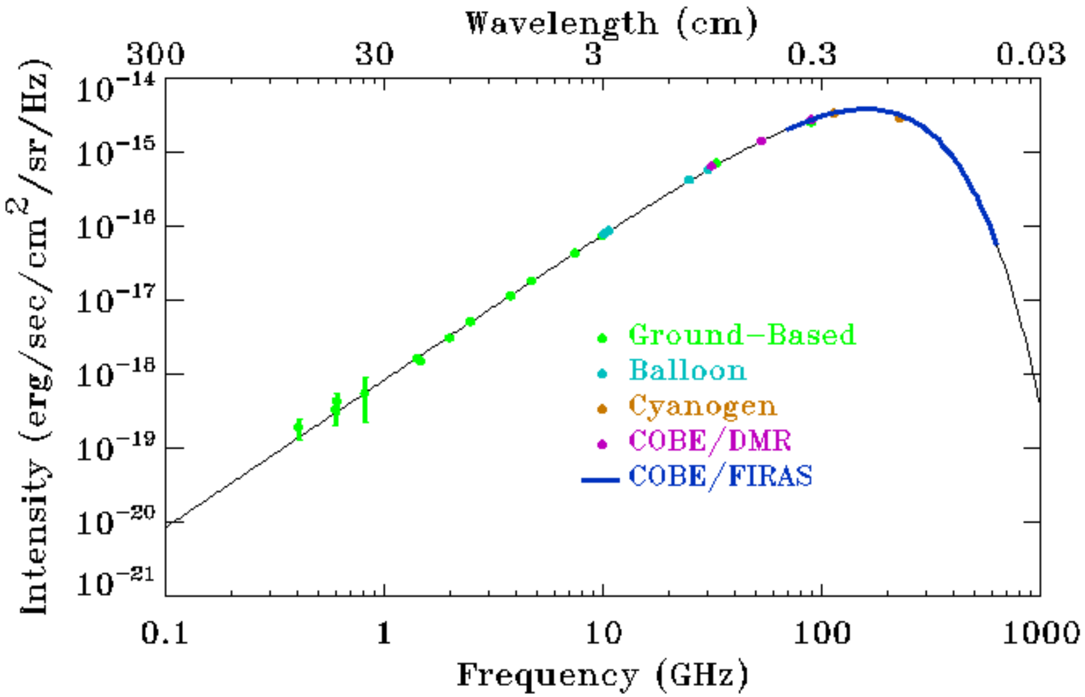
Optically thin regime
typically at high ν

From a thick source we are seeing the emission from a layer of width S_0/τ_ν, which is optically thin

Blackbody radiation

Black body radiation arises when matter is optically thick and photons scatter many times before encountering the observer.
Particles and photons share their kinetic energy continually.

In perfect thermal equilibrium: **(3/2) kT=<1/2 mv²>=h<v>**
Local thermodynamic equilibrium (LTE) in the atmospheres of stars.



CMB spectrum measured by COBE (adapted from Mather+1990)

$$I(\upsilon,T) = \frac{2h\upsilon^3}{c^2}\frac{1}{e^{h\upsilon/kT}-1}$$

Planck radiation law (Planck function)
Specific intensity
[W/m^2/Hz/sr]

Rayleigh-Jeans approximation
hv<<kT (Taylor expansion:
$e^{hv/kT} \approx 1+hv/kT$)

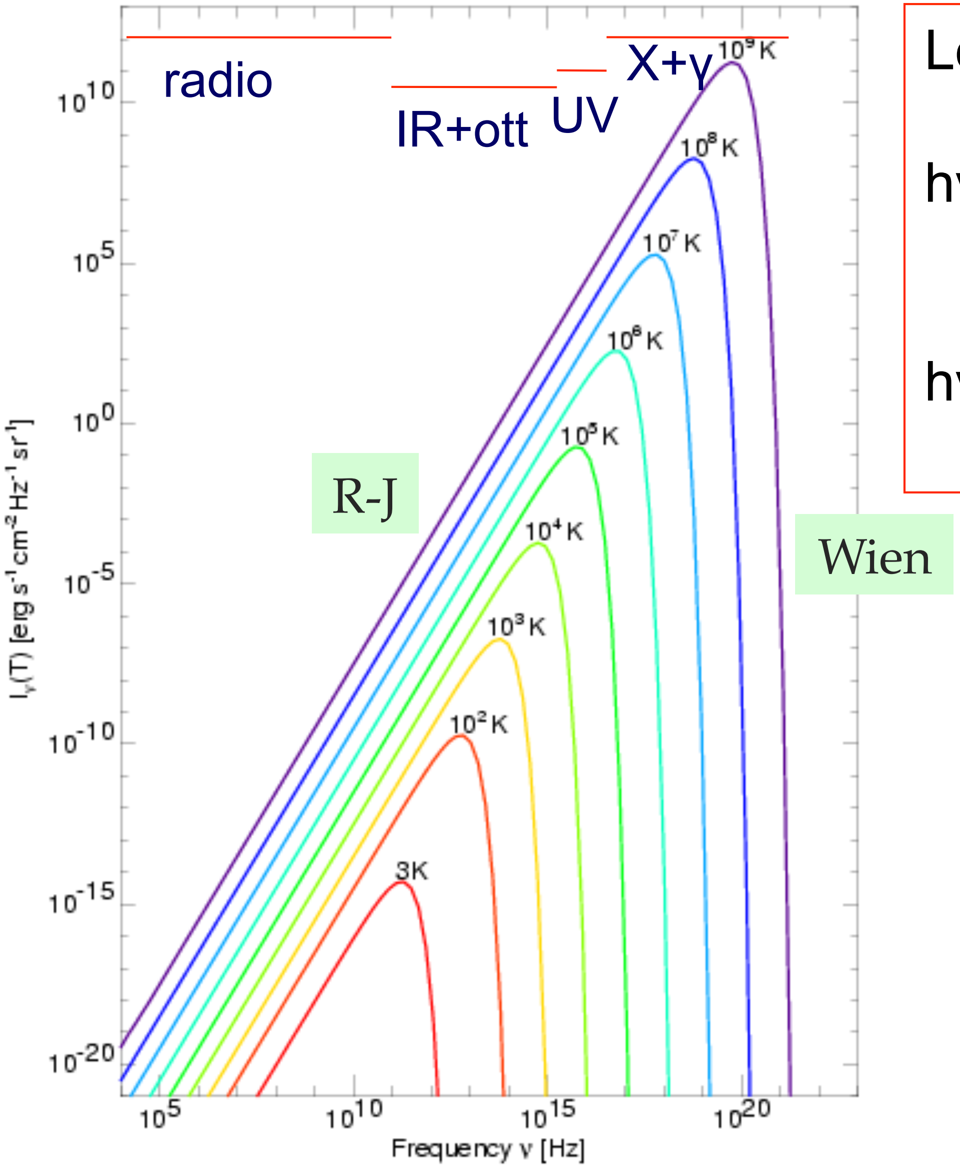
$$I(\upsilon,T) = \frac{2\upsilon^2}{c^2} kT \propto \upsilon^2 T$$

Wien approximation (law)
hv>>kT, $e^{hv/kT}>>1$

$$I(\upsilon,T) = \frac{2h\upsilon^3}{c^2} e^{-h\upsilon/kT}$$

Peak frequency
$hv_{peak}=2.82kT$
$v_{peak}(Hz)=5.88\times10^{10}\,T(K)$

CMB peak=1.06 mm



Log-Log plot of the black body law

$h\nu \ll kT$: R-J approx.:
power-law with slope=2

$h\nu \gg kT$: Wien approx.:
$I \approx \nu^3 e^{-h\nu/kT}$

Spectral energy density = sum of the energies of photons contained in a 1 m^3 at a fixed time $[J/m^3/Hz]$ or impimping on a 1-m^2 surface in 1 s

$$\mu(\upsilon,T) = I(\upsilon,T)\frac{4\pi}{c} = \frac{8\pi h\upsilon^3}{c^3}\frac{1}{e^{h\upsilon/kT}-1}$$

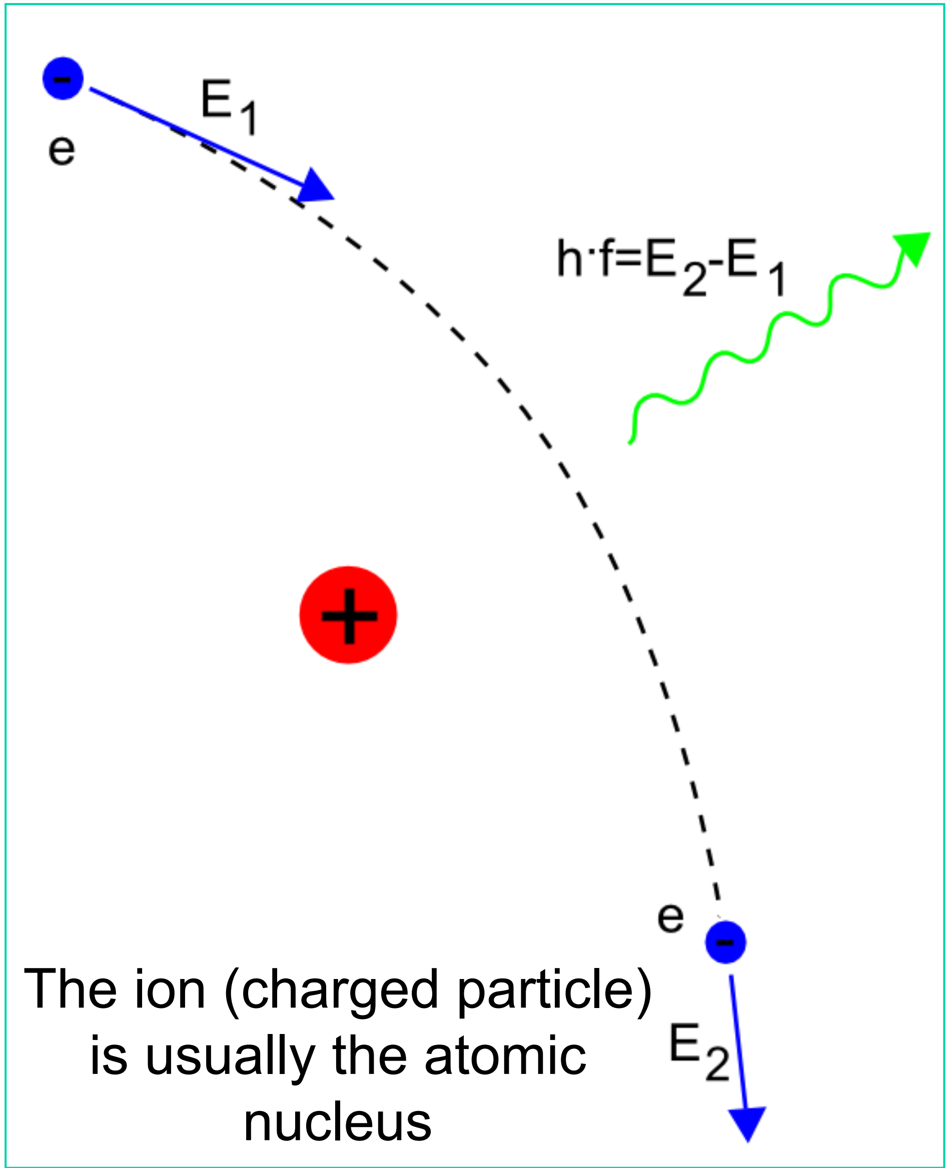
isotropic: $I \approx \mu/4\pi$

Real objects never behave as full-ideal black bodies, and instead the emitted radiation at a given frequency is a fraction of what the ideal (black body) emission would be.

Black body emission occurs when $\tau \rightarrow \infty$, deviations are due to finite opacities and surface layers effects

Grey body (capacity of emission and absorption at any frequency less than 100%): $S(\nu) \approx \nu^{3+\beta}$, where β=emissivity index≈1−2

Bremsstrahlung
("braking radiation" – free-free emission)



Coulomb collisions between electrons and ions in a plasma produce photons
Electrons are accelerated by the Coulomb forces and lose energy via radiation

This is the **bremsstrahlung** (or **free-free**) **emission** (because the electron is free before and after the collision)
➔ ***deflection*** (acceleration) of the e^-
If the gas is in thermal equilibrium, ion and electron velocities obey to the Maxwell-Boltzmann distribution.

$$j_\upsilon(\upsilon,T) \propto g(\upsilon,T) n_e^2 T^{-1/2} e^{-h\upsilon/kT}$$

Volume emissivity:
power emitted per unit volume of the plasma at some frequency in unit of frequency interval

hydrogen plasma
[W/m^3/Hz]

h=Planck constant
k=Boltzmann constant
G(v,T)=Gaunt factor: takes into account exact quantum mechanics calculations (quantum effects+Debye screening) ≈ unity for a large interval of param.

≈√3/π ln(2.25 kT/hv)
if Z=1, T>3×10^5 K and hv<<kT

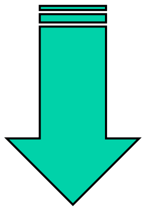
$$I(\upsilon,T) \propto g(\upsilon,T) n_e^2 T^{-1/2} e^{-h\upsilon/kT} \Lambda$$

Specific intensity
hydrogen plasma
[W/m^2/Hz/sr]

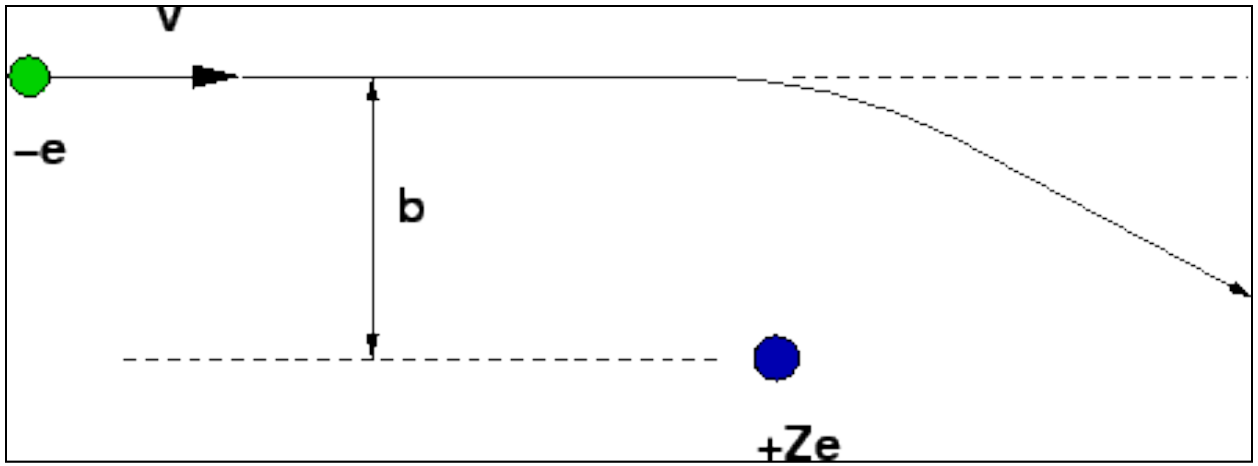
Λ=thickness of the plasma along the line of sight
4πI=j_vΛ

Classical approximation: the energy loss of an electron, integrated over an entire collision, is only a small fraction of its kinetic energy

