

Bulk of the IGM Missing Baryons:

Current Evidence and Future
Prospects

Overview

- The Warm-Hot Intergalactic Medium in X-rays
- Detectability with Current Instrumentation
- Current Evidence: tentative Ω_b^{OVII} and $(dN/dz)^{\text{OVII}}$ estimates
- The Best Observational Strategy: Gaseous Signposts, FUV, X-Ray and Radio bright sources
- The Pilot Chandra Observation of “The Best WHIM Target in the sky”
- The “Missing Baryons” around the Milky Way: Local-Group WHIM, Extended Galactic Halo, or WHIM in local LSS
- The Future for the X-ray WHIM



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Summary and Baryon Budget at $z < 2$

Baryon Budget at $z < 2$

$$\Omega_b^{\text{WMAP}h^{-2}} = 0.0226 h^{-2} = 0.0456$$

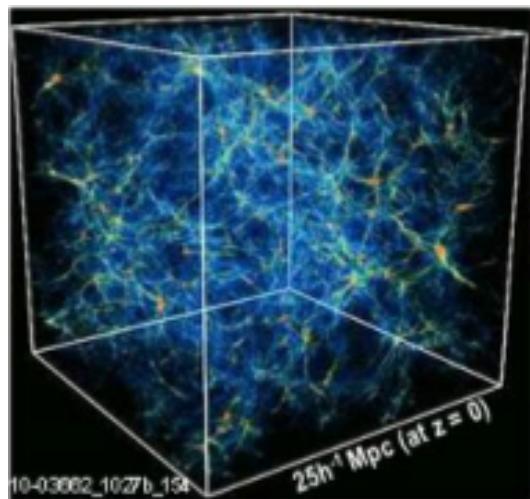
$$\begin{aligned}\Omega_b^{z < 2} &= \Omega_b^{\text{Stars}} + \Omega_b^{\text{HI+HeI+H2}} + \Omega_b^{\text{Clusters}} + \Omega_b^{\text{Ly}\alpha} + \Omega_b^{\text{OVI}} + \Omega_b^{\text{BLA}} = \\ &= 0.0287 = 64\% \Omega_b^{\text{WMAP}}\end{aligned}$$



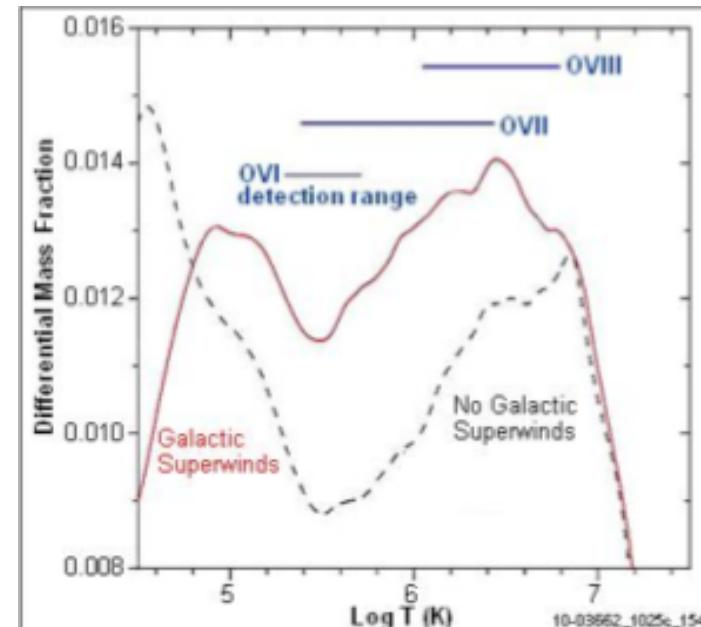
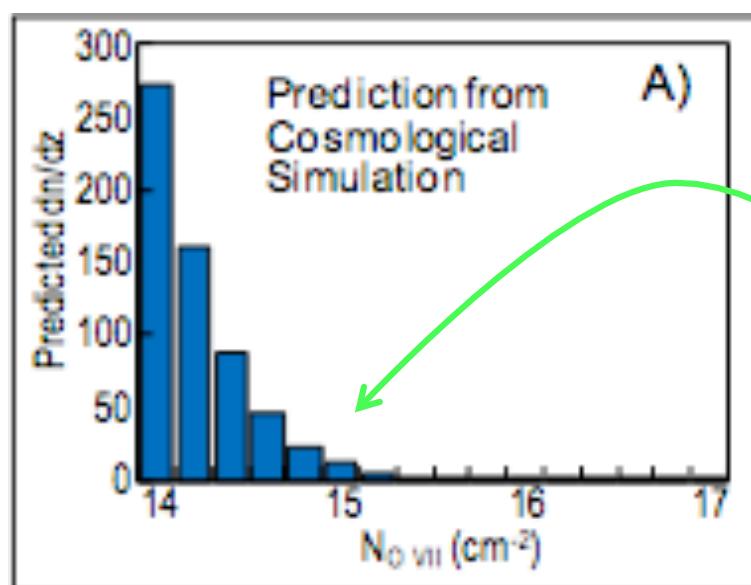
$\sim 35\%$ of Baryons Still Missing at $z \sim 0$

Searching for the Missing Baryons in the X-Ray WHIM

The WHIM in Hydro-dynamical simulations



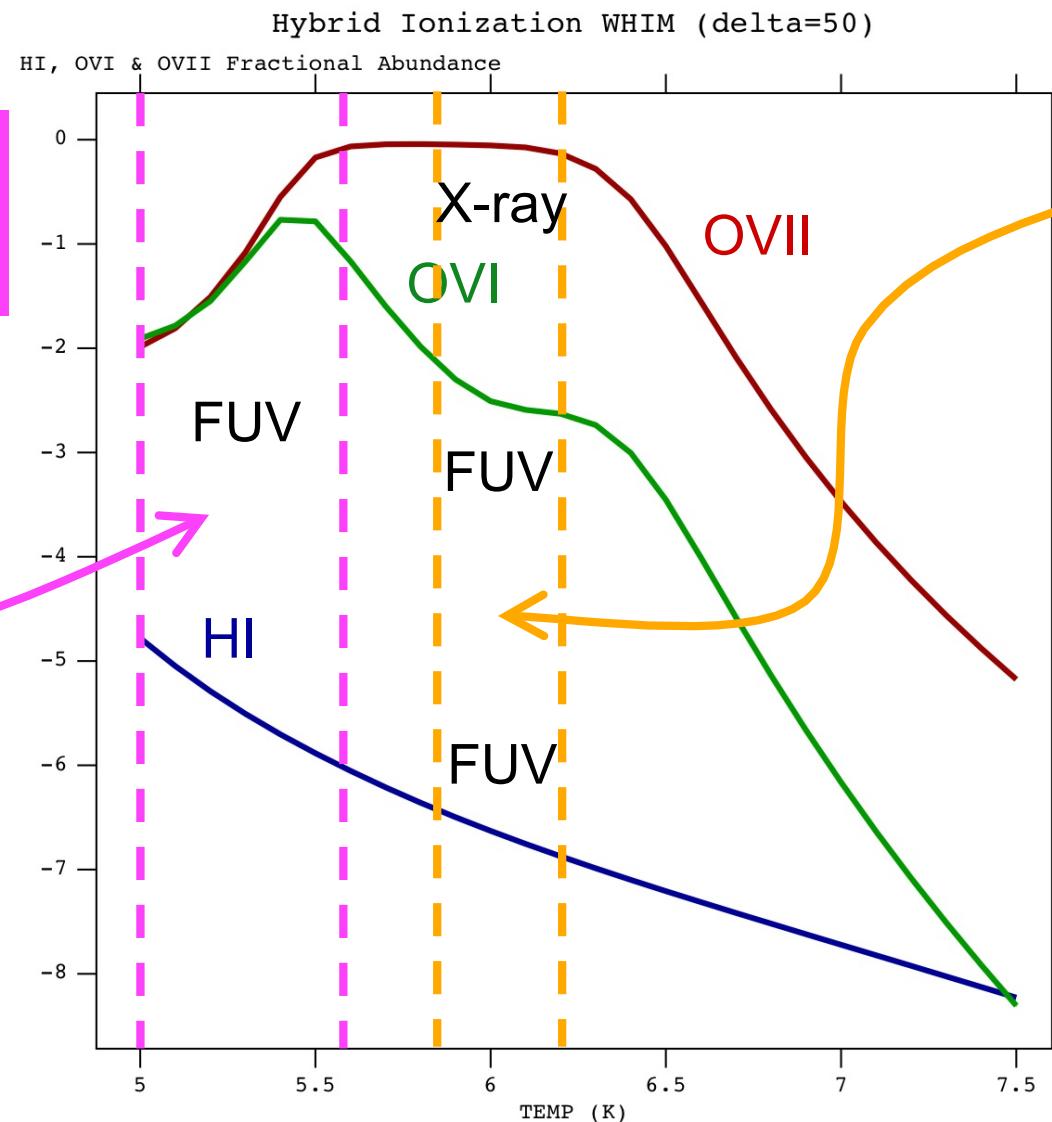
Britton+11



$$dN/dz(N_{\text{OVII}} > 10^{15} \text{ cm}^{-2}) = 10$$

Ionization Balance in the WHIM

Warm Phase:
Needs Moderate
S/N FUV



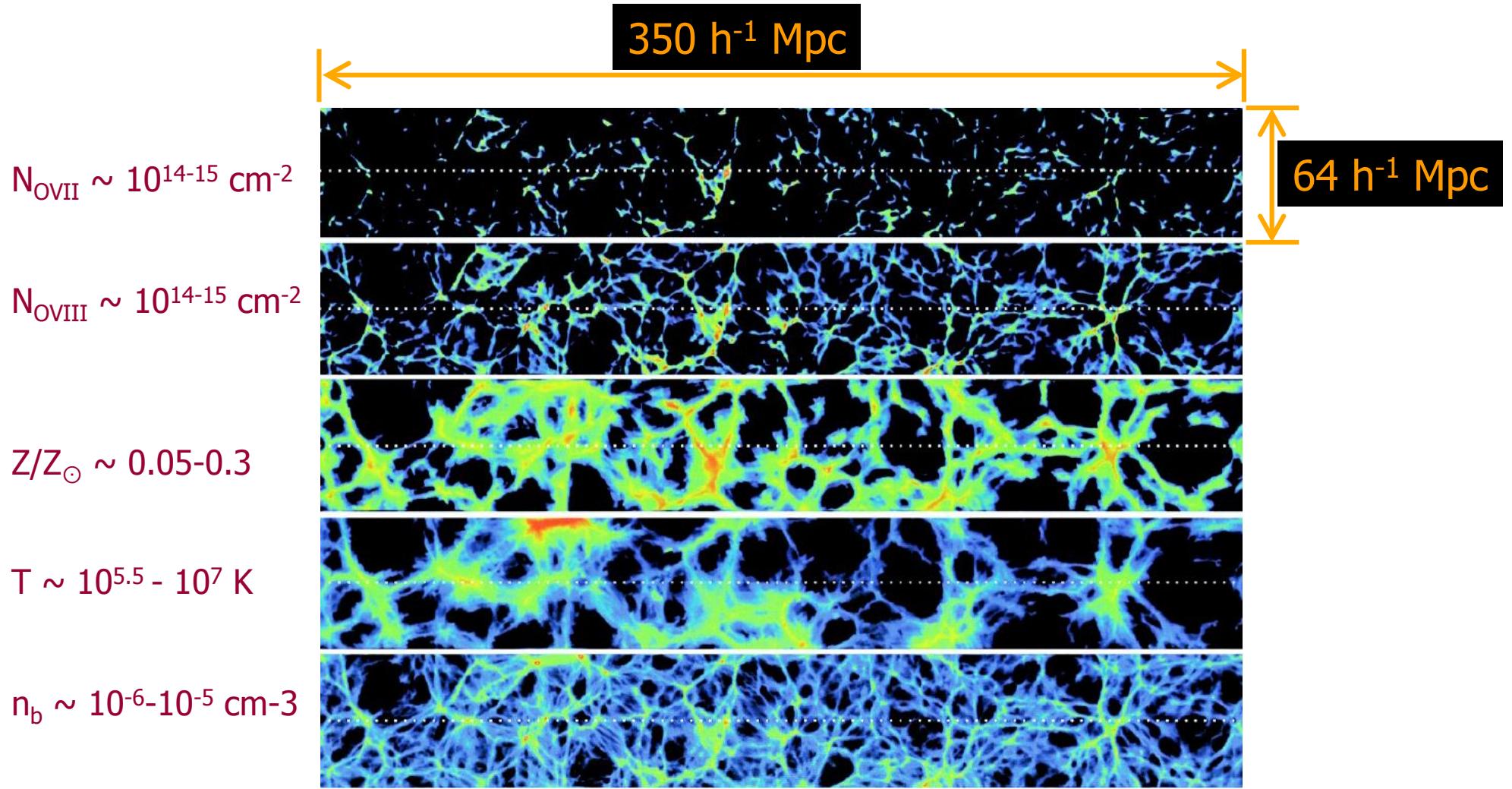
WHIM Phase:
Needs Deep
and very high S/N
FUV & X-ray