- τ=optical depth <<1 (optically thin plasma)
- ion acceleration<<electron acceleration:
  e^- traveling through a region of static electric field (ions move slowly)



The interaction can be treated as elastic collision of duration Δt≈(2)b/v
The power radiated by the accelerating particle is given by the *Larmor formula*



b=impact parameter:
projected distance
of the closest approach
of the e^- to the ion at
a given encounter

Larmor formula: emitted power (integrated over all angles)

$$P(t) = \frac{1}{6\pi\varepsilon_0} q^2 a(t)^2 / c^3$$

Total power [W=J/s=energy/time]
v<<c

ε_0=permissivity of the vacuum
q=charge of the particle
a(t)=acceleration of the particle

⇓

$$Q(b, \mathrm{v}) = \frac{1}{(4\pi\varepsilon_0)^3} \frac{2}{3} \frac{Z^2 e^6}{c^3 m^2 b^3 \mathrm{v}}$$

Total energy (J/collision) radiated by a single electron of speed v as it passes an ion of charge Ze (Z=atomic number) with impact parameter b

Faster e^- spend less time close to the ion

Some details

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r^2}$$

Coulomb force ➔

$$a = \frac{F}{m} = \frac{1}{4\pi\varepsilon_0} \frac{(Ze)e}{mr^2}$$

Small deflection: the e^- loses only a small part of its E_{kin}
Closest approach ≈ b ➔

$$a_{\max} \approx \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{mb^2}$$

Collision time: $\tau_b \approx b/v$
assuming constant acceleration=max
all happens at small distances (but Debye screening from other electrons…)

$$Q(b,\mathrm{v}) = \int_{-\infty}^{\infty} P(t)dt = \frac{1}{6\pi\varepsilon_0} \frac{e^2}{c^3} \int_{-\infty}^{\infty} a^2(t)dt \approx \frac{1}{6\pi\varepsilon_0} \frac{e^2}{c^3} a_{\max}^2 \tau_b$$

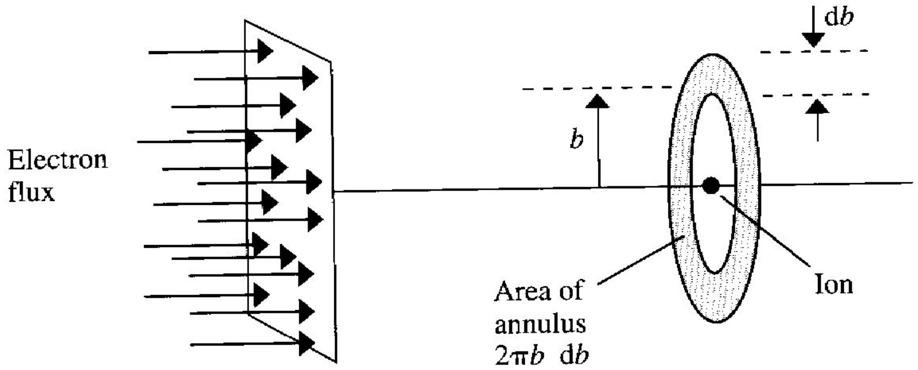
➔

$$Q(b,\mathrm{v}) = \frac{1}{(4\pi\varepsilon_0)^3} \frac{2}{3} \frac{Z^2 e^6}{c^3 m^2 b^3 \mathrm{v}}$$

Energy radiated per collision

Case of single-speed electron beam

Case of one ion and a parallel beam of electrons of speed v intersecting a narrow annulus of radius b and width db



Electron flux (electrons/m²/s) $n_e \,\mathrm{v}$

Number of e- per second striking this area $n_e \,\mathrm{v}\, 2\pi \mathrm{b}\, db$

Emitted power/ion $Q(b,\mathrm{v})\, n_e \mathrm{v}\, 2\pi \mathrm{b}\, db$

Angular frequency ω

$$\omega \approx {1}/{\tau_b} = {\mathrm{v}}/{\mathrm{b}} \rightarrow \nu = {\omega}/{2\pi} = {\mathrm{v}}/{2\pi b} \rightarrow db = -{\mathrm{v}}/{2\pi \nu^2}\, d\upsilon$$

Case of electrons of many speeds

Maxwell-Boltzmann distribution describes the probability of having e^- of speed v for a non-degenerate gas

$$P(\mathrm{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m\mathrm{v}^2/2kT}$$

Probability of an e^- having speed v in the interval dv

P(v) × (volume of a shell of velocity v)

$$= P(\mathrm{v}) \times 4\pi \mathrm{v}^2\, d\mathrm{v}$$

Total power summed over all speeds=
power emanating from each ion at frequency ν in the band dν…

$$Q(b,\mathrm{v})\, n_e \mathrm{v}\, 2\pi b\, db \approx \frac{1}{(4\pi\varepsilon_0)^3} \frac{2}{3} \frac{Z^2 e^6}{c^3 m^2 b^3 \mathrm{v}} n_e \mathrm{v}\, 2\pi \frac{\mathrm{v}}{2\pi\upsilon} \frac{\mathrm{v}}{2\pi\upsilon^2}$$

…× Maxwell-Boltzmann distribution of velocities

Spectrum of emitted photons

Multiple ions and electrons, having a Maxwell-Boltzmann distribution of speeds

$$j_v(\upsilon)d\upsilon = g(\upsilon,T,Z)\frac{1}{(4\pi\varepsilon_0)^3}\frac{32}{3}\left(\frac{2\pi^3}{3m^3k}\right)^{1/2}\frac{Z^2e^6}{c^3}n_e n_i\frac{e^{-h\upsilon/kT}}{T^{1/2}}d\upsilon$$

Volume emissivity [W/m^3/Hz] – power emitted in all directions

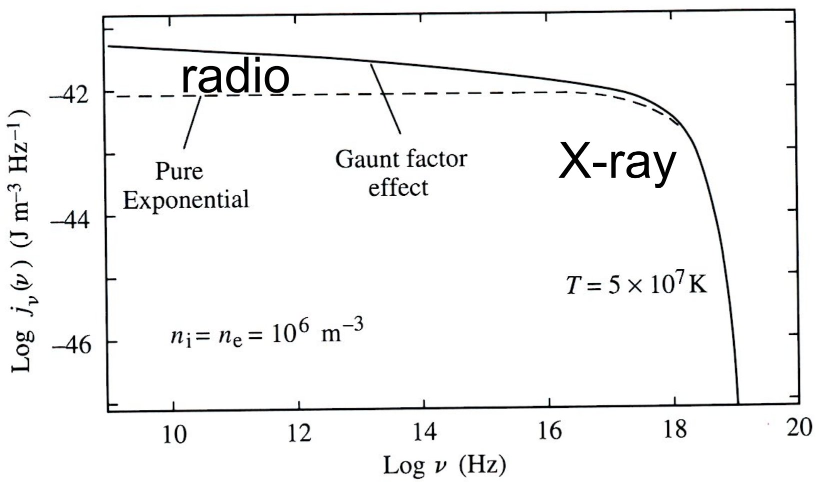


Figure 5.5. Theoretical continuum thermal bremsstrahlung spectrum. The volume emissivity (37) is plotted from radio to x-ray frequencies on a log-log plot with the Gaunt factor (38) included. The specific intensity $I(\nu, T)$ would have the same form. Note the gradual rise toward low frequencies due to the Gaunt factor. We assume a hydrogen plasma ($Z = 1$) of temperature $T = 5 \times 10^7$ K with number densities $n_i = n_e = 10^6$ m^{-3}.

Theoretical continuum thermal bremsstrahlung emission

(from H. Bradt, "Astrophysics Processes")

If τ>>1 ➔ black body emission

Integrated volume emissivity

Total power from unit volume integrated over frequencies

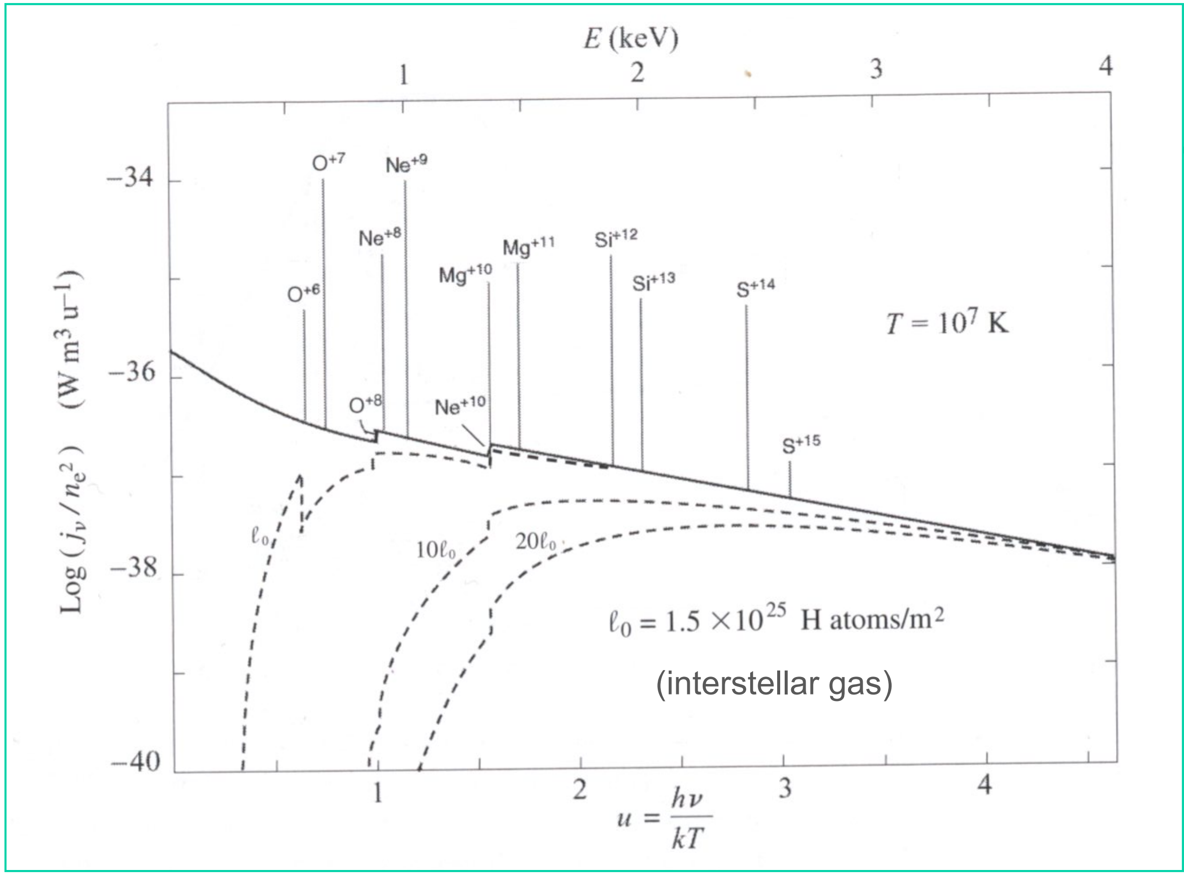
$$j_\upsilon(\upsilon,T) \propto g(\upsilon,T,Z)Z^2 n_e n_i e^{-h\upsilon/kT} T^{-1/2}$$

$$j(T) = \int_0^\infty j_\upsilon(\upsilon)d\upsilon = cons \times \overline{g}(T,Z)Z^2 n_e n_i T^{-1/2} \int_0^\infty e^{-h\upsilon/kT} d\upsilon$$

$$j(T) = \int_0^\infty j_\upsilon(\upsilon)d\upsilon \propto \overline{g}(T,Z)Z^2 n_e n_i T^{1/2}$$

[W/m^3]

An example of bremsstrahlung spectrum



Theoretical calculation of the volume emissivity/n^2_e bremsstrahlung emission

(from H. Bradt, "Astrophysics Processes")

Free-free absorption

A photon can be absorbed by a free electron in the Coulombian field of an atom: it is the free-free absorption, which is the absorption mechanism corresponding to bremsstrahlung.
Thus, for thermal electrons (equilibrium):

$$j_\upsilon = \alpha_\upsilon S_\upsilon = \alpha_\upsilon B_\upsilon$$

Kirchhoff's Law (relating emission to absorption for a thermal emitter) + Planck function (B_ν)

$$\alpha_\upsilon(\upsilon,T) = j_{br}(\upsilon,T) / B(\upsilon,T) \propto Z^2 n_e n_i T^{-1/2}(1-e^{-h\upsilon/kT})\upsilon^{-3}$$

Absorption coefficient for a plasma emitting via Bremsstrahlung (how effectively the gas absorbs its own radiation)

hν>>kT

$$\alpha_\upsilon \propto \upsilon^{-3} T^{-1/2}$$

hν<<kT
RJ regime

$$\alpha_\upsilon \propto \upsilon^{-2} T^{-3/2}$$

optical depth

$$\tau_\upsilon = \int \alpha_\upsilon dl$$

At low frequencies, matter in thermal equilibrium is optically thick to free-free aborption, becoming thin at high frequencies.

***Bremsstrahlung + Equation of transport* (I)**

$$I_{\upsilon,brem} = B_{\upsilon}(T)(1 - e^{-\tau_{\upsilon}})$$

$$\tau_{\nu} >> 1 \rightarrow I_{\nu} = B_{\nu}$$ Opt. thick regime

$$\tau_{\nu} << 1 \rightarrow I_{\nu} = B_{\nu}\tau_{\nu}$$ Opt. thin regime

hv/kT<<1, opt. thick

$$I_{\nu} = B_{\nu} \propto \upsilon^{2}$$

hv/kT<<1, opt. thin

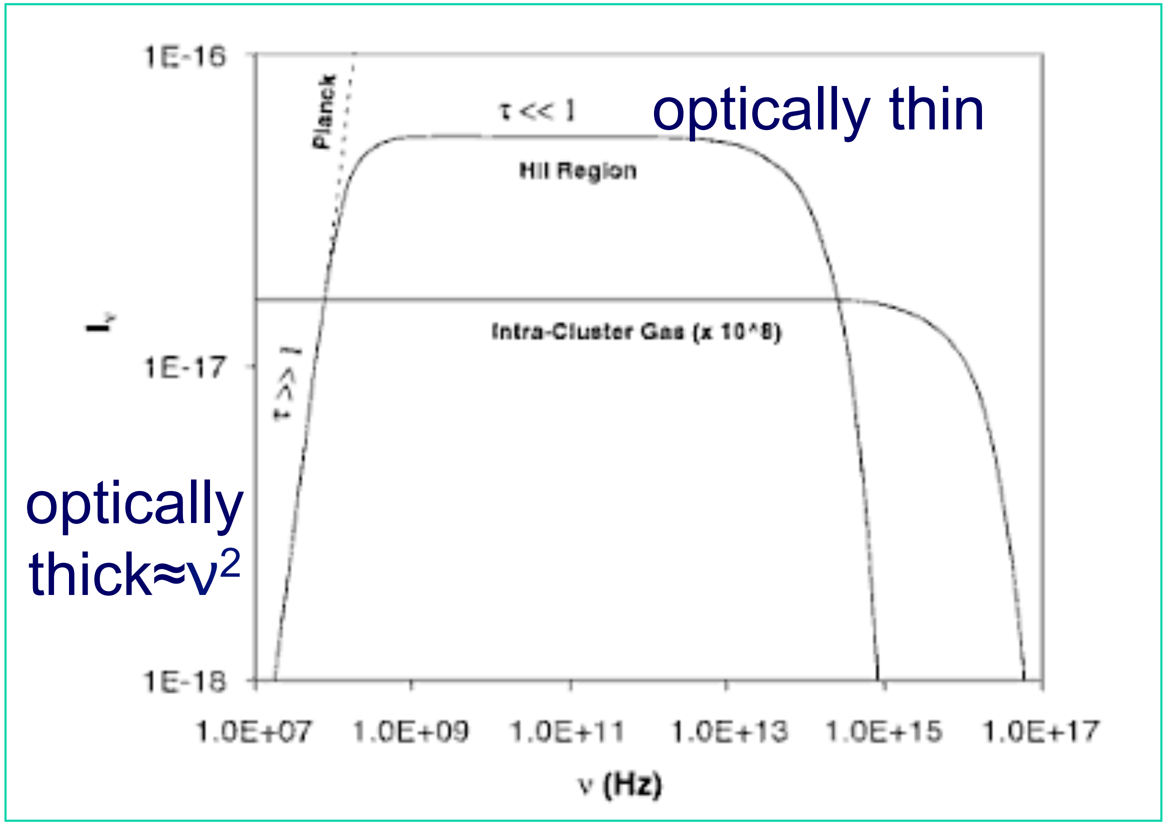
$$I_{\nu} = B_{\nu}\tau_{\nu} \propto \upsilon^{2}\tau_{\nu} \propto \upsilon^{2}(\upsilon^{-2}T^{-3/2}) = \text{cons.}$$

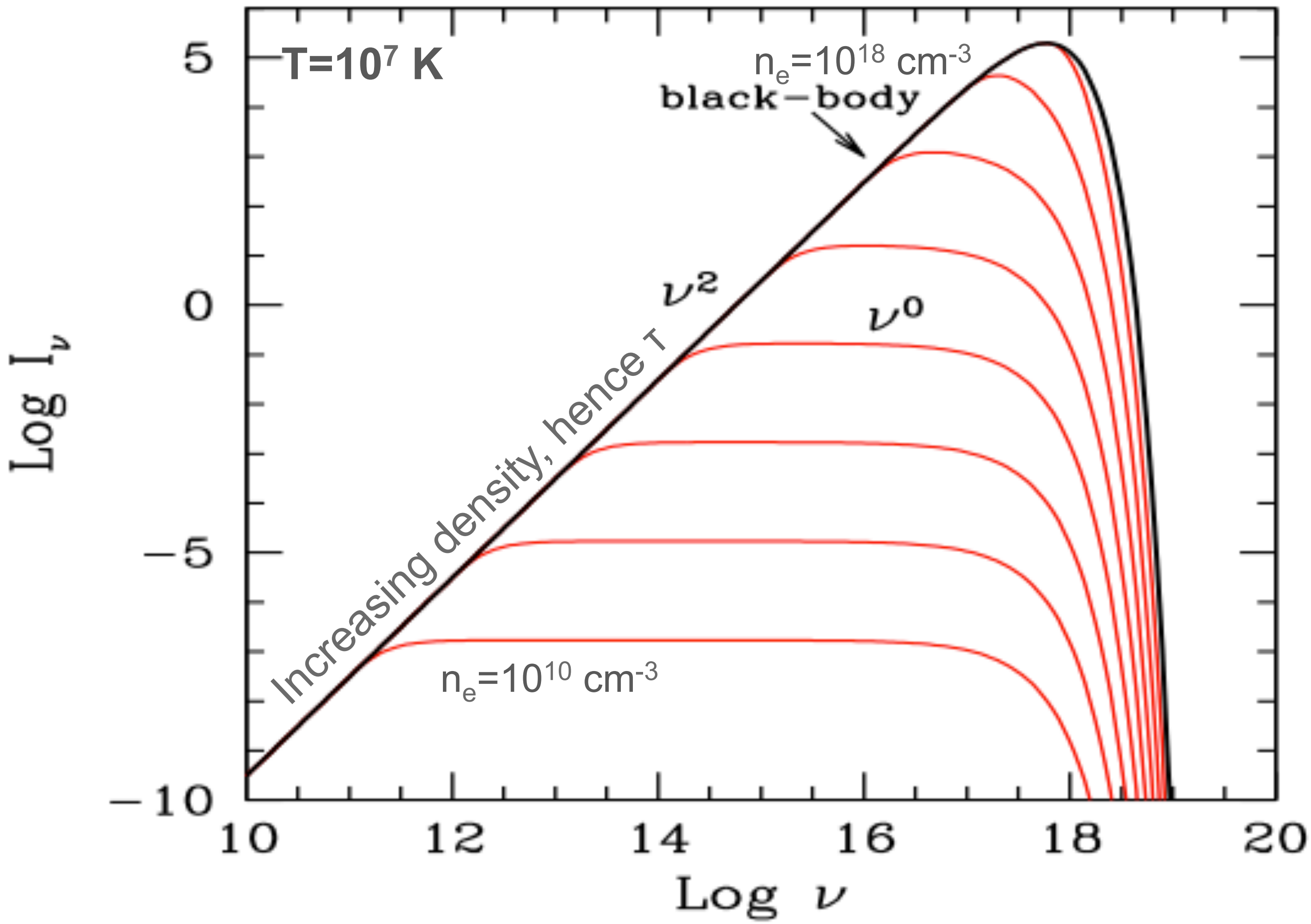
hv/kT>>1, opt. thin

$$I_{\upsilon} \approx B_{\upsilon}(T)\tau_{\upsilon} \approx (\upsilon^{3}e^{-h\upsilon/kT})\ (\upsilon^{-3}T^{-1/2}) \propto e^{-h\upsilon/kT}$$

τ_{ν} in the two regimes from previous slide

Bremsstrahlung + Equation of transport (II)





Absorption coefficient for a plasma emitting via Bremsstrahlung (how effectively the gas absorbs its own radiation)

At increasing densities, the spectrum is self-absorbed up to larger and larger frequencies

Bremsstrahlung: cooling time

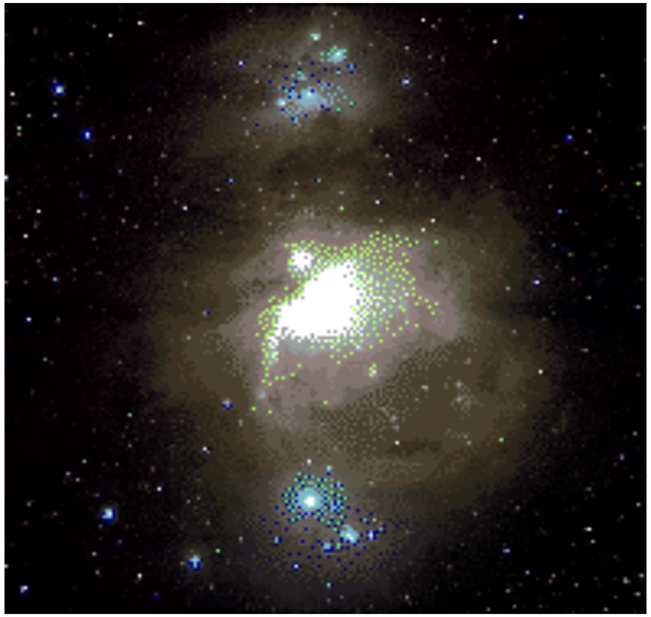
For any emission mechanism, the cooling time is defined as:

$$t_{cool} = \frac{E}{|dE/dt|} = \frac{3/2(n_e + n_i)kT}{j_\nu(T)}$$

where E is the energy of the emitting particle (thermal) and dE/dt the energy lost by radiation.

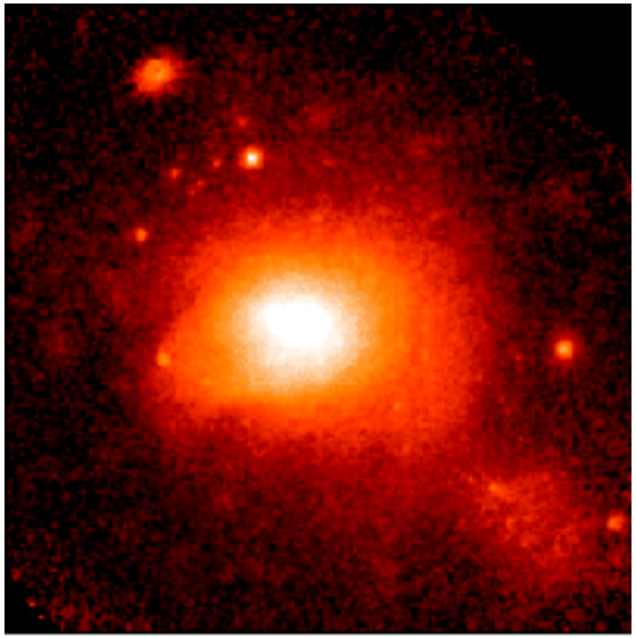
$$t_{cool} \approx \frac{6 \times 10^3}{n_e Z^2 \bar{g}_{ff}} T^{\frac{1}{2}} yr$$

Bremsstrahlung emission



← Radio image of the Orion Nebula

X-ray emission of the Coma Cluster →



HII regions: $t_{cool} \approx 1000$ years

($n_e \approx 100\text{-}1000\ cm^{-3}$, $T \approx 1000\text{-}10000$ K)

Clusters of galaxies: a few times 10^{10} years

($n_e \approx 10^{-3}\ cm^{-3}$, $T \approx 10^8$ K)

Bremsstrahlung: polarization

Bremsstrahlung photons are polarized with the electric vector perpendicular to the plane of interaction.

In most astrophysical situations, and certainly in case of thermal bremsstrahlung, the planes of interaction are randomly distributed, resulting in null net polarization.

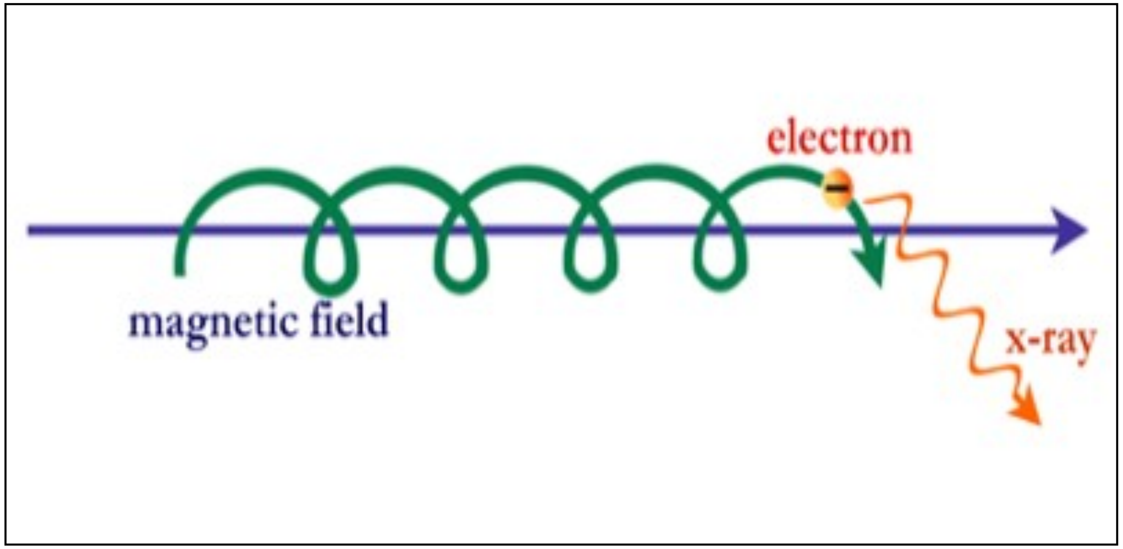
For an anisotropic distribution of electrons, however, bremsstrahlung emission can be polarized.

Synchrotron emission

Synchrotron radiation (or magnetic bremsstrahlung) is due to relativistic electrons spiraling around magnetic fields. The magnetic force q(v×B) causes the electrons to be accelerated, then to lose some of their kinetic energy via photon emission. While in bremsstrahlung emission the electric fields (Coulombian forces) provide the accelerating forces, in synchrotron emission magnetic fields do the same.

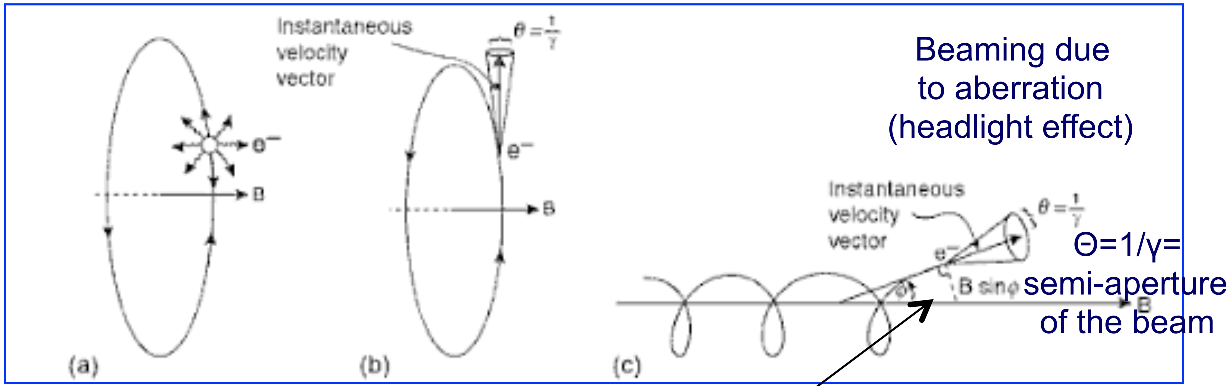
$$\vec{F} = q(\vec{v} \times \vec{B})$$

Lorentz force



$$\vec{F} = q(\vec{v} \times \vec{B})$$

(S.I. system)



φ: *pitch angle* between v and B

The force is always perpendicular to the particle velocity, so it does not do work. Therefore, the particle moves in a *helical path with constant |v|* (if energy losses by radiation are neglected). The *radius of gyration and* the *gyro-frequency* (**cyclotron frequency**) of the orbit are:

$$r_B = \frac{\gamma m v_\perp}{qB} = \frac{\gamma m v \sin\varphi}{qB}$$

$$\omega_B = \frac{v \sin\varphi}{r_B} = \frac{qB}{\gamma m} \ \text{[rad/s]}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$$

Lorenz factor

Synchrotron properties

This emission depends on the electrons' energy properties

It is not possible to associate a temperature to the electrons ➔ **non-thermal emission**

The originating electrons are not in thermal equilibrium with the surroundings

Cyclotron radiation

Radiation from a non-relativistic electron in a magnetic field
(centripetal force)

$$P(t) = -dE/dt = (2/3)\frac{q^2}{c^3}a(t)^2$$

Larmour formula

$$a(t) = F/m = q\mathrm{v}_\perp B/cm$$

(c.g.s. system)