Why in X-Rays

$$f_{12} = \frac{g_2}{g_1} \frac{mc}{2\pi e^2 v^2} A_{21}$$

Ion	Transition	Wavelength (Å)	Energy (eV)	f
OI 0.118	K α	23.52	527.1	
OII	K α	23.29	532.3	0.198
OIII	K α	23.01-23.08	538.8-537.2	0.292
OIV	K α	22.76	544.7	0.409
OV 0.533	K α	22.37	554.2	
OVI	K α	22.03	562.8	0.538
OVII	K α	21.60	574.0	0.696
OVII	K β	18.63	665.5	0.146
OVIII	K α	18.97	653.6	0.416

The X-Ray Forest



(Hellsten et al., 1998)

WHIM Strength in X-Rays

Transverse Size: $\Delta R \sim 0.5 \text{ Mpc}$,

Density: $n_b \sim 10^{-5} \text{ cm}^{-3}$ ($\delta \sim 50$)

Temperature: $T \sim 10^6 \text{ K}$,

Metallicity: $Z \sim 0.1 Z_\odot$ ($A_O = 8.5 \times 10^{-4}$)

Ion Fraction: $\xi_{\text{OVI}} = 0.87$, $\xi_{\text{OVII}} = 0.13$, $\xi_{\text{OVIII}} = 0.003$,
 $\xi_{\text{CV}} = 0.27$, $\xi_{\text{CVI}} = 0.53$

$$\Rightarrow N_{\text{OVI}} \sim n_b \xi_{\text{OVI}} Z \Delta R \sim 8 \times 10^{13} \text{ cm}^{-2}$$

$$\Rightarrow N_{\text{OVII}} \sim n_b \xi_{\text{OVII}} Z \Delta R \sim 10^{15} \text{ cm}^{-2}$$

$$\Rightarrow N_{\text{OVIII}} \sim n_b \xi_{\text{OVIII}} Z \Delta R \sim 2 \times 10^{14} \text{ cm}^{-2}$$

$$\Rightarrow N_{\text{CV}} \sim n_b \xi_{\text{CV}} Z \Delta R \sim 4 \times 10^{14} \text{ cm}^{-2}$$

$$\Rightarrow N_{\text{CVI}} \sim n_b \xi_{\text{CVI}} Z \Delta R \sim 2 \times 10^{14} \text{ cm}^{-2}$$

Line Saturation

Line Saturates when: $\Delta \lambda \sim W_\lambda$

$$\Delta \lambda = \lambda (\Delta v/c) \sim \lambda (100/c) \sim 3 \times 10^{-4} \lambda$$

→ $\Delta \lambda (O) \sim 5\text{-}7 \text{ m}\text{\AA}$ (20-25 Å)

→ $\Delta \lambda (C) \sim 12\text{-}13 \text{ m}\text{\AA}$ (40-45 Å)

WHIM Strength & Detectability in X-Rays

$$EW_{X^i} \cong 8.9 \times 10^{-21} (N_H A_X \xi_{X^i}) f_{lu} \lambda^2 \text{ } \overset{o}{\text{\AA}}$$

$$\begin{aligned}W_{\text{OVI}} &\sim 0.2 (1+z) \text{ mA} \\W_{\text{OVII}} &\sim 3 (1+z) \text{ mA} \\W_{\text{OVIII}} &\sim 0.3 (1+z) \text{ mA} \\W_{\text{CV}} &\sim 2 (1+z) \text{ mA} \\W_{\text{CVI}} &\sim 2 (1+z) \text{ mA}\end{aligned}$$

Contrast:

$$20/0.003 \sim 7000 \sim 175 \times R_{\text{chandra/XMM}}$$

$$40/(0.002) \sim 20000 \sim 250 \times R_{\text{chandra}}$$

OVII/CV/CVI Need High-Contrast Spectra

For a Dispersive Spectrometer with FWHM Resolution of $\Delta \lambda^{Spect}$, and an absorption line with FWHM $\Delta \lambda^{Line}$:

$$EW_{Thresh}^{N\sigma} \geq \frac{N \Delta \lambda^{Spect}}{(S/N)_{RE}}$$

$$EW(BLA)_{Thresh}^{3\sigma} \geq \frac{3(50mA)}{(S/N)_{RE}}$$

(Chandra LETG)



$(S/N)_{RE(LETG)} \geq 50 \Rightarrow \sim 2500$ CPRE

to detect at 3σ CV/OVII with $EW=3$ mA

Extremely Demanding with Current Spectrometers

Chandra LETG $A_{\text{eff}} = 10 \text{ cm}^2$ at 20 Å (XMM: 30 cm²)

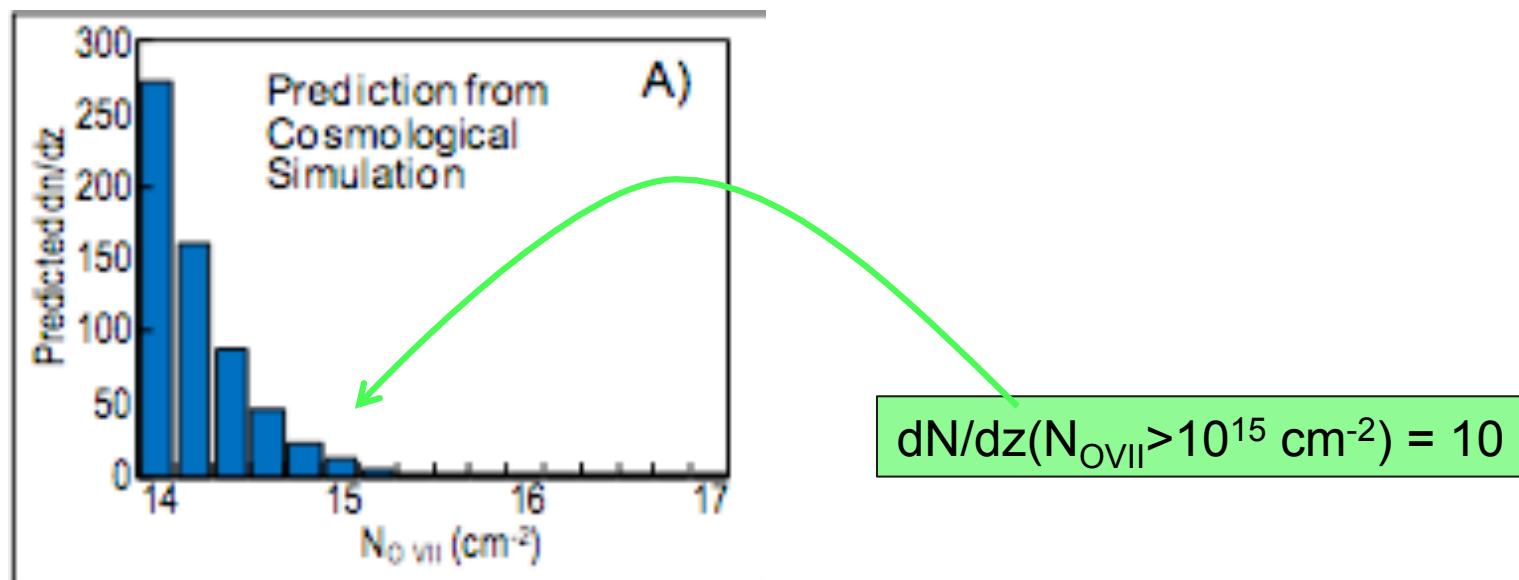
Chandra FWHM Resolution: $\Delta \lambda = 0.05 \text{ \AA}$ (XMM: 0.075 Å)

Brightest extragalactic targets $F_{20\text{\AA}} = 0.001 \text{ ph s}^{-1} \text{ cm}^{-2} \text{\AA}^{-1}$, typically at $z < 0.1$

$$(S/N)_{\text{RE}} = (F_{20\text{\AA}} \Delta \lambda A_{\text{eff}} T)^{0.5} = 50$$



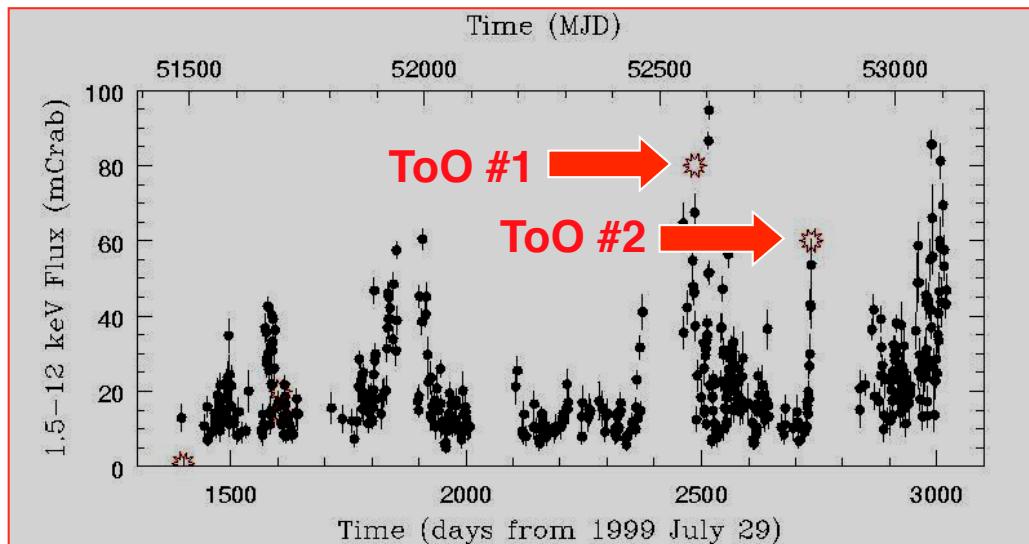
$T = 5 \text{ Ms} !!!$ (about half with XMM)



Waiting for Outbursts

Mkn 421 ($z=0.03$)

RXTE ASM (0.5-12) keV light curve



- Blazars flare to > 10 times normal
- Trigger ToO (from Rossi-XTE ASM)
- Outbursts last days to 1-2 weeks