Lorentz

$$-dE/dt = (2/3)\frac{q^4}{m^2c^5}\mathrm{v}_\perp^2 B^2$$

Synchrotron radiation

Radiation from a *relativistic* electron in a magnetic field

$$P(t) = -dE/dt = (2/3)\frac{q^2}{c^3}\gamma^2 a(t)^2$$

Larmour (relativistic case)

$$-dE/dt = (2/3)\frac{q^4}{m^2c^3}\beta^2\gamma^2 B_\perp^2$$

$$\beta = \mathrm{v/c},\ B_\perp = B\sin(\varphi)$$

Thomson cross-section:

$$\sigma_T = (8/3)\pi r_0^2 = (8/3)\pi\left(\frac{e^2}{m_e c^2}\right)^2$$

$$\Rightarrow \quad -dE/dt = (2/3)\frac{q^4}{m^2c^3}\beta^2\gamma^2 B_\perp^2 = (c/4\pi)\sigma_T\beta^2\gamma^2 B_\perp^2$$

$$\langle \sin_\varphi^2 \rangle_{av} = \frac{1}{4\pi}\int_{sphere}\sin_\varphi^2\, d\Omega = 2/3 \quad \Rightarrow \quad -dE/dt = \frac{c}{6\pi}\sigma_T\beta^2\gamma^2 B^2$$

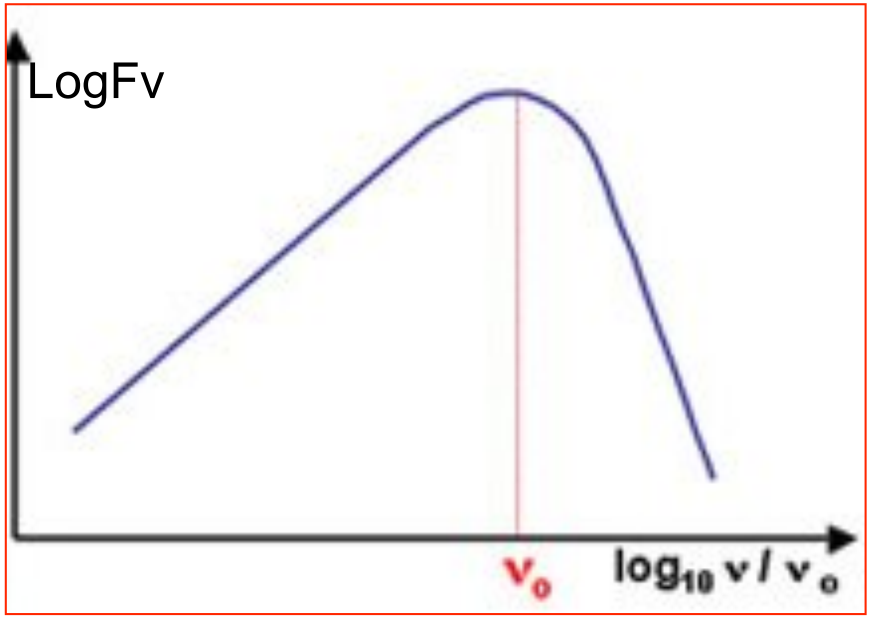
Energy loss by a relativistic electron in terms of magnetic energy density

$$P = -dE/dt = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$$

$$U_B = B^2/8\pi$$ magnetic energy density (J/m^3)

Critical (**characteristic**) **frequency:** the shortest time period (the pulse duration) sets the maximum frequency as seen by the observer

➔ at $\nu > \nu_{crit}$ there is "negligible" emission



$$E = \gamma m_e c^2$$

$$\upsilon_{crit} = \frac{3}{4\pi}\gamma^2 \frac{eB_\perp}{m_e c} = \frac{3}{4\pi}\frac{eB_\perp}{m_e^3 c^5}E^2$$

➔ $\nu_{max} = \nu_0 = (1/3)\nu_{crit}$

Synchrotron emission from an ensamble of e⁻

Radiation from a *multiple relativistic* electrons in a magnetic field

Emission from one electron

$$-dE/dt = \frac{4}{3} c\sigma_T \beta^2 \gamma^2 U_B \propto E^2 B^2$$

Number of particles (e⁻) as a function of energy (energy distribution)

$$N(E) = N_0 E^{-\delta}$$

[δ≈2.5 for cosmic rays over a large range of ν]

Approximation: $\nu \approx \nu_{crit}$

$$\upsilon_{crit} \approx \upsilon \propto E^2 B \rightarrow E \propto \left(\frac{\upsilon}{B}\right)^{1/2}$$

Emission from multiple electrons

$$j_E(E)dE = j_\upsilon(\upsilon)d\upsilon = = (-dE/dt) \times N(E)dE \propto \propto B^2 E^2 N_0 E^{-\delta} dE$$

$$\Rightarrow \quad j_E(E) \propto B^2 E^2 N_0 E^{-\delta} = N_0 B^2 E^{(2-\delta)} \propto N_0 B^{\frac{2+\delta}{2}} \upsilon^{\frac{2-\delta}{2}}$$

$$j_E(E)dE = j_\upsilon(\upsilon)d\upsilon$$

1:1 relation energy-frequency of the photons

$$\frac{dE}{d\upsilon} = \frac{d}{d\upsilon}\left(\frac{\upsilon}{B}\right)^{1/2}$$

$$dE = B^{-1/2}\upsilon^{-1/2}d\upsilon$$

relation between dE and dv

+

$$j_E(E) \propto N_0 B^{\frac{2+\delta}{2}} \upsilon^{\frac{2-\delta}{2}}$$

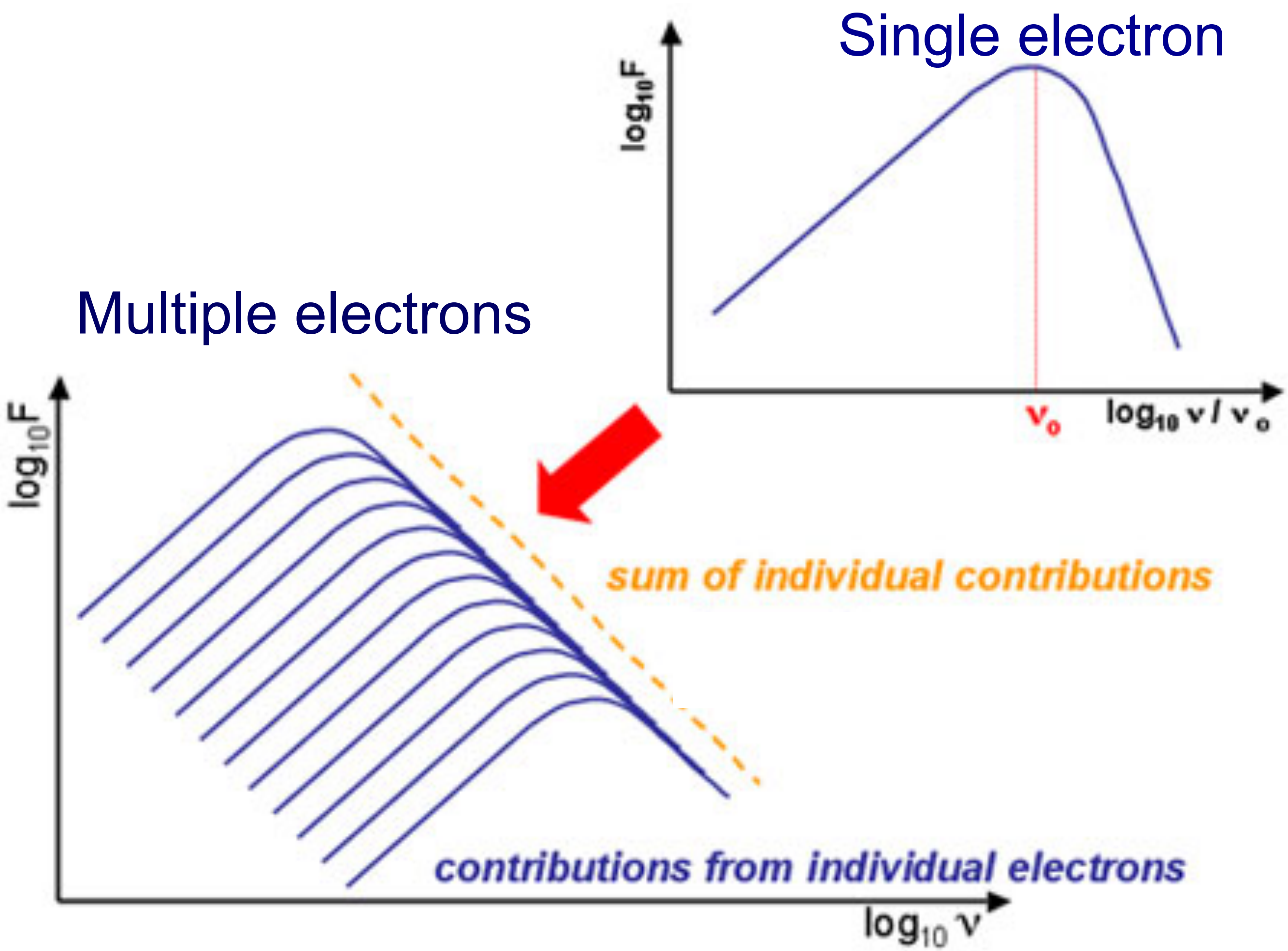
Volume emissivity per Joule [W/m^3/J]

⇒

$$j_\upsilon(\upsilon) = j_E(E)\frac{dE}{d\upsilon} \propto N_0 B^{\frac{2+\delta}{2}} \upsilon^{\frac{2-\delta}{2}} B^{-1/2}\upsilon^{-1/2} =$$

$$= N_0 B^{\frac{\delta+1}{2}} \upsilon^{\frac{1-\delta}{2}} = N_0 B^{\alpha+1}\upsilon^{-\alpha}$$

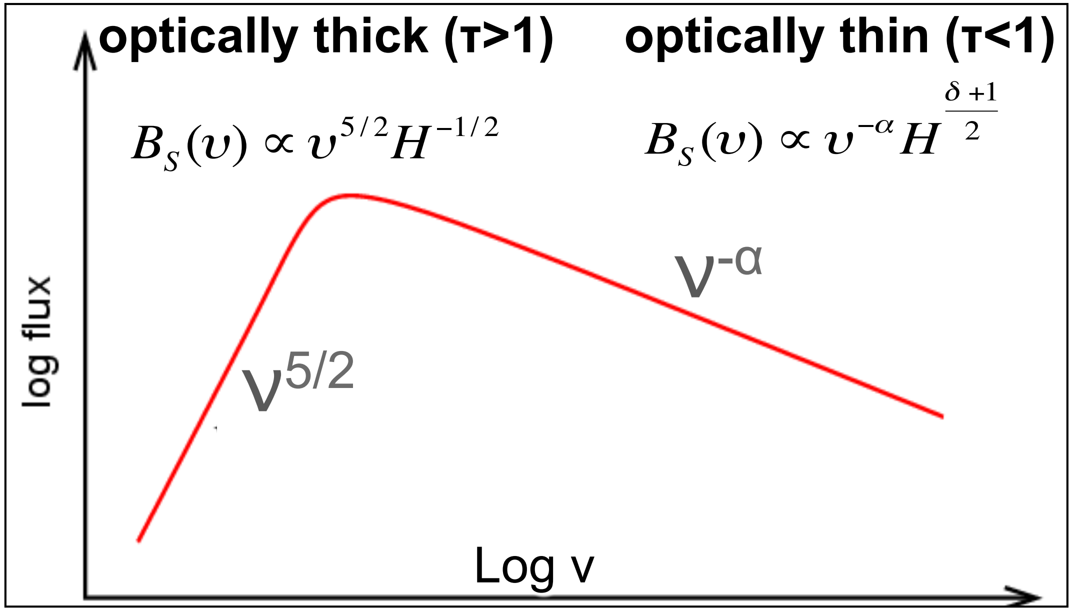
where **α=(δ-1)/2=spectral index**=log slope of the photon-energy spectrum



$$I(\nu) \propto N_0 B^{(\delta+1)/2} \upsilon^{-\alpha} = N_0 B^{\alpha+1} \upsilon^{-\alpha}$$

Synchrotron self-absorption

If the energy distribution of the electrons is non-thermal, e.g. a power law, $N(E)=N_0E^{-\delta}$, the **absorption coefficient** cannot be derived from the Kirchhoff's law. The direct calculation using Einstein's coefficient yields, describing the transitions among energy levels in atoms, leads to:



In the optically thick region, the spectrum is independent of δ

$$\mu_S(\nu) \propto N_0 B^{\frac{\delta+2}{2}} \nu^{-\frac{\delta+4}{2}}$$

Radiative transfer

$$B_S(\upsilon) = \frac{J_S(\upsilon)}{4\pi\mu_S(\upsilon)}\left(1 - e^{-\tau_S(\upsilon)}\right)$$

Brightness of a synchrotron source

The transition frequency (thick/thin regime) is related to B and can be used to determine it

Optically-thick regime

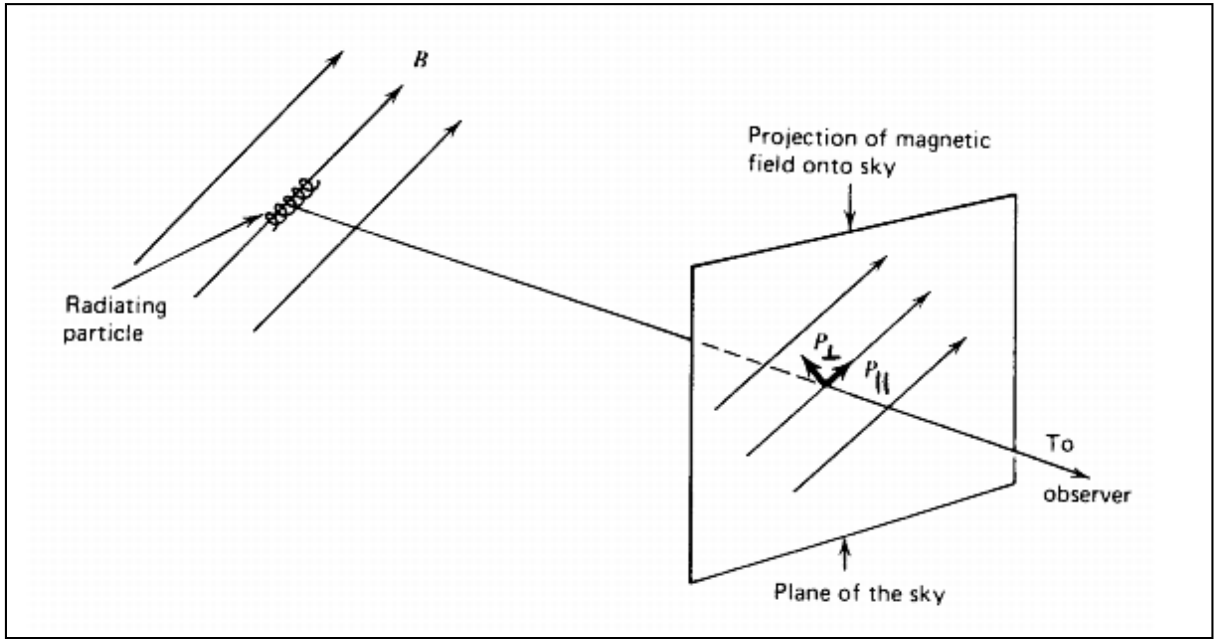
$$\tau_\nu >> 1 \rightarrow B_S(\nu) = I_\nu \propto \frac{B^{\alpha+1}\nu^{-\alpha}}{B^{\frac{\delta+2}{2}}\nu^{-\frac{\delta+4}{2}}} = \frac{B^{\frac{\delta+1}{2}}\nu^{\frac{1-\delta}{2}}}{B^{\frac{\delta+2}{2}}\nu^{-\frac{\delta+4}{2}}} = B^{-1/2}\nu^{5/2}$$