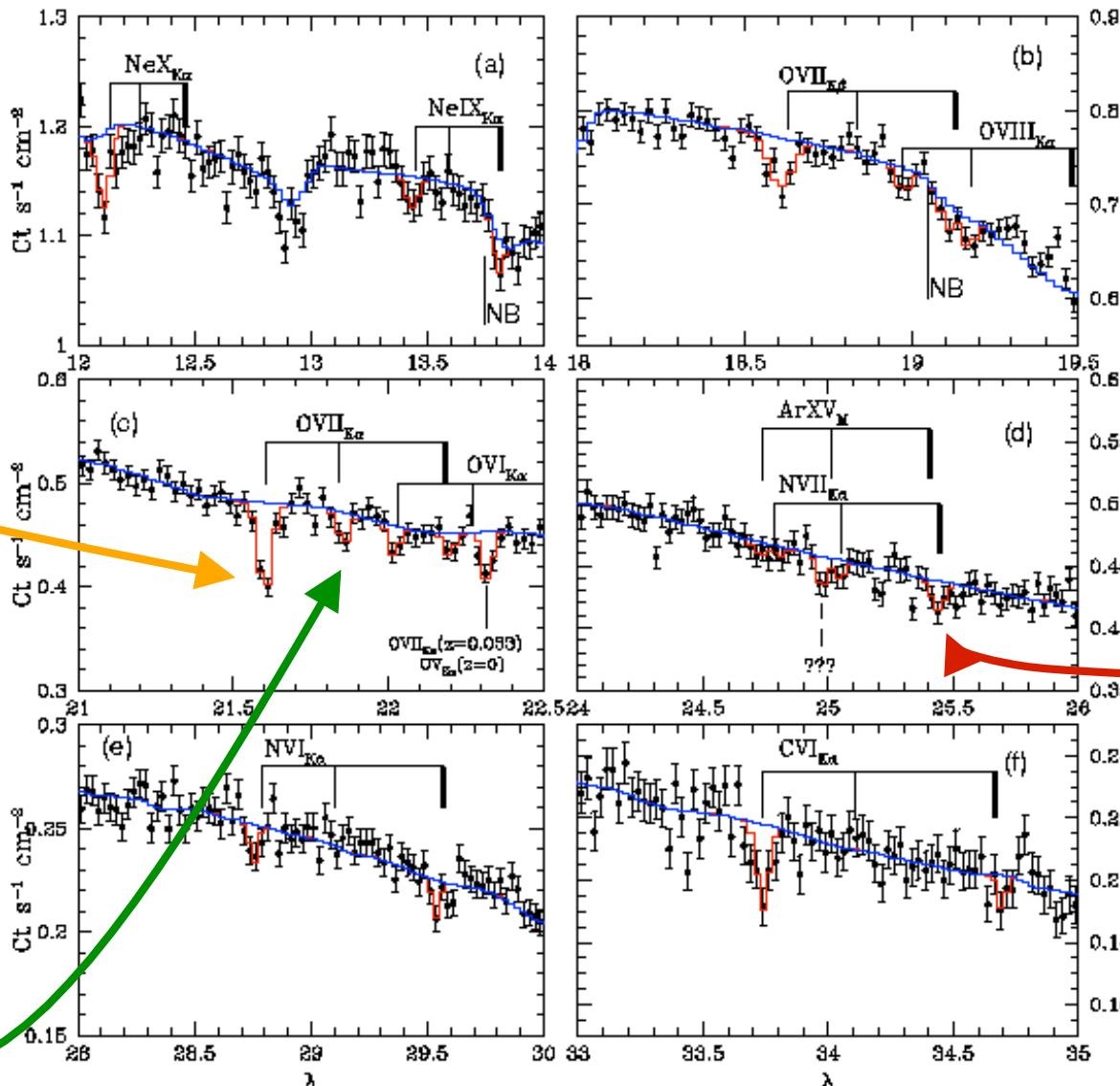
- ✓ 1st ToO **80mCrab** 2002 October 27: 100 ks ACIS-LETG
- ✓ 2nd ToO **60mCrab** 2003 June 6: 100 ks HRC-LETG

Mkn 421 in Outburst LETG-HRC Spectrum

The highest signal-to-noise
grating spectrum taken by Chandra
6000 CPRE

WHIM and ISM Absorption

$z \sim 0$ Ionized
Absorption



(Nicastro+05a; Nicastro+05b; Williams+05)

10/18/11

Bologna: PhD Courses 'Missing Baryons'

$\langle z \rangle = 0.011$
WHIM

$\langle z \rangle = 0.027$
WHIM

WHIM Observables

Absorption Line Equivalent Widths:

→ Ion Column Densities: N_{ion}

Equivalent Width Ratios:

→ Ionization Balance: T, n_b

Equivalent Widths and Saturation

- Same-Ion Line ratios: direct check
- Curve of Growth technique:

$$W_{X^i} = \int_0^\infty (1 - e^{-\tau_\nu}) d\nu \approx \int_0^\infty \tau_\nu d\nu + \frac{1}{2} \int_0^\infty \tau_\nu^2 + \dots$$

$$\tau_\nu = N_{X^i} \frac{\pi e^2}{mc} f_{lu} \Phi(\nu)$$

$$\Rightarrow W_{X^i} \approx \frac{\pi e^2}{mc} f_{lu} N_{X^i} + \frac{1}{2} \left(\frac{\pi e^2}{mc} f_{lu} N_{X^i} \right)^2 \int_0^\infty \Phi^2(\nu) d\nu + \dots$$

Ionization Balance

- Most metal absorption Lines fall in the linear branch of the Curves of Growth (CoG)

→ EW \propto Ion Column Density :

$$EW_{X^i} \cong 8.9 \times 10^{-21} (N_H A_X \xi_{X^i}) f_{lu} \lambda^2 \text{ } \text{\AA}$$



EW ratios \propto Ionization Balance:

For 2 ions from the same element

$$\frac{\xi_{X^i}}{\xi_{X^{[i+n]}}} = \frac{EW_{X^i}}{EW_{X^{[i+n]}}} \times \frac{f_{lu}(X^{[i+n]})}{f_{lu}(X^i)} \times \left(\frac{\lambda_{X^{[i+n]}}}{\lambda_{X^i}} \right)^2$$

For 2 ions from different elements

$$\frac{\xi_{X^i}}{\xi_{Y^j}} = \frac{EW_{X^i}}{EW_{Y^j}} \times \frac{f_{lu}(Y^j)}{f_{lu}(X^i)} \times \frac{A_Y}{A_X} \times \left(\frac{\lambda_{Y^j}}{\lambda_{X^i}} \right)^2$$

Mass and Metal Content of the WHIM

$$\Omega_b = \left(\frac{1}{\rho_c} \right) \left(\frac{\mu m_p \sum_i N_H^i}{d_{Tot}} \right)$$

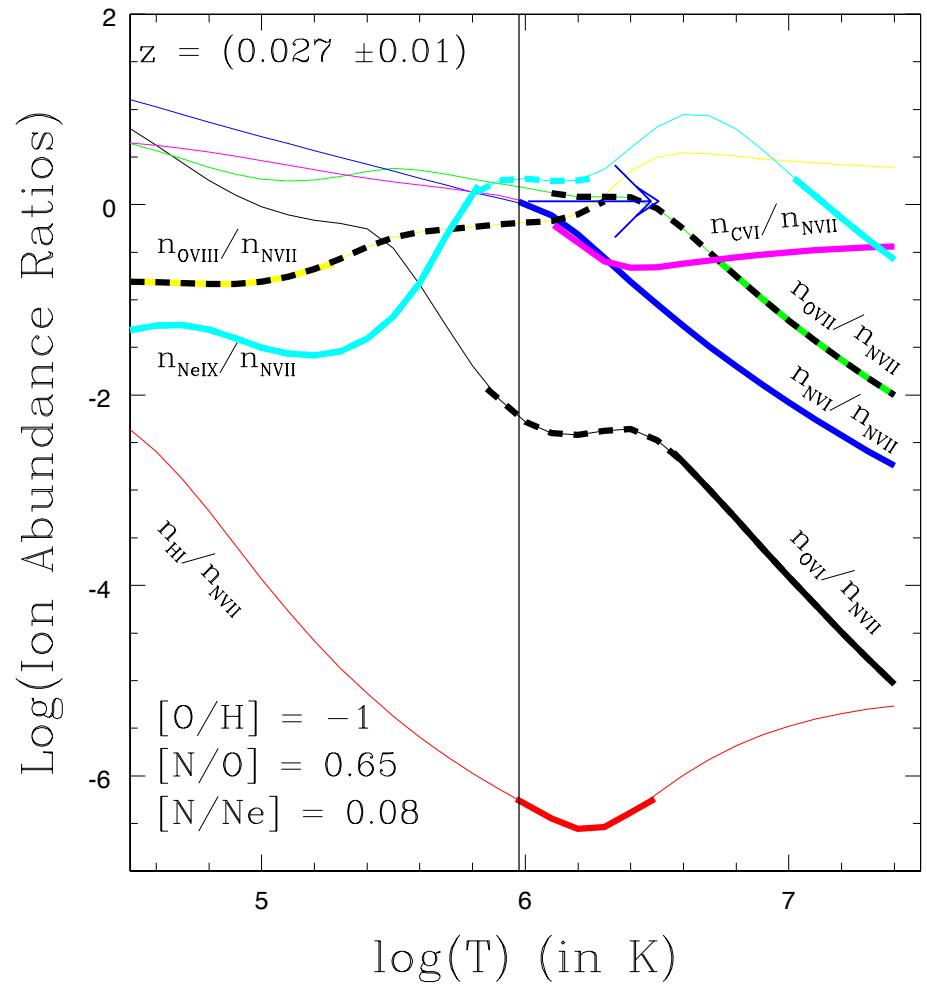
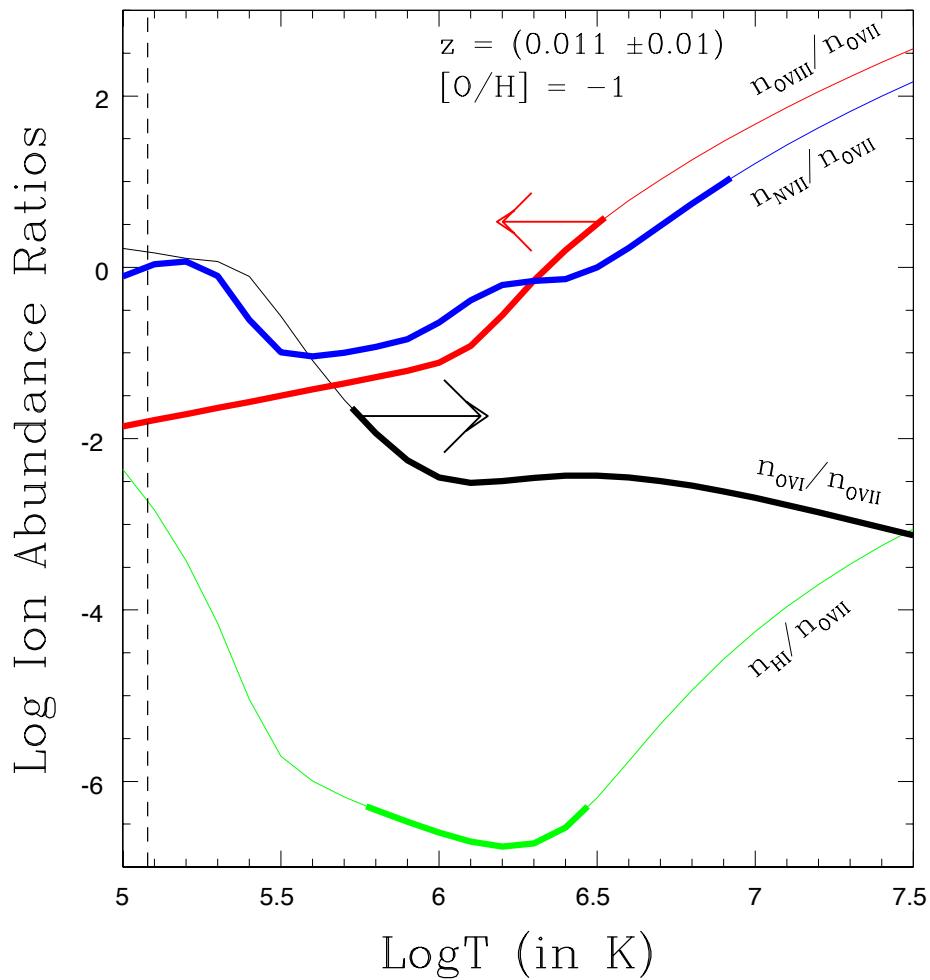
$$N_H = N_{ion} \times A_{element}^{-1} \times \xi_{ion}^{-1}$$