Optically-thin regime

$$\tau_\nu << 1 \rightarrow B_S(\nu) = I_\nu \propto \frac{B^{\alpha+1}\nu^{-\alpha}}{\mu_S(\nu)}\tau_S(\nu) = B^{\frac{\delta+1}{2}}\nu^{-\alpha}$$

$$\alpha = \frac{\delta - 1}{2}$$

Synchrotron polarization

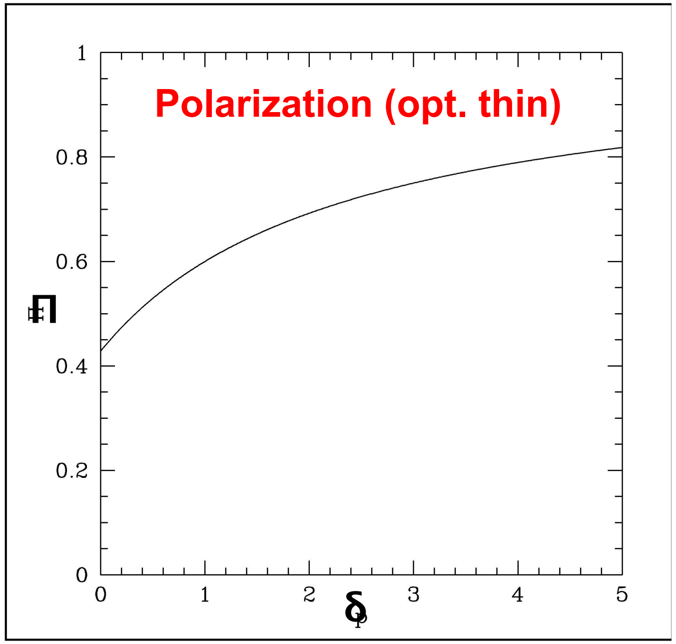


The radiation is (linearly) polarized perpendicularly to the projection of B on the plane of the sky

For a power law distribution of emitting particles, the degree of polarization is $\Pi = P/I = (3\delta+3)/(3\delta+7)$ for the optically thin region

This is actually un upper limit, because the magnetic field is never perfectly ordered.

$\Pi = 3/(16\delta+13)$ for the optically thick region



Synchrotron cooling time

(Electron's rest mass is irreducible)

The cooling time is:

$$t_{cool} \approx \frac{(\gamma - 1)mc^2}{\frac{4}{3}\sigma_T c U_B \gamma^2 \beta^2} \approx (\gamma >> 1) \approx \frac{7.75 \times 10^8}{B^2 \gamma} s$$

For the interstellar matter
(B ~ a few μG, $\gamma \sim 10^4$): $\tau \sim 10^8$ yr

For a radio galaxy (B~10^3 G, $\gamma \sim 10^4$):
$\tau \sim 0.1$ s → continuous acceleration

Thomson scattering

Thomson scattering: elastic scattering of photons from free electrons in an ionized or partially ionized gas. The *scattering* is *elastic* provided the energy of the incoming photon is much less the rest-mass energy of the electron, otherwise some energy would be imparted to the electron. Thomson scattering cross-section is independent of wavelength ("grey").

$$E_{ph} << m_e c^2$$

Condition for Thomson scattering

Thomson cross-section

$$\sigma_T = (8/3)\pi r_0^2 = (8/3)\pi\left(\frac{e^2}{m_e c^2}\right)^2 =$$

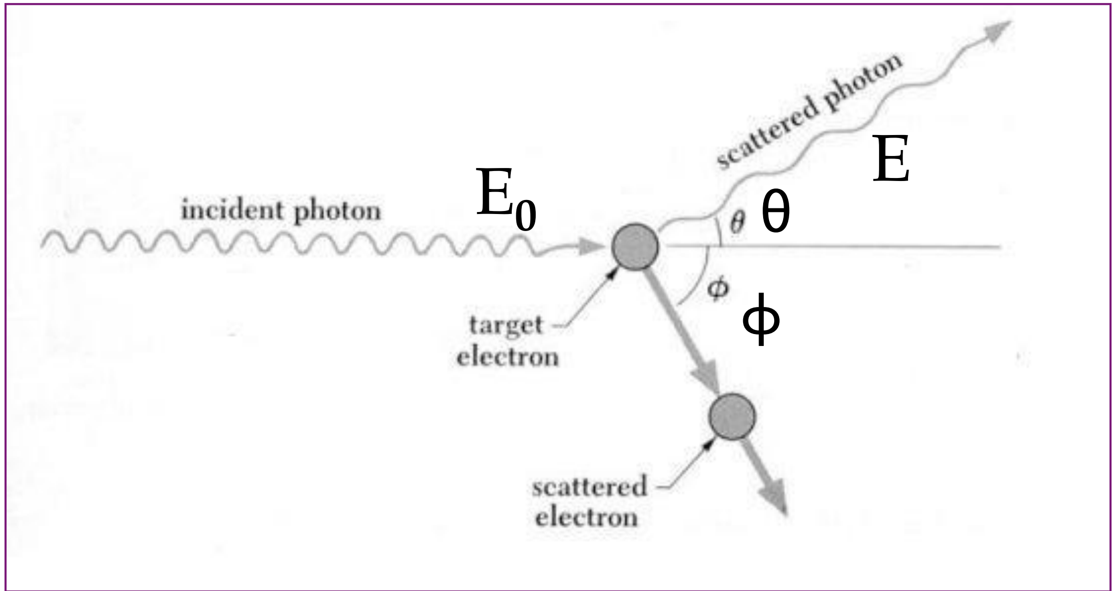
$$= 6.65 \times 10^{-25} cm^2$$

Compton scattering

Compton scattering treats the collision between an assumed stationary electron and a photon, which provides energy to the electron. The photon moves to longer wavelengths because it loses energy via the interaction.

$$E_{ph} \approx m_e c^2$$

Condition for Compton scattering



Wavelength shift
(energy/momentum conservation)

$$\lambda - \lambda_0 = \frac{h}{m_e c}(1 - \cos\vartheta)$$

Compton wavelength=
=2.43×10^{-14} cm

Fractional shift $(\lambda-\lambda_0)/\lambda$ substantial only at short incoming wavelengths

Energy conservation (where the electron rest energy is $m_e c^2$, and its recoil energy is $\gamma m_e c^2$)

$$h\nu_0 + m_e c^2 = h\nu + \gamma m_e c^2$$

Longitudinal momentum conservation (where the momentum of the electron is $p=\gamma m_e v=\gamma\beta m_e c$)

$$\longrightarrow \quad \frac{h\upsilon_0}{c} = \frac{h\upsilon}{c}\cos\vartheta + \gamma\beta m_e c\cos\Phi$$

Transverse momentum conservation

$$\uparrow \quad 0 = \frac{h\upsilon}{c}\sin\vartheta - \gamma\beta m_e c\sin\Phi$$

In the electron rest frame, the photon changes its energy as:

$$E = \frac{E_0}{1 + \frac{E_0}{m_e c^2}(1 - \cos\vartheta)}$$

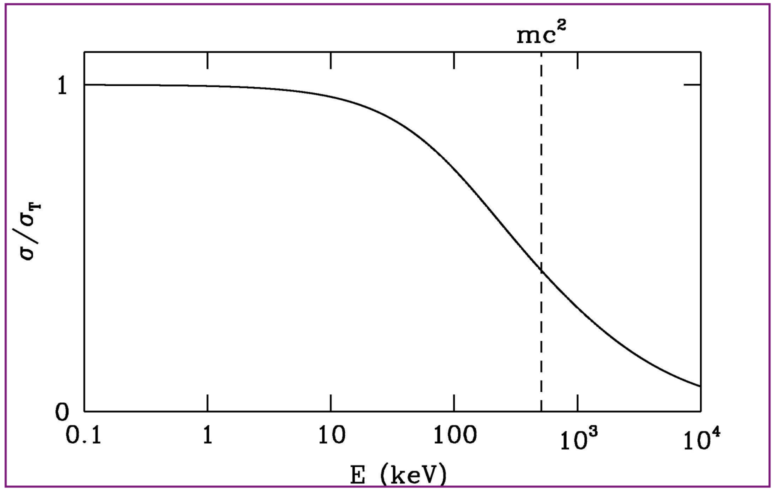
← before scattering

← larger E_0 (comparable to $m_e c^2$), larger scattering

The cross section is the Klein-Nishina

$$\sigma_{KN} = \sigma_T \frac{3}{4}\left[\frac{1+x}{x^3}\left\{\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right\} + \frac{\ln(1+2x)}{2x} - \frac{1+3x}{(1+2x)^2}\right]$$

$$x \equiv \frac{E}{mc^2}$$



$$\sigma_{KN} \to \sigma_T \quad for \quad x \to 0$$

$$\sigma_{KN} \approx \frac{3}{8}\sigma_T \frac{1}{x}(\ln 2x + 1/2) \quad for \quad x >> 1$$

$$\Rightarrow \sigma_{KN} \propto {}^1\!/_E$$

Inverse Compton scattering

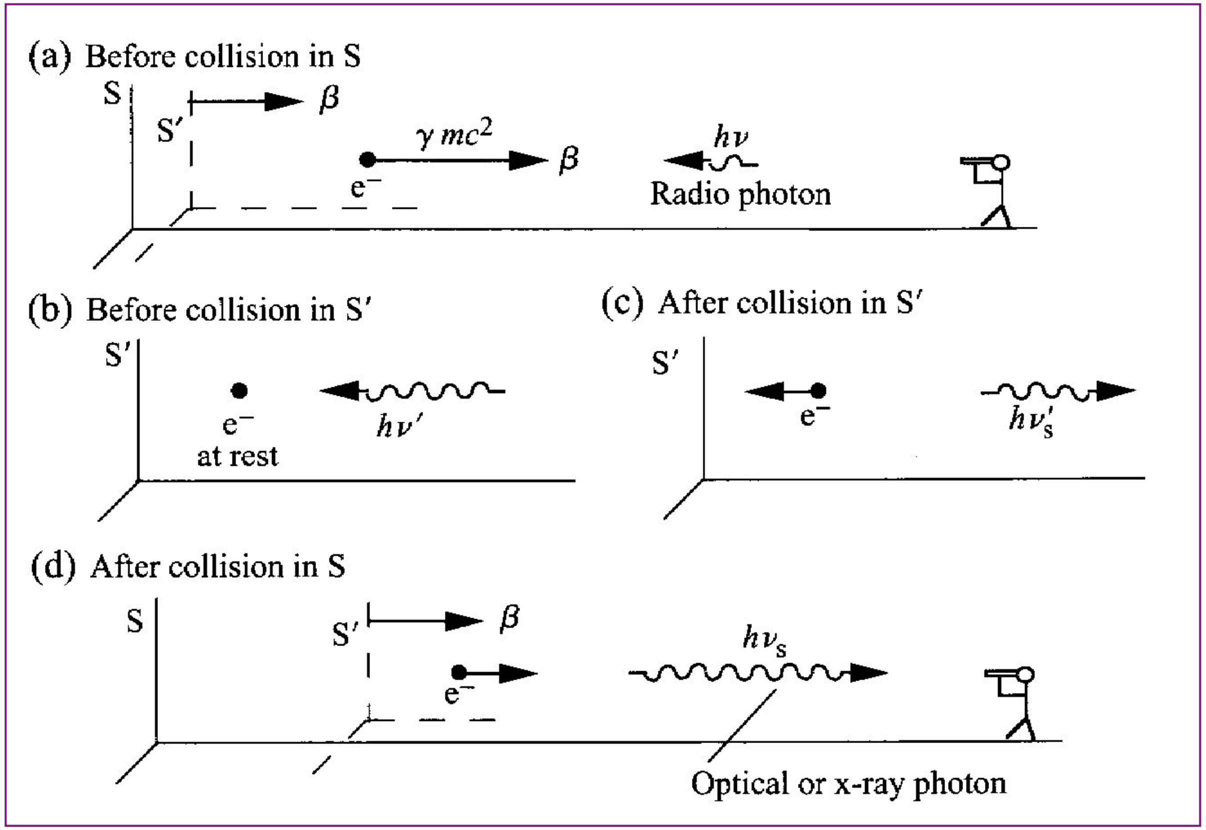
Inverse Compton scattering: a high-energy electron inelastically scatters off a lower energy photon, the result being a loss of kinetic energy for the electron and a gain of energy for the photon (moving to higher frequencies) ➔ *photon upscattering* (mechanism at work in e.g. binaries and AGN)

$$E_{ph} << \gamma m_e c^2$$

Condition for Inverse Compton scattering

Electron rest-frame (S') – moves with the electron
➔ Energy of the scattered photon determined

Transformation back to the lab rest frame (S)
➔ Final energy of the scattered photon



S
before collision

S'
Before/after collision

S
After collision

Electron rest frame
Head-on collision
Photon Doppler shifted

$$h\upsilon' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon$$ (b)

Electron at rest
Classic Compton scattering
in case of back-scattering
(Θ=π), energy given to e⁻

$$h\upsilon_s' = \frac{h\upsilon'}{1+\frac{2h\upsilon'}{m_e c^2}}$$ (c)

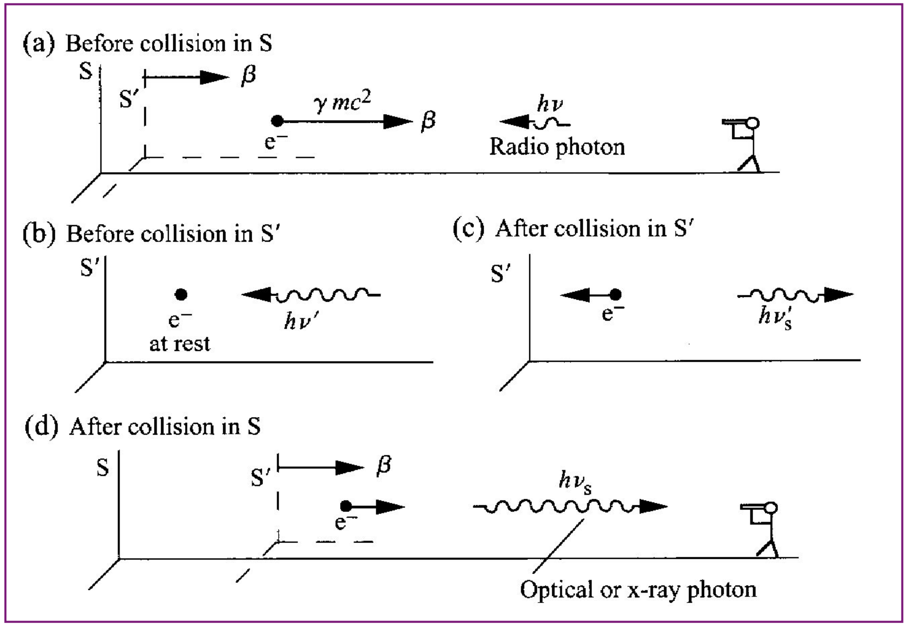
$$h\upsilon_S = \frac{h\nu}{1+\frac{h\upsilon}{m_e c^2}\left(1-\cos\vartheta\right)}$$

$$\xrightarrow{\theta=\pi} h\upsilon_S = \frac{h\nu}{1+\frac{2h\upsilon}{m_e c^2}}$$

$$\Rightarrow h\upsilon_S{}' = \frac{h\upsilon'}{1+\frac{2h\upsilon'}{m_e c^2}}$$