FUV | X

FUV & X

X

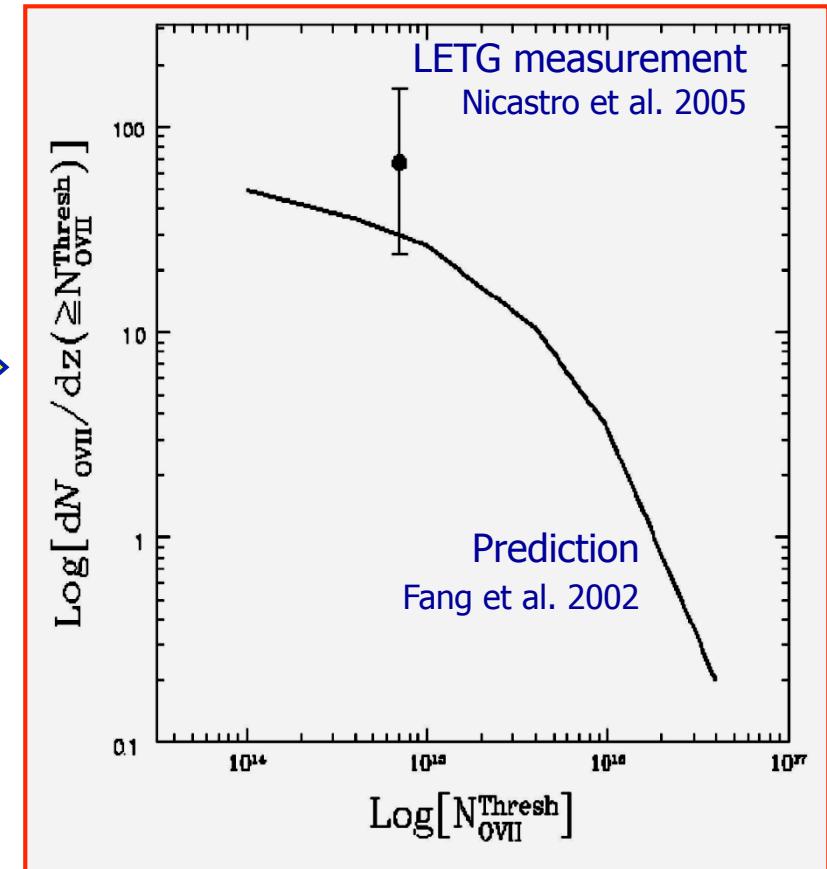
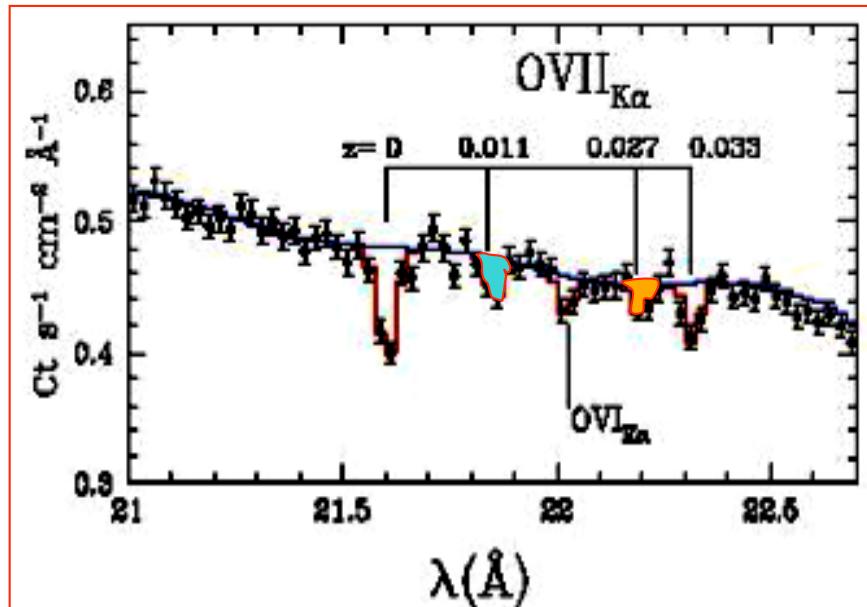
Modeling the WHIM with Hybrid-Ionization Models



X-Ray WHIM toward Mkn 421

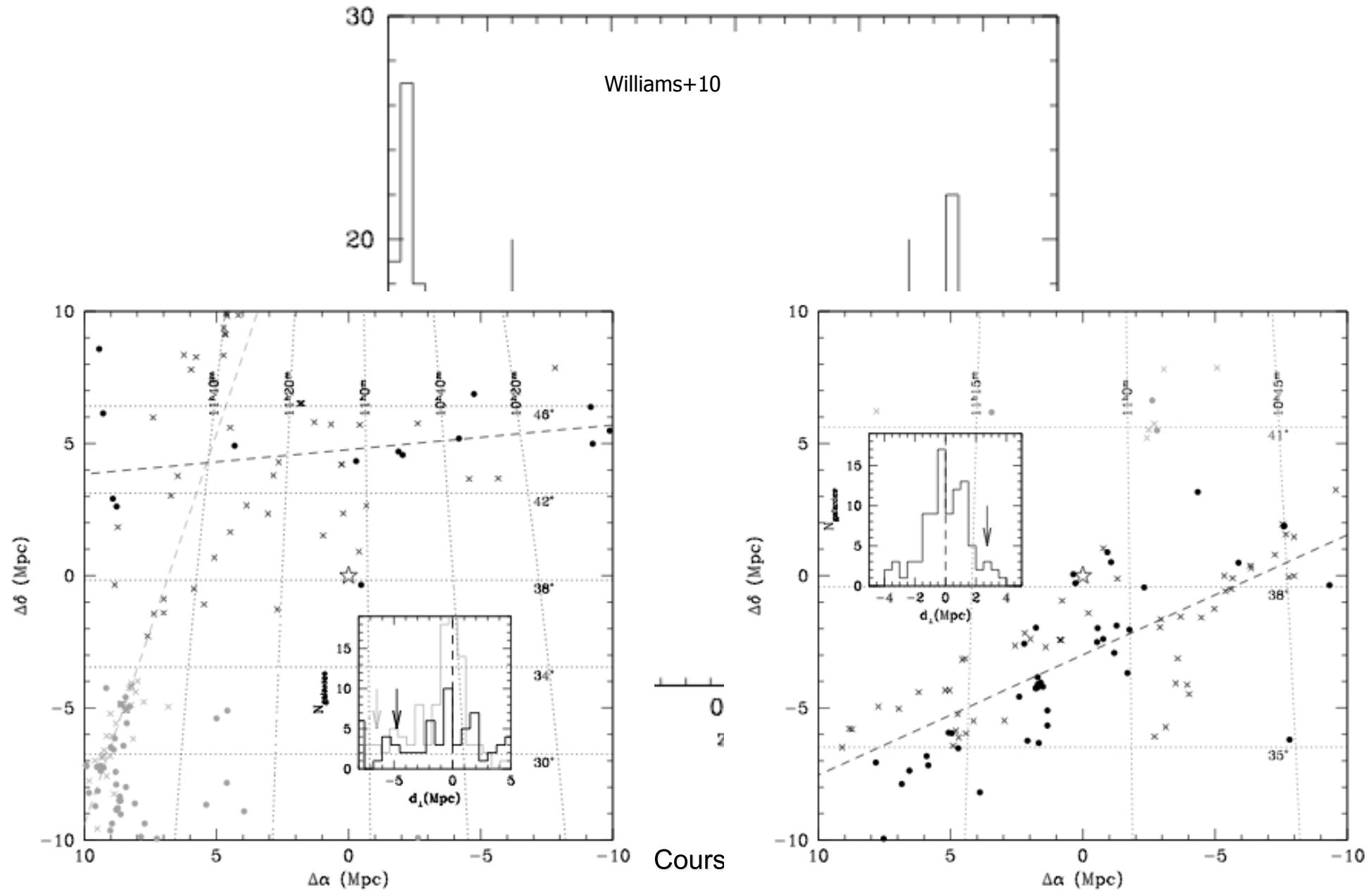
Nicastro+05, *Nature*

9 $z > 0$ lines: 2 systems
 $z = 0.011$ (3.5σ), 0.027 (4.9σ)

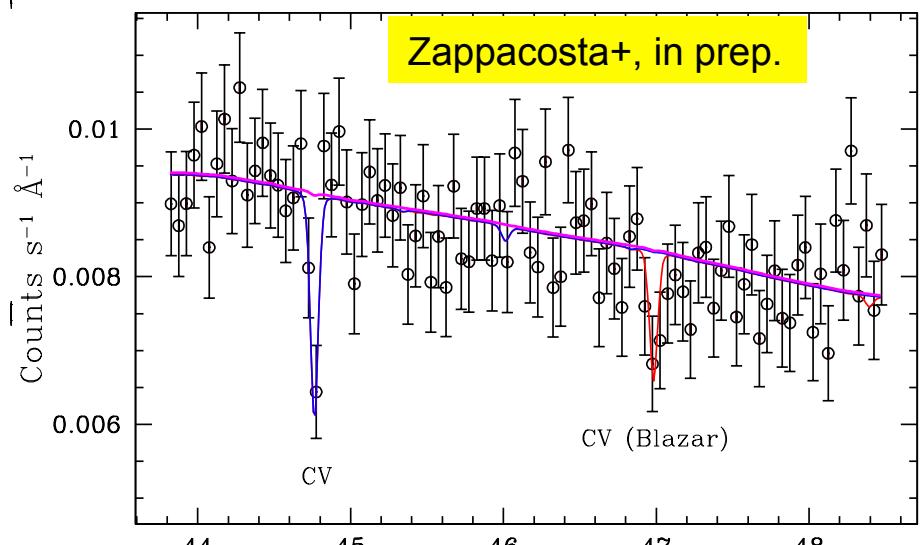
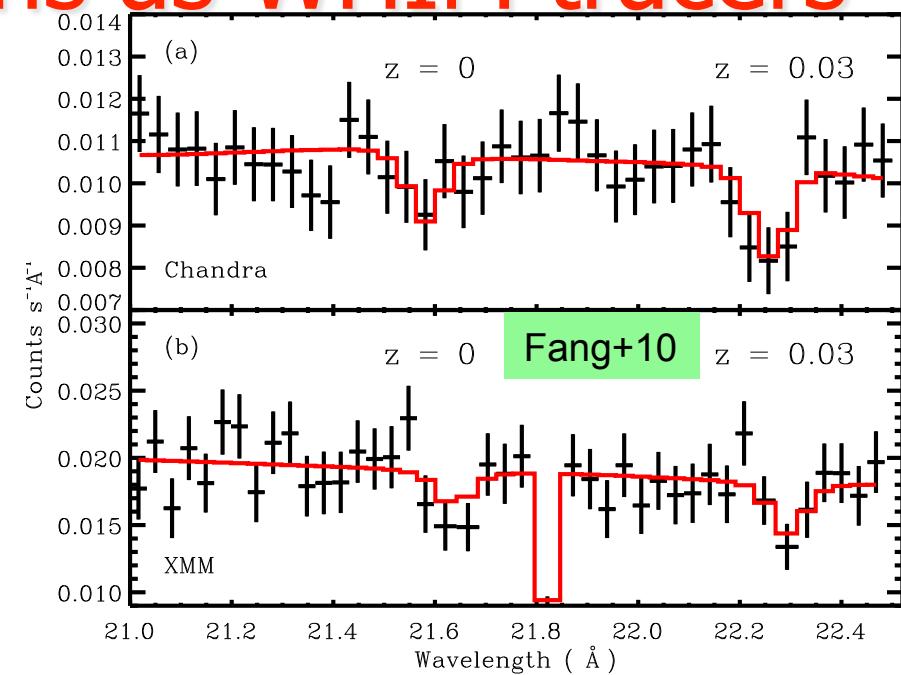
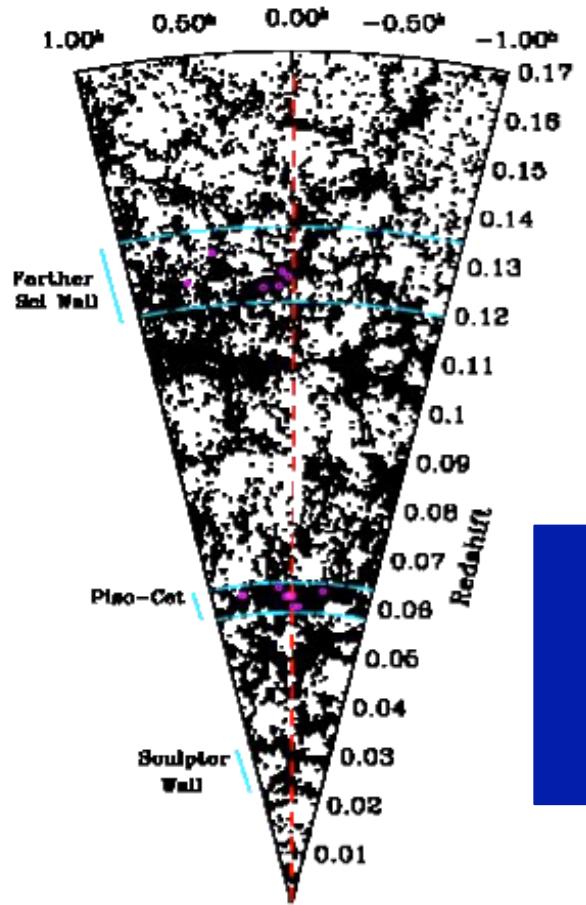


$$\Omega_b(N_{O VII} > 7 * 10^{14}) = \left(\frac{1}{\rho_c} \right) \left(\frac{\mu m_p \sum_i N_H^i}{d_{Mkn421}} \right) = 2.7_{-1.9}^{+3.8} * 10^{-[O/H]_{-1}} \%$$