(c)

v_S: frequency of the scattered photon



Lab rest frame
Second Doppler shift
(the scattered photon velocity is in the direction of S')

$$h\upsilon_s = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon_s' \qquad \text{(d)}$$

Energy gain in the two Doppler shifts >> loss of energy

Highly relativistic electron
($\beta \approx 1$ or $\gamma >> 1$)

$$\left(\frac{1+\beta}{1-\beta}\right)^{1/2} \xrightarrow[\beta\approx 1]{} 2\gamma$$

Some calculations (I)

$$\left(\frac{1+\beta}{1-\beta}\right)^{1/2} = \frac{(1+\beta)^{1/2}}{(1-\beta)^{1/2}}\frac{(1+\beta)^{1/2}}{(1+\beta)^{1/2}} = \frac{1+\beta}{(1-\beta^2)^{1/2}} = \gamma(1+\beta) \xrightarrow[\beta\approx 1]{} 2\gamma$$

$$h\upsilon_S' = \frac{h\upsilon'}{1+\frac{2h\upsilon'}{m_e c^2}} = \frac{h\upsilon' m_e c^2}{m_e c^2 + 2h\upsilon'} = \frac{\left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon m_e c^2}{m_e c^2 + 2\left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon}$$

$$h\upsilon_S = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon_S' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \frac{\left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon m_e c^2}{m_e c^2 + 2\left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon} =$$

$$= \left(\frac{1+\beta}{1-\beta}\right) \frac{h\upsilon m_e c^2}{m_e c^2 + 2\left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\upsilon} \approx 4\gamma^2 \frac{h\upsilon m_e c^2}{m_e c^2 + 2h\upsilon 2\gamma} = \frac{4\gamma^2 h\upsilon m_e c^2}{m_e c^2 + 4\gamma h\upsilon}$$

(b,c,d)

Some calculations (II)

$$h\upsilon_S = \frac{4\gamma^2 h\upsilon m_e c^2}{m_e c^2 + 4\gamma h\upsilon} = 4\gamma^2 h\upsilon\left(\frac{m_e c^2}{m_e c^2 + 4\gamma h\upsilon}\right) =$$

$$= 4\gamma^2 h\upsilon\left(\frac{m_e c^2 + 4\gamma h\upsilon}{m_e c^2}\right)^{-1} = 4\gamma^2 h\upsilon\left(1 + \frac{4\gamma h\upsilon}{m_e c^2}\right)^{-1}$$

$$h\upsilon_S = 4\gamma^2 h\upsilon\left(1+\frac{4\gamma h\upsilon}{m_e c^2}\right)^{-1}$$

Single IC back-scatter ($\gamma>>1$)

$4\gamma h\nu << m_e c^2$ often satisfied

$$h\upsilon_S \approx 4\gamma^2 h\upsilon = 4\left(\frac{U}{m_e c^2}\right)^2 h\nu$$

Single head-on scatter ($\gamma>>1$ and $4\gamma h\nu << m_e c^2$)

U=total initial (rest+kinetic) energy of the electron=$\gamma m_e c^2$

The final energy of the photon is increased by a factor γ^2
(Doppler shift transformations)

Average over all directions: in a real source, the electron will collide with many photons with various directions and impact parameters

$$h\upsilon_{S,iso} = \frac{4}{3}\gamma^2 h\upsilon$$

Scattered photon average energy
(γ>>1 and $4\gamma h\nu << m_e c^2$)

Example: *Crab Nebula*

$\gamma^2 \approx 4\times10^8$

milli-meter photon of ν=300 GHz

➔ $\nu_S \approx 1.6\times10^{20}$ Hz (γ-ray band)

Electron up-scatters (boosts) the photon

Rate of electron energy loss
Single electron and many photons

Observer's frame (approximate case)

$$-\left(\frac{dE}{dt}\right)_{IC} \approx \sigma_T n_{ph} c \left(h\upsilon_{S,iso}\right)$$

Number of interactions per second

Energy of the photon (averaged over all angles)

$$\Rightarrow \quad -\left(\frac{dE}{dt}\right)_{IC} \approx \frac{4}{3}\gamma^2 h\upsilon_{av}\sigma_T n_{ph} c$$

σ_T fine until E_{ph} in the electron rest frame S' << $m_e c^2$: $2\gamma h\nu << m_e c^2$

$h\upsilon_{av}$ to consider that photons have a distribution in energy

Energy density of the photons

$$U_{ph} = n_{ph} h\upsilon_{av}$$

$$-\left(\frac{dE}{dt}\right)_{IC} \approx \frac{4}{3}\sigma_T c\gamma^2 \beta^2 U_{ph}$$

Rate of energy loss by electron due to IC [W]

($\gamma h\nu \ll m_e c^2$; $h\nu_{av} \ll h\nu_{s,iso}$)

Loss of energy similar to that of the synchrotron emission

to obtain the 'correct' expression

Assuming a thermal M-B distribution for the electrons ($E_{kin}=\langle mv^2\rangle/2=3kT_e/2$), the mean percentage energy gain of the photons is

$$\frac{\Delta E}{E_0} = \frac{4kT - E_0}{mc^2}$$

$4kT > E_0 \rightarrow$ *Energy transferred from electrons to photons*

$4kT < E_0 \rightarrow$ *Energy transferred from photons to electrons*

Inverse Compton scattering: volume emissivity

Population of relativistic electrons, each of energy $\gamma m_e c^2$, with $\gamma >> 1$, in a sea of photons with energy density U_{ph}, and photon energies negligible compared with the IC upscattered energies

$$j_{IC} = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 n_e U_{ph}$$

Integrated volume emissivity [W/m³]

($\gamma h\nu << m_e c^2$, $h\nu_{av} << h\nu_{S,iso}$ → significant energy increase)

$$N(E) = N_0 E^{-\delta} \longrightarrow j_{IC}(\nu) \propto \nu^{-(\delta - 1)/2} = \nu^{-\alpha}$$

IC cooling time

The formula for the energy losses by a single electron is identical to the synchrotron one, once U_{ph} replaces U_B.
Therefore $P_{IC}/P_{syn}=U_{ph}/U_B$

IC losses dominate when the energy density of the radiation field is larger than that of the magnetic field ($U_{ph}>U_B=B^2/8\pi$) – "Compton catastrophe"
$T>10^{12}$ K: the Compton luminosity dominates over the synchrotron luminosity: the source irradiates via Inverse Compton emission and t_{cool} is very short

Cooling time

$$t_{cool} \approx \frac{(\gamma-1)mc^2}{\frac{4}{3}\sigma_T c U_{ph} \gamma^2 \beta^2 n_e}$$

$U_{rad}=U_{sync}+U_{ph}$, which may be very short for relativistic electrons in a strong radiation field

Comptonization

A population of photons encountering a region of free electrons will find its spectrum modified as a result of IC scatters.
If the electrons are more energetic, the photons will, on average, be scattered to higher energies.
If electrons are less energetic, photons will be down-scattered.
The modification of the photon spectrum by Compton scatters is called **Comptonization**.
Electrons may not be relativistic.

Let us define the *Comptonization parameter* as: $y = \Delta E/E_0 \times N_{scatt}$

Assuming non relativistic electrons, the mean energy gain of the photons is $E = E_0 e^y$

To derive the spectral shape, one has to solve the diffusion equation, also known as the *Kompaneets equation*

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x} x^4 (n + n^2 + \frac{\partial n}{\partial x})$$

In general , it should be solved numerically.

In case of *unsaturated Comptonization (i.e., not very optically thick matter):*

$$n = I(E)\frac{(hc)^2}{8\pi E^3}$$ Photon occupation number

$$x = E / kT_e$$ Photon energy

$$y = \frac{4kT_e}{m_e c^2}\sigma_T N_e ct$$ **Kompaneets parameter**

$\partial n / \partial x$ Doppler motion

n Recoil effect

n^2 Induced/stimulated emission

$$I_\nu \propto \nu^{3+m} e^{-\frac{h\nu}{kT}}$$

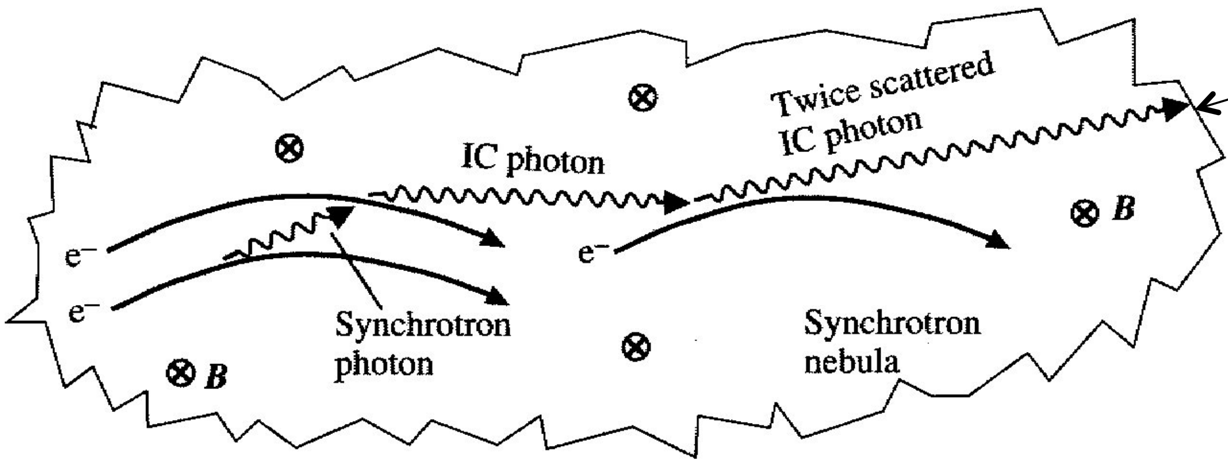
$$m = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{4}{y}}$$

$$(+ \; for \quad y >> 1 \quad ; \quad - \; for \quad y << 1$$

$$m \approx 1.5 \quad for \quad y \approx 1$$

Synchrotron Self-Compton (SSC)

SSC: relativistic electrons in a magnetic field produce synchrotron radiation. If the radiation energy density of synchrotron photons is intense, the photons will undergo IC scattering by the same electrons that emitted them in the first place.

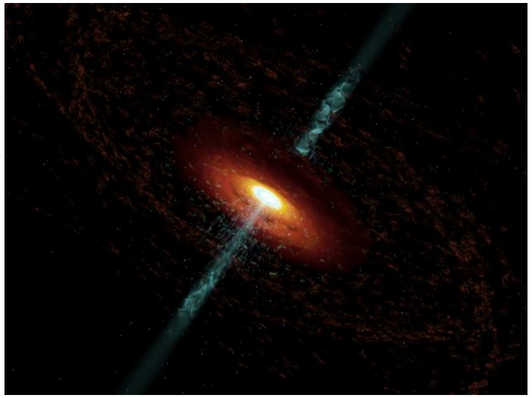


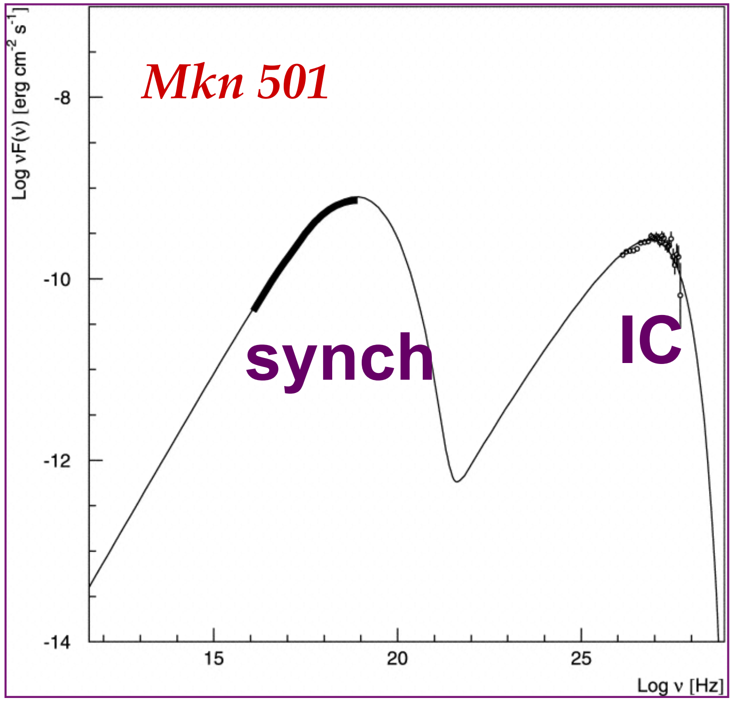
Twice scattered IC photons: the photon is boosted by a second γ^2 factor

$T_{br}=10^{12}$ K is the maximum allowed T for a compact source emitting via incoherent synchrotron

Limit to the maximum T of e^- in the nebula emitting via synchrotron ($T_{br}=10^{12}$ K) ➔ *Compton limit (catastrophe)* ➔ Inverse Compton dominates the radiative processes, the source emits most of the emission in X-rays, and the average life of electrons is very short ➔ fast cooling

$$\frac{(\frac{dE}{dt})_{IC}}{(\frac{dE}{dt})_{synch}} = \frac{U_{ph}}{U_B}$$



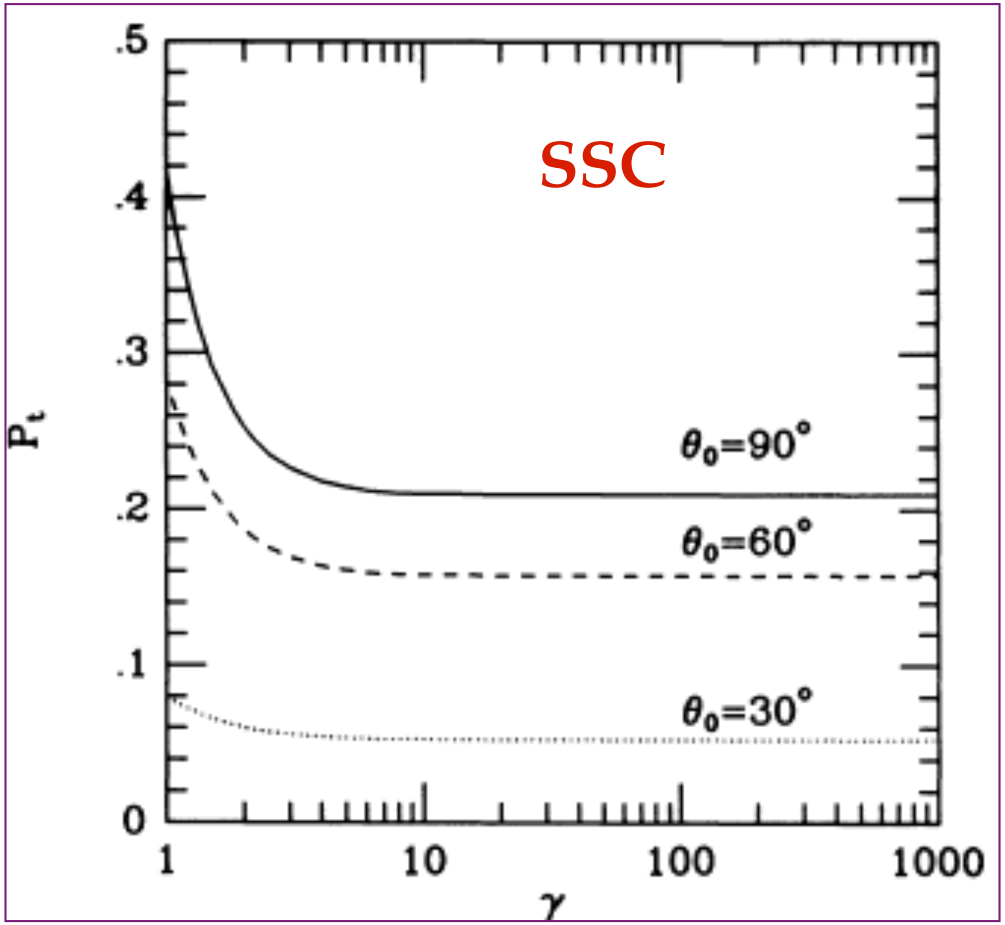


SSC emission may be relevant in *Blazars,* where two peaks are actually observed. The first peak is due to Synchrotron, the second to IC (either SSC or external IC, i.e., the "nature" of the photons can be varied)

Compton scattering: polarization

Compton scattering radiation is polarized (*but less than Thomson scattering. Polarization degree decreases with hν/mc^2 in the reference frame of the electron*).

The degree of polarization depends on the geometry of the system.

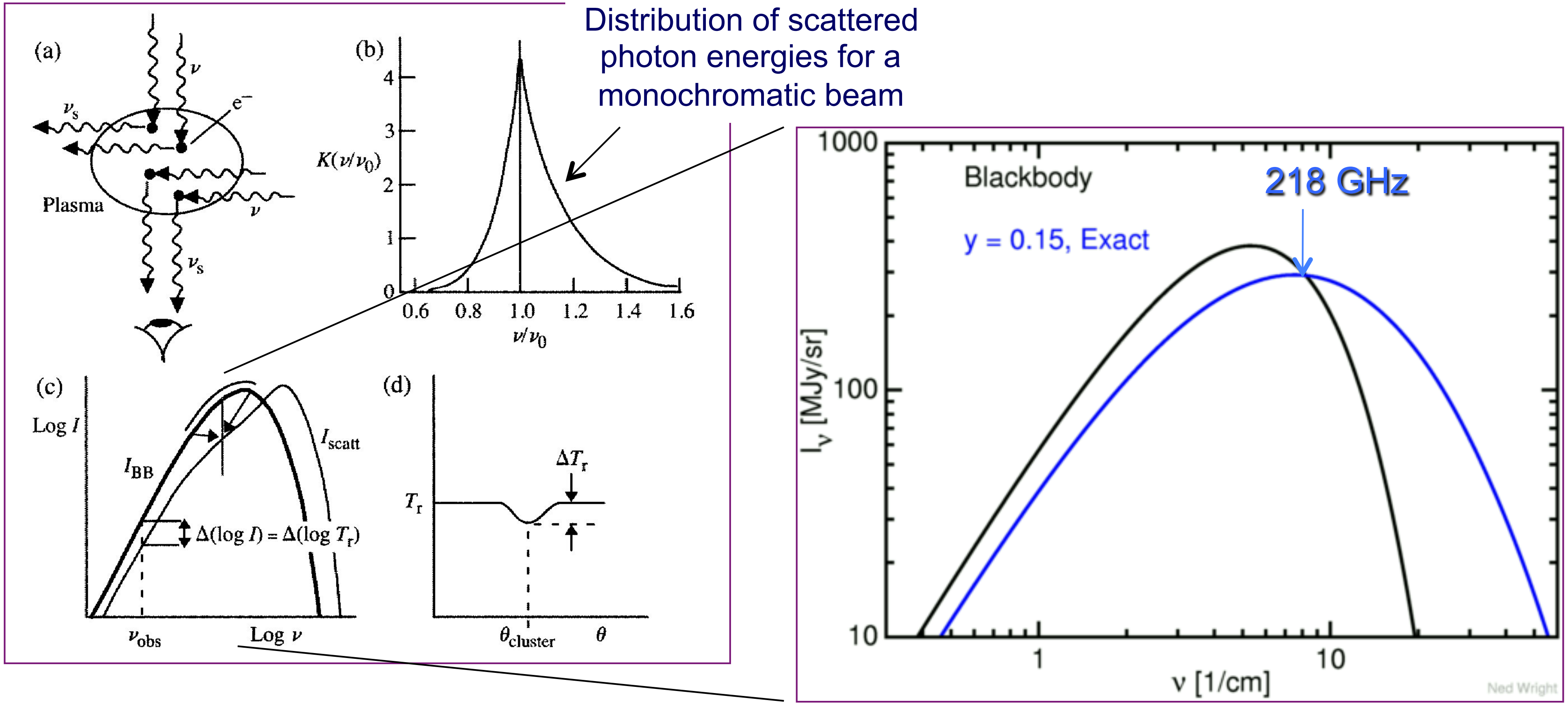


In *Blazars*, the radiation field may be either the synchrotron emission (SSC) or the thermal emission from the accretion disc (external IC).

The polarization properties are different in the two cases: while in the SSC the pol. angle of IC and Synch. are the same, in the external IC the two are no longer directly related.

Sunyaev-Zeldovich effect

SZ effect: distortion of the black body spectrum of the CMB owing to IC interaction of the low-energy CMB photons with the energetic electrons in the plasma of a cluster of galaxies



CMB photons are Comptonized by the IGM in Clusters of Galaxies. As a result, the CMB spectrum in the direction of a cluster is shifted by

$\Delta I_\nu / I_\nu \approx -2y$

(R-J regime)

$$\tau \approx \sigma_T \langle n_e \rangle R \approx 10^{-2} - 10^{-3}$$

$$\frac{\Delta \nu}{\nu} \approx \frac{4kT}{mc^2} \approx 5 \times 10^{-2}$$

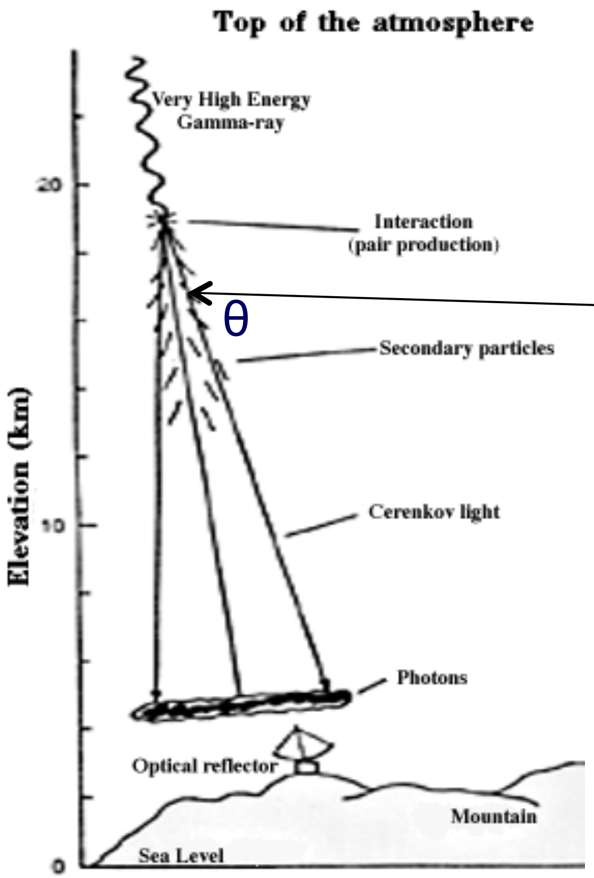
$$y = \int n_e \sigma_T dz \left(\frac{kTe}{m_e c^2} \right) \approx 10^{-3} - 10^{-4}$$

y=Comptonization parameter
measures the strength of Compton scattering

The S-Z effect is potentially a very efficient tool to search for clusters and, when combined with X-ray observations, can be used to estimate the baryonic mass fraction and even the Hubble constant (see lessons on X-ray clusters of galaxies)

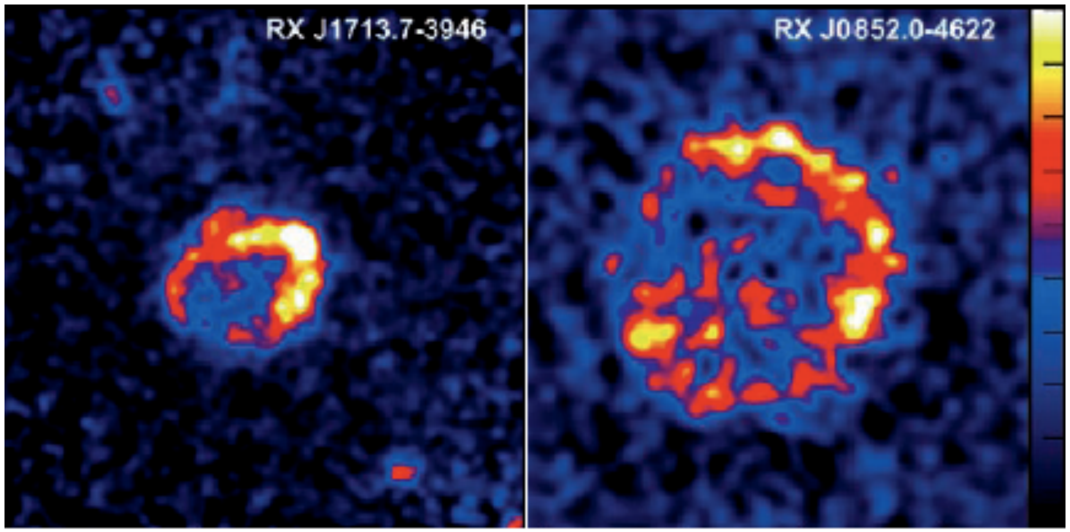
Cherenkov radiation. I

It occurs when a *charged particle passes through a medium at a speed larger than the speed of light in that medium*

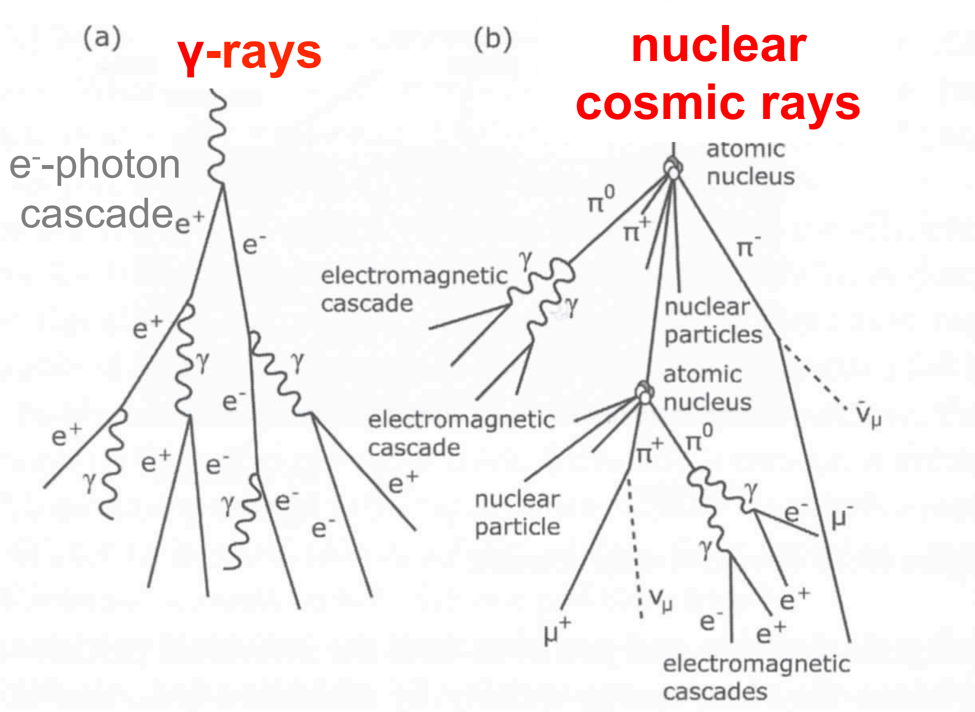


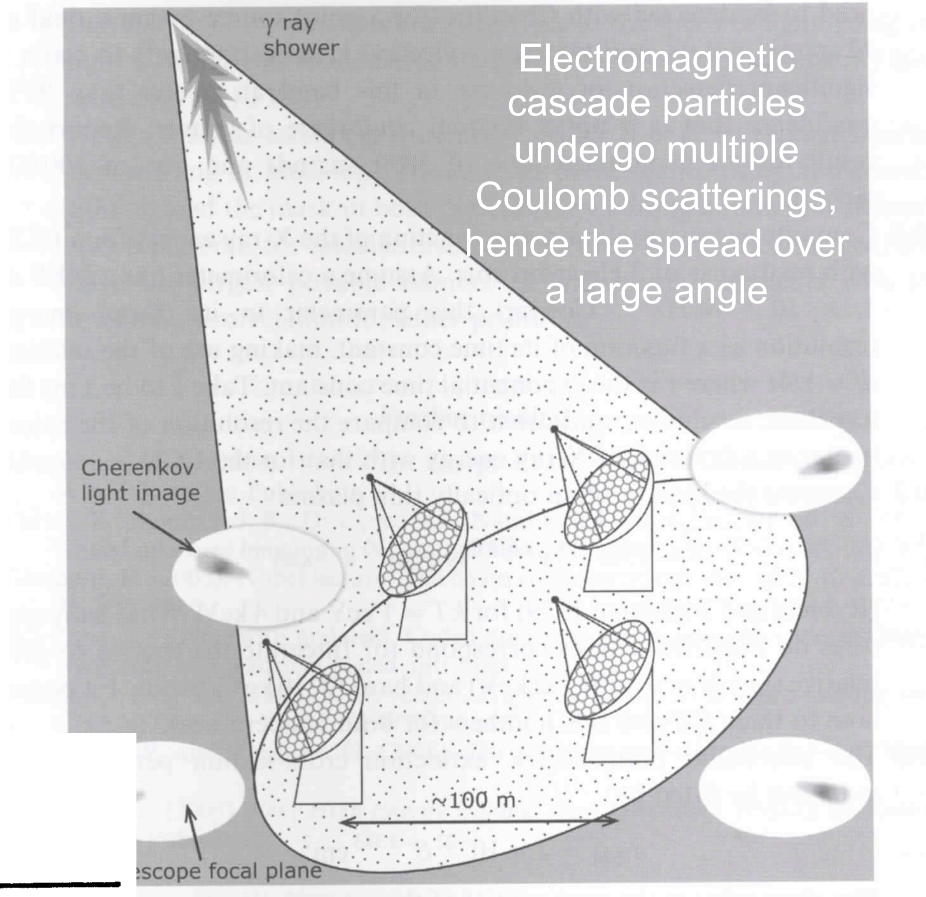
γ-ray ➔ *pair production* and secondary particles (*air shower*) ➔ faint *short-lasting* (3−5 ns) beam of blue light (Cherenkov radiation) detected by large telescopes on the ground
$\cos\theta = c/(v \times n)$ [v=velocity of e.g. the electron, n=refraction index in the atmosphere]