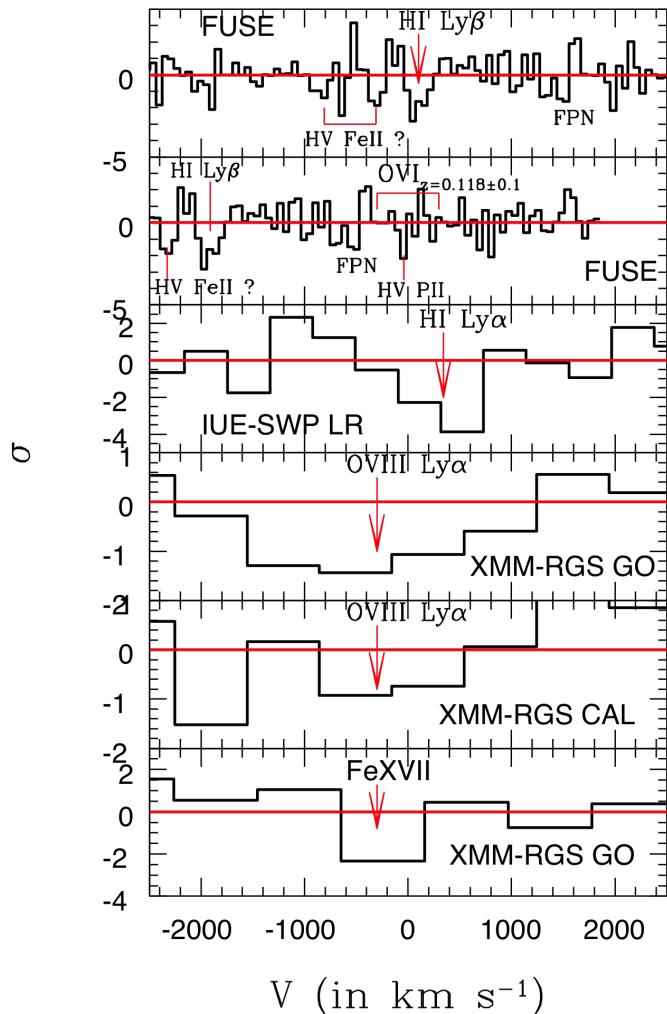
Galaxy Environment of Mkn 421 Absorbers



Galaxy concentrations as WHIM tracers



A Serendipitous hot X-Ray/BLA Filament at $z=0.118$?



From X-rays → $\log T < 6.75$;
 $\log N_H = 21.5 \pm 0.3 (Z/Z_{0.01\odot})^{-1}$

From lack of OVI → $\log T > 6.52$

From HI and T → $Z = (1-4)\% Z_\odot$

From Z and T
and Theory → $\delta \approx 300$

From δ and N_H → $D=4-7 \text{ Mpc}$

Combined Statistical Significance = 5.2σ
(5σ if FUSE systematics are included)
[Nicastro+10]



3

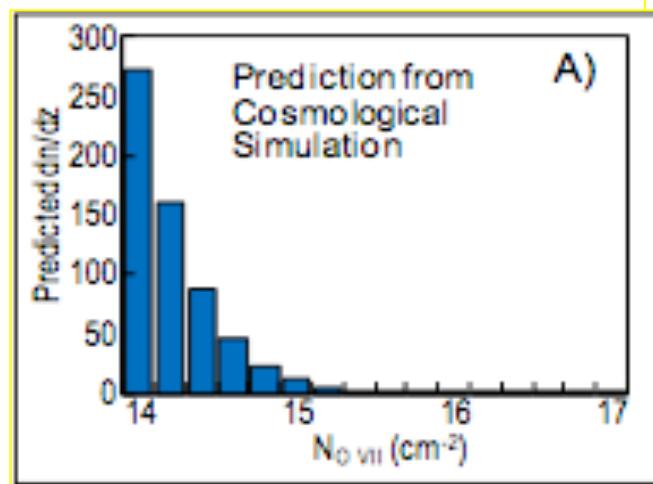
Optimizing the Observational Strategy

The WHIM is hard to detect

- Only 4 systems at $z < 0.4$ and $N_{\text{OVII}} > 10^{15} \text{ cm}^{-2}$
→ Needs high-z targets
- $N_{\text{OVII}} = 10^{15} \text{ cm}^{-2} \Leftrightarrow \text{EW(OVII K}\alpha\text{)} = 3 \text{ mA}$
→ Needs bright X-ray targets ($> 1 \text{ mCrab}$)
- ~ 170 REs at $z < 0.4$ in Chandra: ~ 10 by chance
at $< -2\sigma$ → Needs “Signposts”

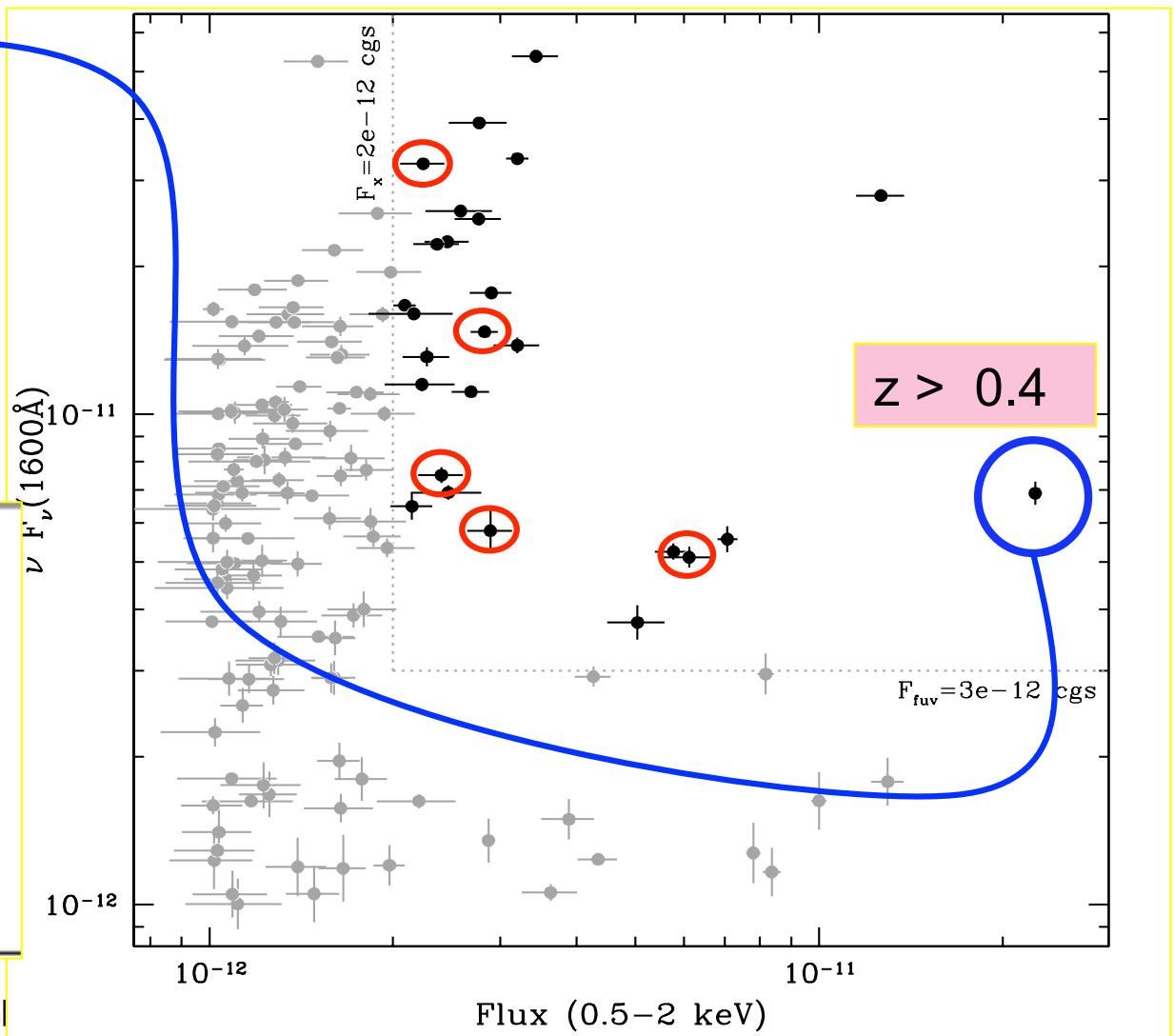
Best WHIM Targets

1ES 1553+115: the Best WHIM Target in the Universe!



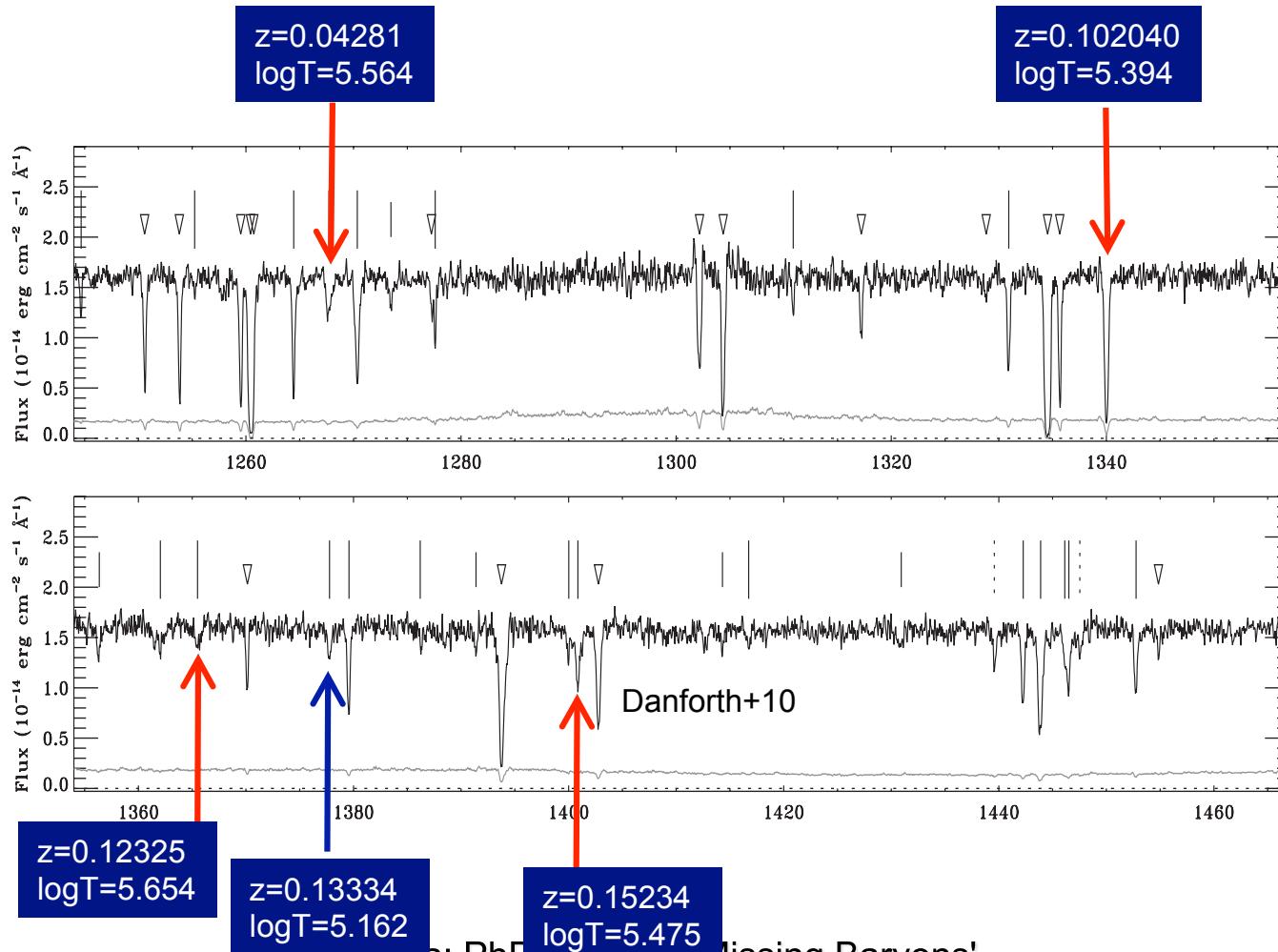
10/18/11

Bol



FUV Signposts: BLAs and Ly α

42 Ly α systems in COS: **4(5) Broad** ($b > 52(46)$ km s $^{-1}$ $\rightarrow b_{\text{th}} > 40(35)$ km s $^{-1}$)

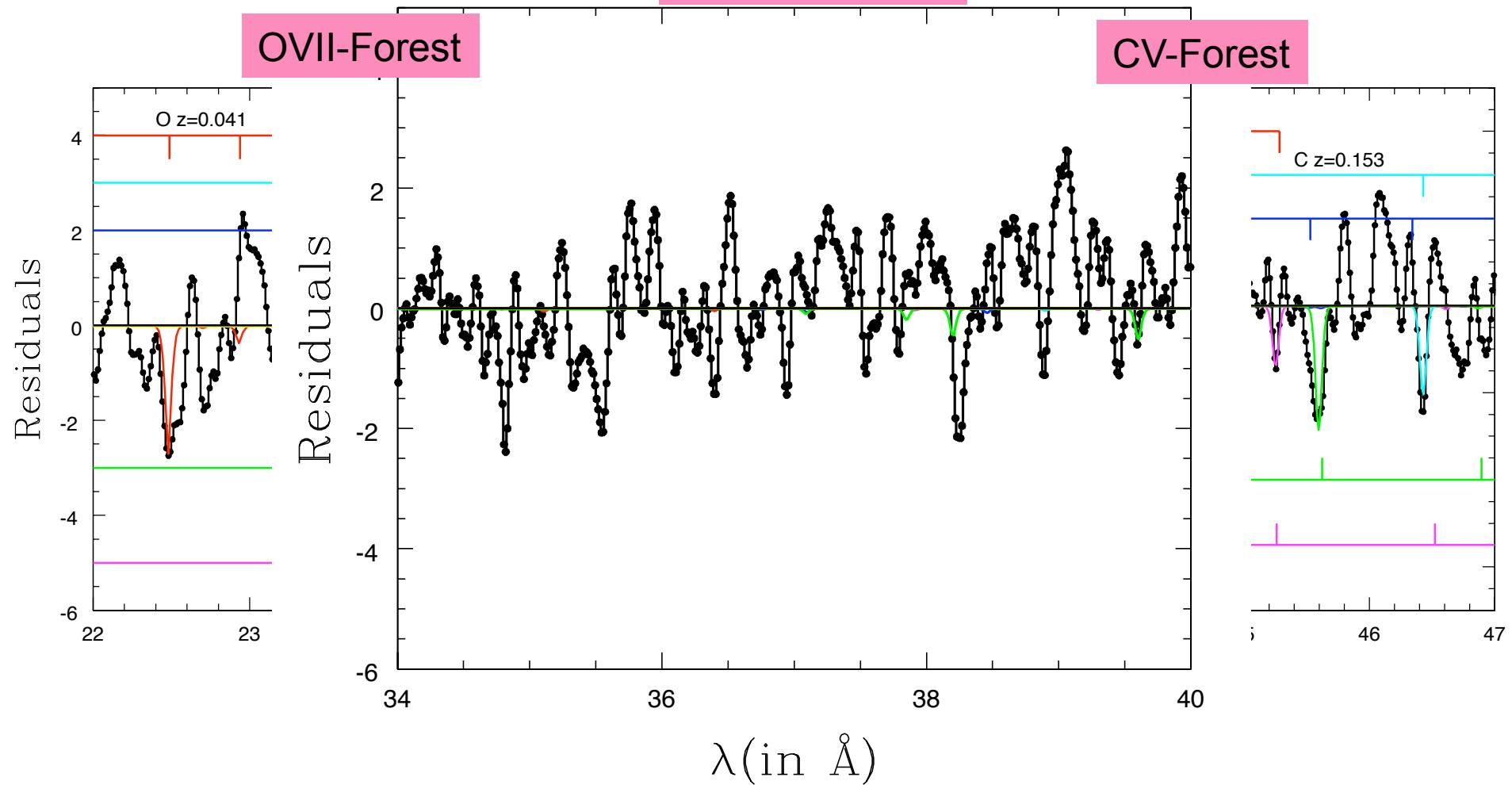


Chandra 500 ks of 1ES 1553+113

- 3 Observations: (a) 2011, May 4-6 (166 ks);
(b) 2011, May 6-9 (175 ks); (c) 2011, June
18-20 (154 ks)
- Total Exposure: 495 ks
- Confirm average flux: $F_{0.5-2}(a)=1.1 \text{ mCrab}$;
 $F_{0.5-2}(b)=1.2 \text{ mCrab}$; $F_{0.5-2}(c)=0.6 \text{ mCrab}$
- Sensitivity to Absorption Line $\text{EW} > 4 \text{ mA}$ (1σ)
in 18-28 Å ($\sim 300 \text{ Ct/R.E.}$)
- Detects several ISM/IGM absorption lines

LETG LSF-Smoothed Residuals: 4 WHIM + 1 Photoionized Systems

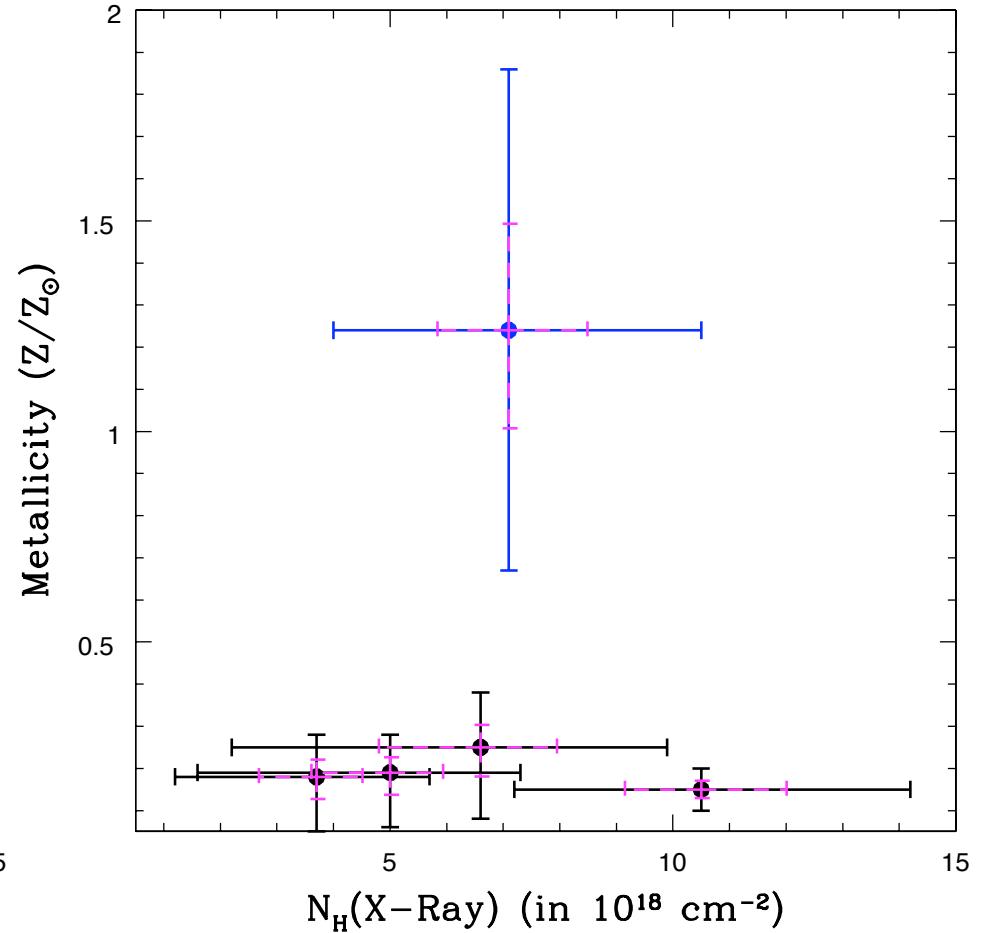
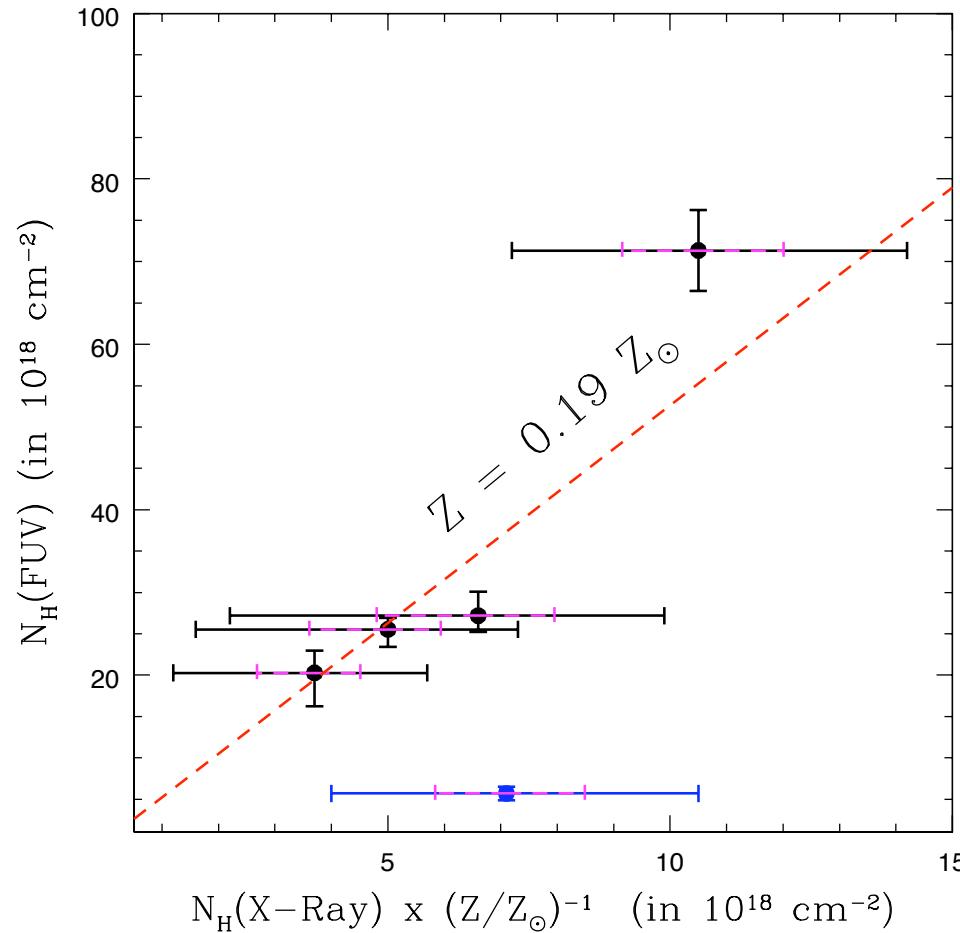
No-Line Region