It is used to detect *high-energy (~TeV) γ-rays* (e.g., *Veritas, HESS* and *Magic telescopes*)

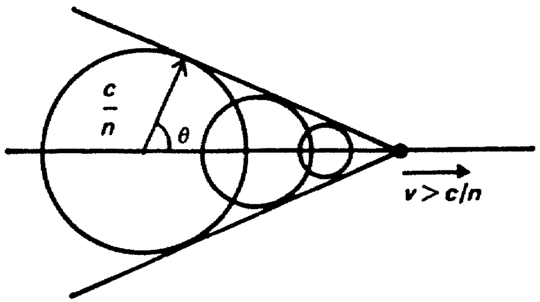


Cherenkov radiation. II





$$\vartheta = \cos^{-1}\left(\frac{1}{n\beta}\right),\ \beta = v/c$$



Huygens' construction for the wavefront of coherent radiation of a charged particle moving at constant v>c/n (n=refrective index)

n=refraction index of the air (>1); $v_{light(atmosphere)}$=c/n

$v_{light(atmosphere)}=v_{phase,light}<v_{relativistic\ particle}$: Cherenkov light

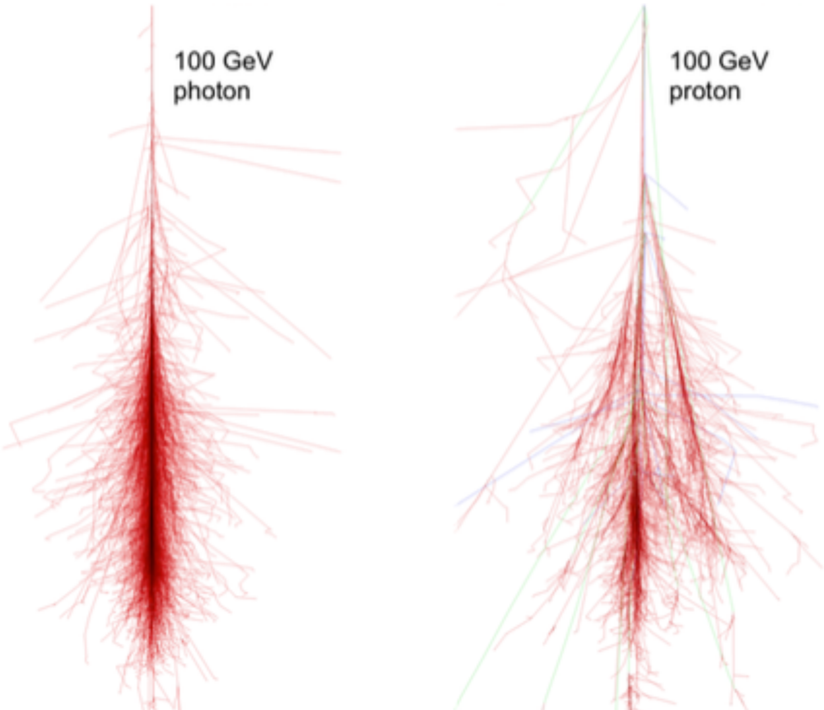
θ≈1 deg; shower maximum at ≈9 km for TeV photons (≈200 m diameter)

Nuclear primary at ≈7 km and penetrate closer to the ground

Cherenkov radiation. III

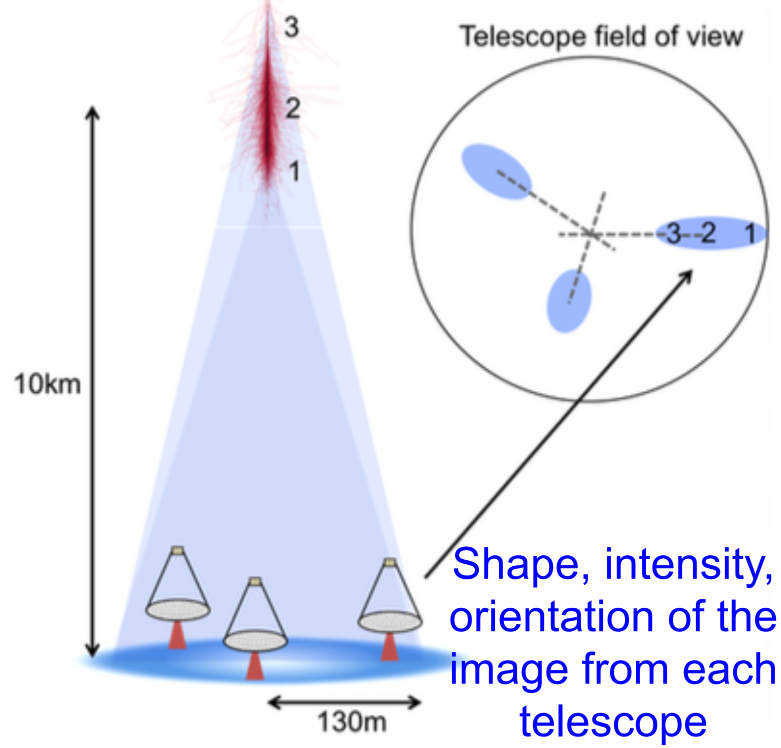
γ-ray photon (E_0) ➔ Electron-positron pair ➔ Bremsstrahlung + pair production interactions lead to the generation of an electromagnetic cascade Below E_C=84 MeV ionization losses dominate ➔ mazimum number of particles in a cascade is given by E_0/E_C

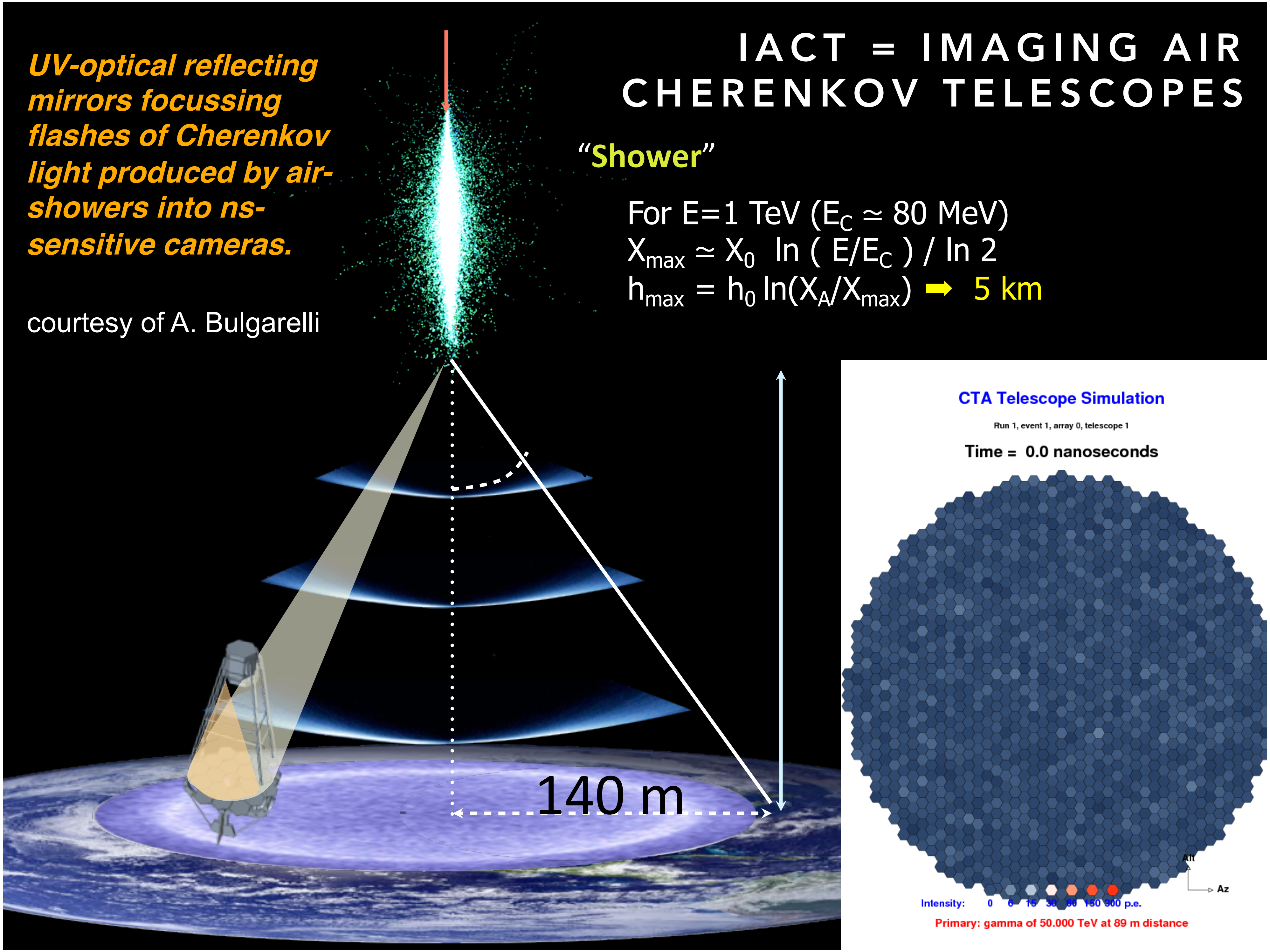
Cosmic rays – charged, relativistic protons and nuclei – produce more complex cascade developments in the atmosphere

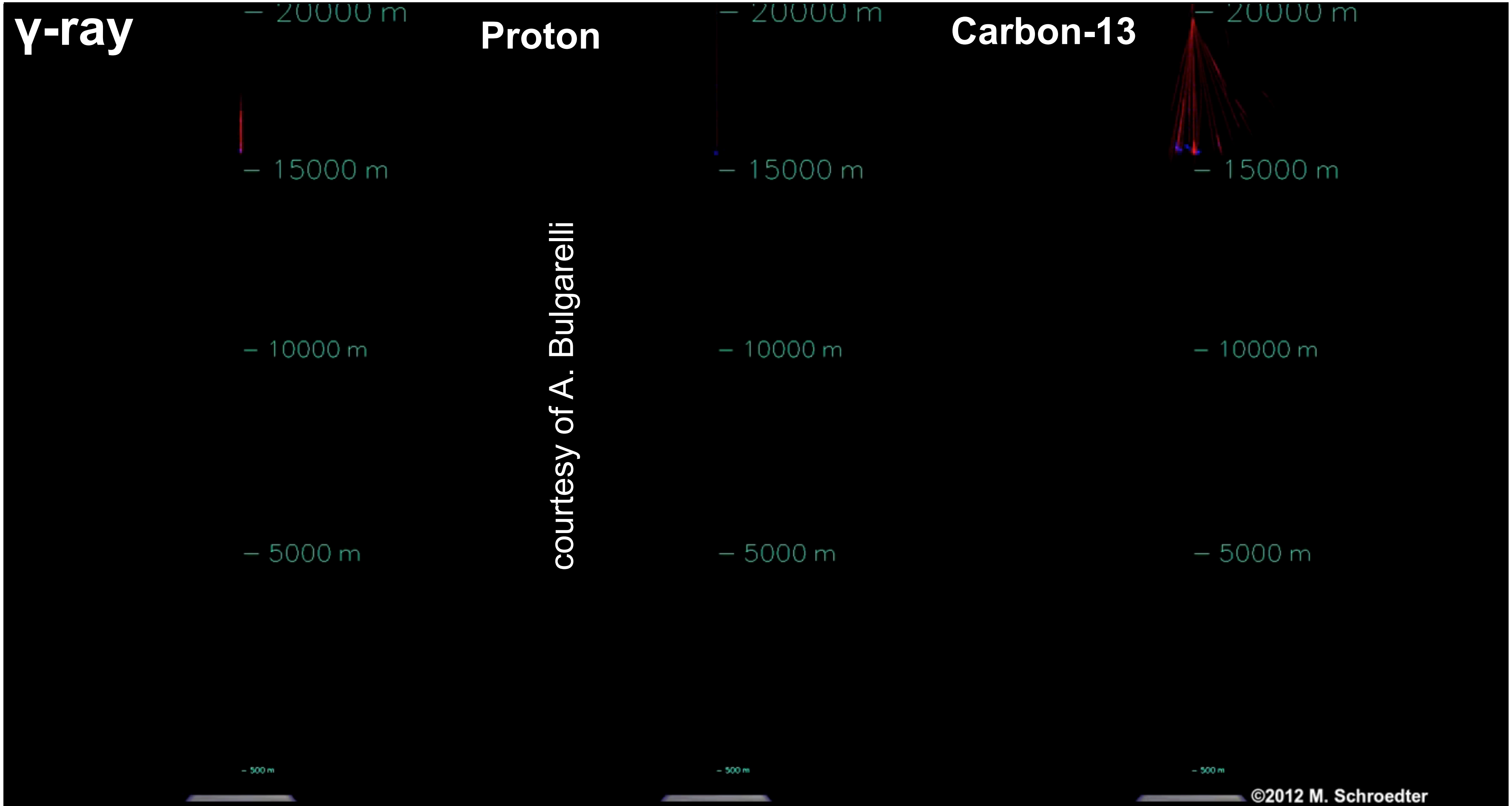


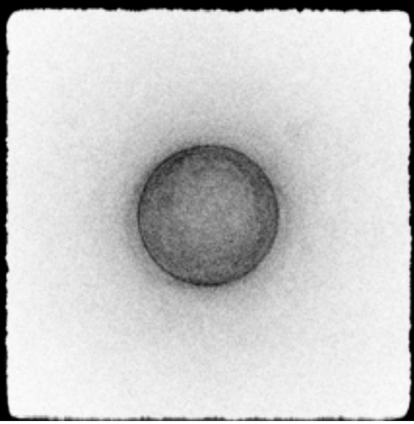
Holder (2015) review

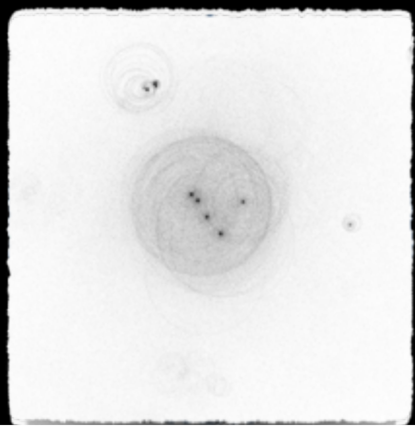
Stereoscopic imaging technique




IACT = IMAGING AIR CHERENKOV TELESCOPES
UV-optical reflecting mirrors focussing flashes of Cherenkov light produced by air-showers into ns-sensitive cameras.
courtesy of A. Bulgarelli
"Shower"
For E=1 TeV ($E_C \simeq$ 80 MeV)
$X_{max} \simeq X_0 \ \ln (E/E_C) / \ln 2$
$h_{max} = h_0 \ln(X_A/X_{max})$ ➡ 5 km
140 m
CTA Telescope Simulation
Run 1, event 1, array 0, telescope 1
Time = 0.0 nanoseconds
Alt
Az
Intensity: 0 6 15 30 60 150 300 p.e.
Primary: gamma of 50.000 TeV at 89 m distance







Intensity of the Image
↳ Shower Energy

Orientation of the image
↳ Shower Direction

Image Shape
↳Particle type