Physical Properties and Metallicities

Redshift X/FUV	b_{th} (km s $^{-1}$)	logT (K)	f(HI) (10 $^{-6}$)	$N_{\text{H}}(X)$ (10 18 cm $^{-2}$)	N_{HI} (10 13 cm $^{-2}$)	Z/Z_{\odot}	Stat. Sign. (σ) X/FUV
0.041	56 ± 5	5.564	1.03	$6.6^{+4.4}_{-3.3}$	$2.8^{+0.2}_{-0.3}$	$0.25^{+0.17}_{-0.13}$	3/23.9
0.04281							
0.100	40 ± 1	5.394	1.85	$10.5^{+3.3}_{-3.7}$	13.2 ± 0.09	0.15 ± 0.05	2.6/90
0.10230							
0.123	62 ± 12	5.654	0.74	$3.7^{+2.5}_{-2.0}$	$1.5^{+0.3}_{-0.2}$	$0.18^{+0.13}_{-0.10}$	2/4.5
0.12325							
0.153	51 ± 4	5.475	1.41	$5.0^{+3.4}_{-2.3}$	$3.6^{+0.3}_{-0.2}$	$0.19^{+0.13}_{-0.09}$	2/13.3
0.15234							
0.132	35 ± 6	5.327	2.46	$7.1^{+3.1}_{-3.4}$	$1.4^{+0.2}_{-0.2}$	$1.24^{+0.57}_{-0.62}$	2/5.4
0.13334							

Physical Properties and Metallicities



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The Missing Baryons in and around the Milky Way

Hot Gas in the Local Group

> 1.5×10^{12} M_⊕ are needed to stabilize the Local Group

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND
ASTRONOMICAL PHYSICS

VOLUME 130

NOVEMBER 1959

NUMBER 3

INTERGALACTIC MATTER AND THE GALAXY

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Received May 18, 1959

ABSTRACT

It is shown that the Local Group of galaxies can be dynamically stable only if it contains an appreciable amount of intergalactic matter. A detailed discussion shows that this matter consists mainly of ionized hydrogen and that stars can contribute only a small fraction to its total mass. The most likely values for the intergalactic temperature and density are found to be 5×10^5 degrees and 1×10^{-4} proton/cm³, respectively. It is thought that this gas confines the halo. The distortion of the disk of the Galaxy, revealed by 21-cm observations, is analyzed. This effect cannot be regarded as a relic from a primeval distortion, which occurred at the time of formation of the Galaxy; a more promising explanation for it can be given in terms of the flow pattern of the intergalactic gas past the Galaxy and of the resulting pressure distribution on the halo.

OVI Velocity Distribution in the LSR

(Nicastro+03, Nature)

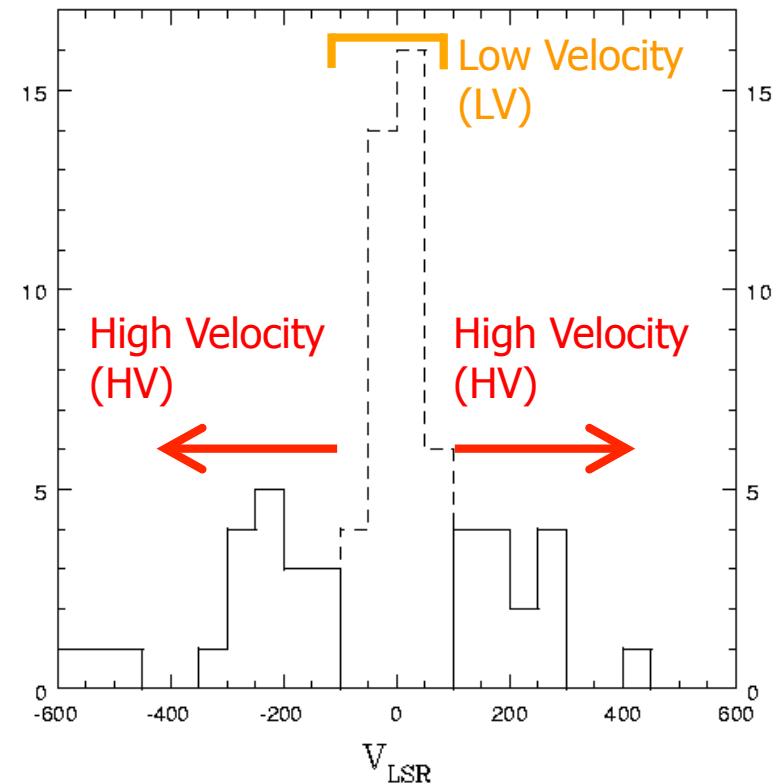
The Sample:

- 103 FUSE Obs. Of AGN
- 54 with $S/N \geq 5$ per Res. El., at 1032 Å
- 45 show at least 1 OVI abs. at $z \sim 0$, at our detection threshold (7 of the remaining 9 have poor S/N spectra)

- 38/45 LV-OVI
- 32/45 HV-OVI
- 21 both

• High Covering Factor

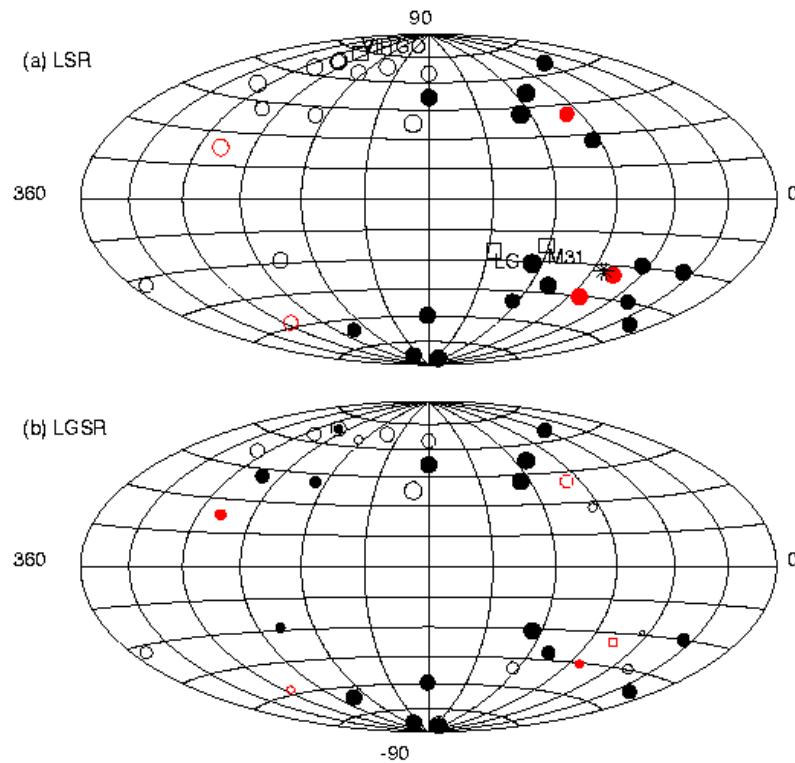
(could be 90 % down to 20 mÅ, Sembach+02)



Velocity Segregation

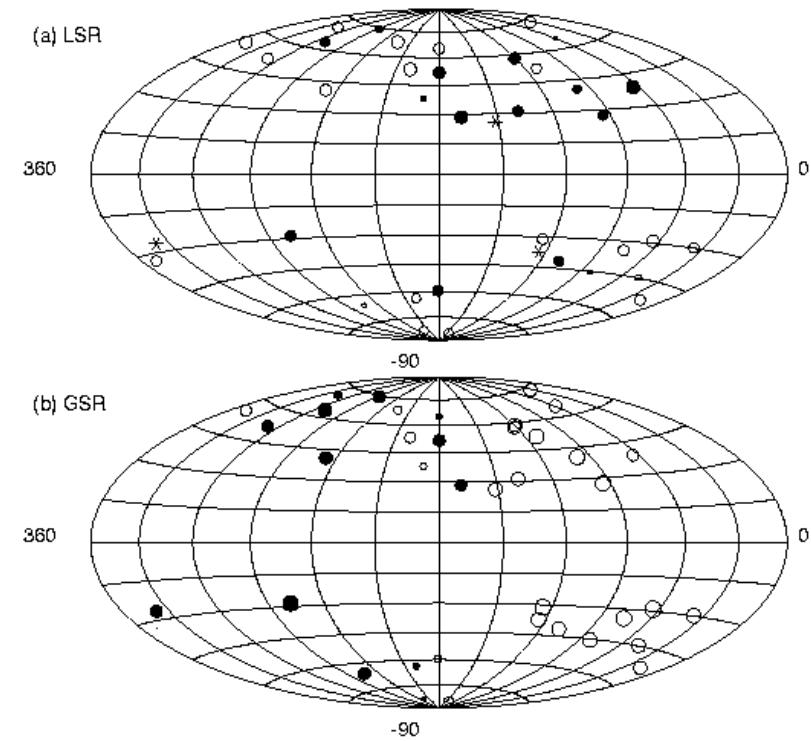
HV-OVI

Strong Segregation in the LSR

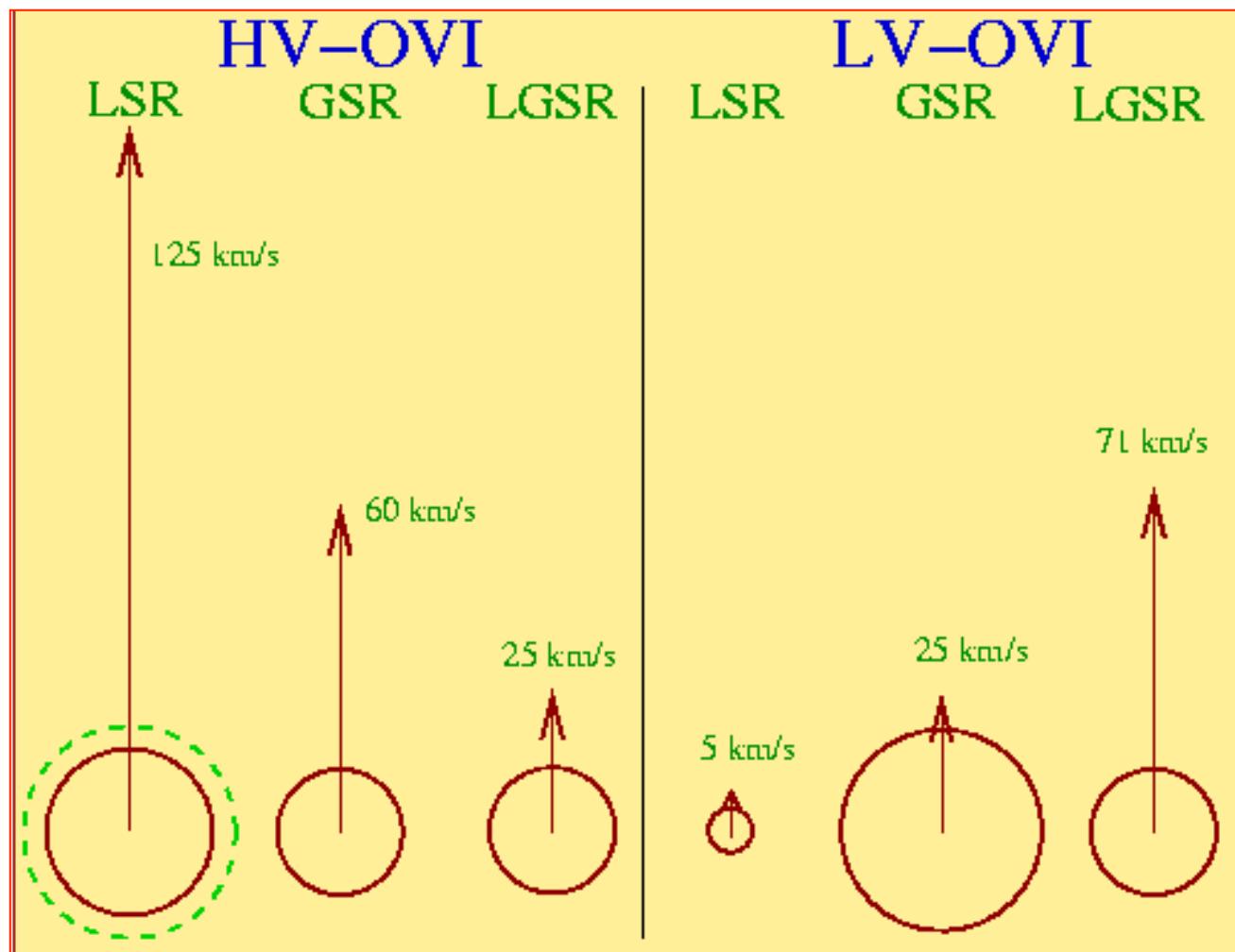


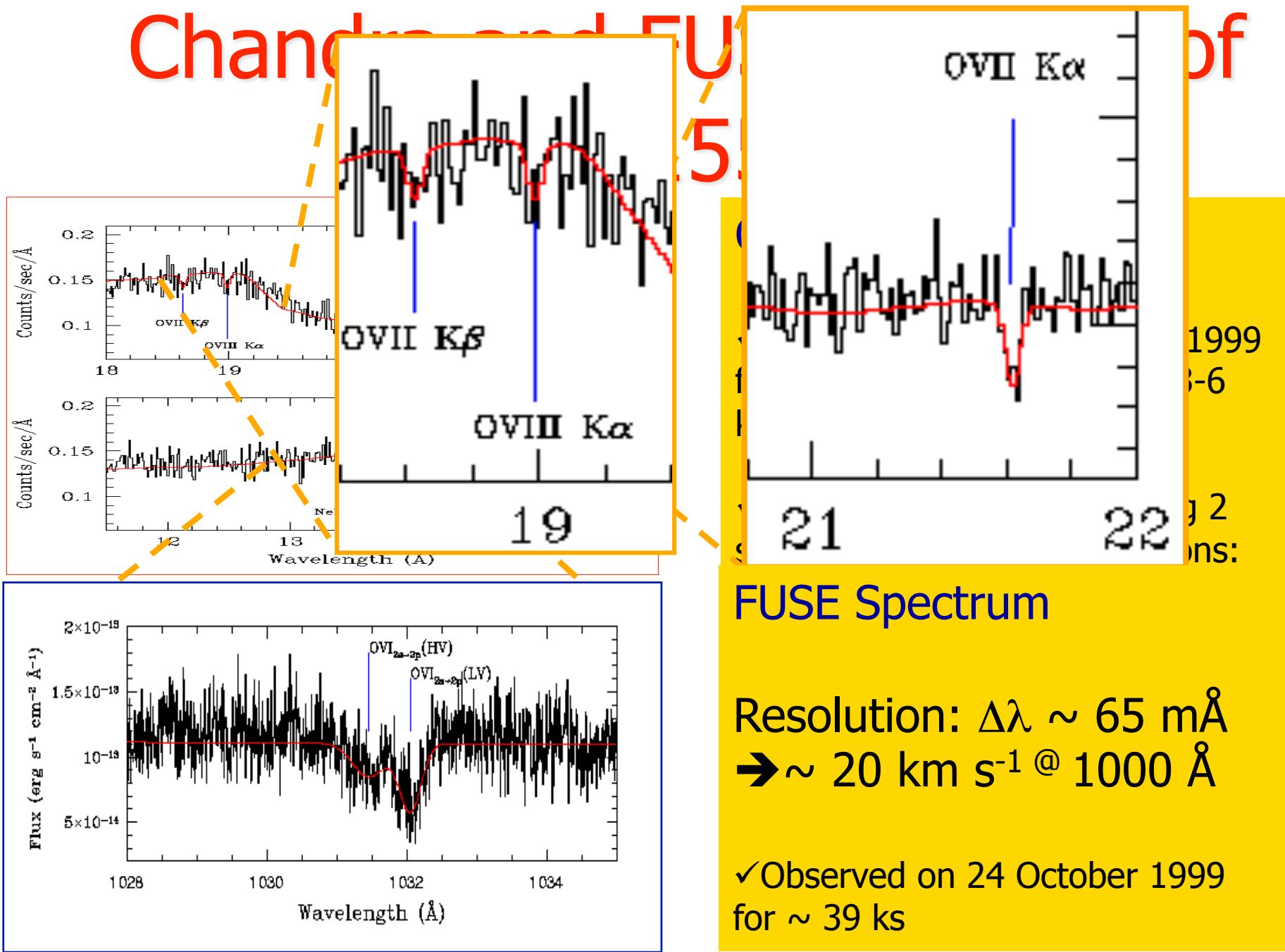
LV-OVI

Strong Segregation in the GSR



Average Velocity Vectors





10/18/11

Bologna: PhD Courses 'Missing Baryons'

Galactic vs Extragalactic Solutions

Galactic = High Density (typical ISM densities) → No Phot. Contr. → Gas in collisional equilibrium: 3-cloud Galactic solution.

Extragalactic = Low Density ($n_e < 10^{-5} \text{ cm}^{-3}$) → XRB contribution becomes important: it populates OVII-OVIII species even at relatively low temperature ($T \sim 10^6 \text{ K}$) → Local Group WHIM solution

WHIM Solution:

$$\log T = 5.8; [\text{Ne}/\text{O}]_{\odot} = 2.5$$

$$n_e = 4-6 \times 10^{-6} \text{ cm}^{-3} = 20-30 \delta$$

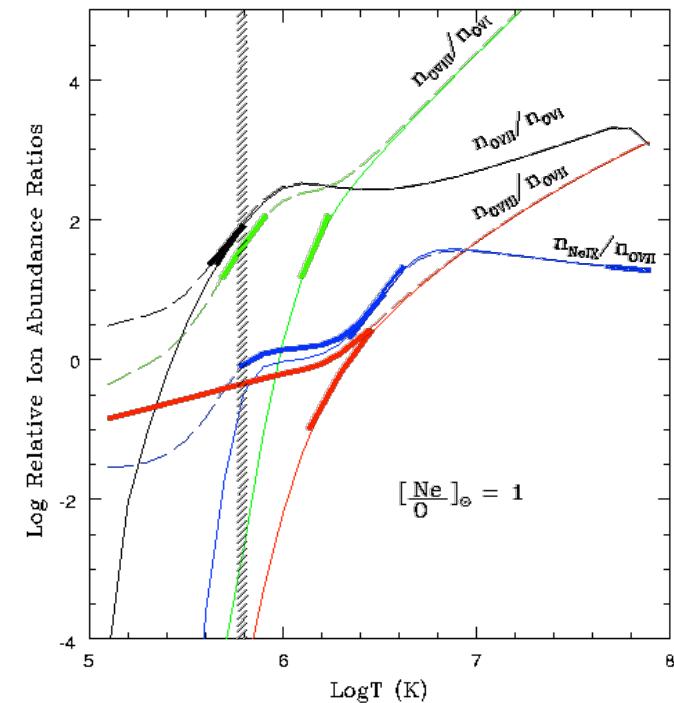
$$N_H = 4.5 \times 10^{19} [\text{H}/\text{O}]_{0.3} \text{ cm}^{-2}$$

All consistent with WHIM predictions

$$==> D = (2-4)[\text{H}/\text{O}]_{0.3} \text{ Mpc}$$

Well Beyond the Milky Way

(Nicastro et al., 2002, ApJ, 573, 157)



Implication

- Assuming a transverse size of $1 \times [H/O]_{0.3}$ Mpc



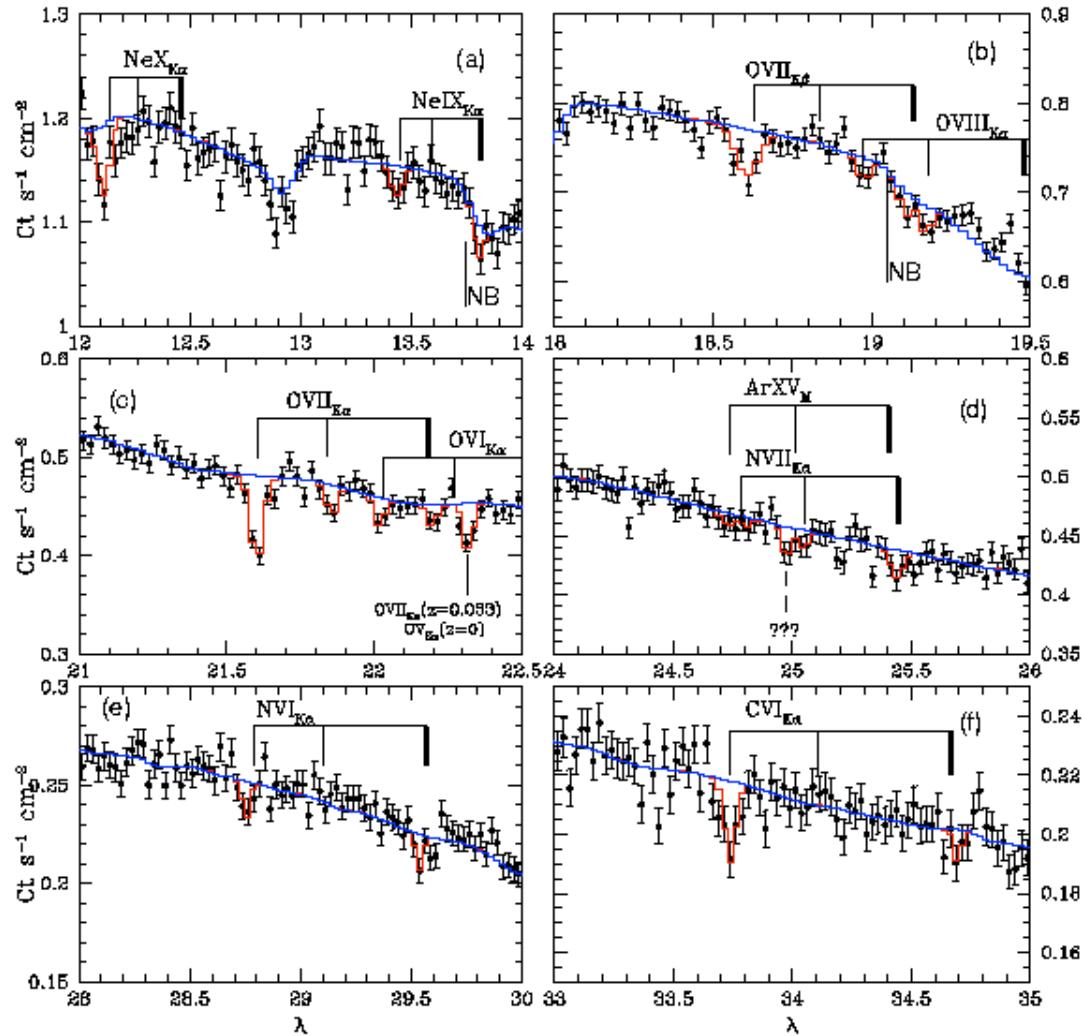
$$M = (0.6 - 2) \times 10^{12} M_\odot$$

Assumes High Covering Factors

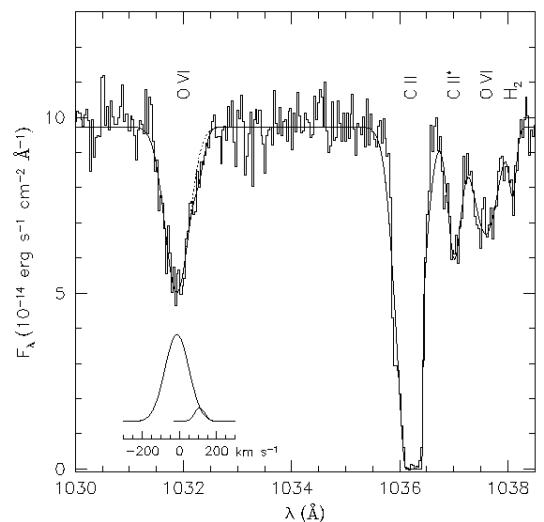
Z=0 Absorption

Chandra-LETG spectrum of Mkn 421

(Williams+05)

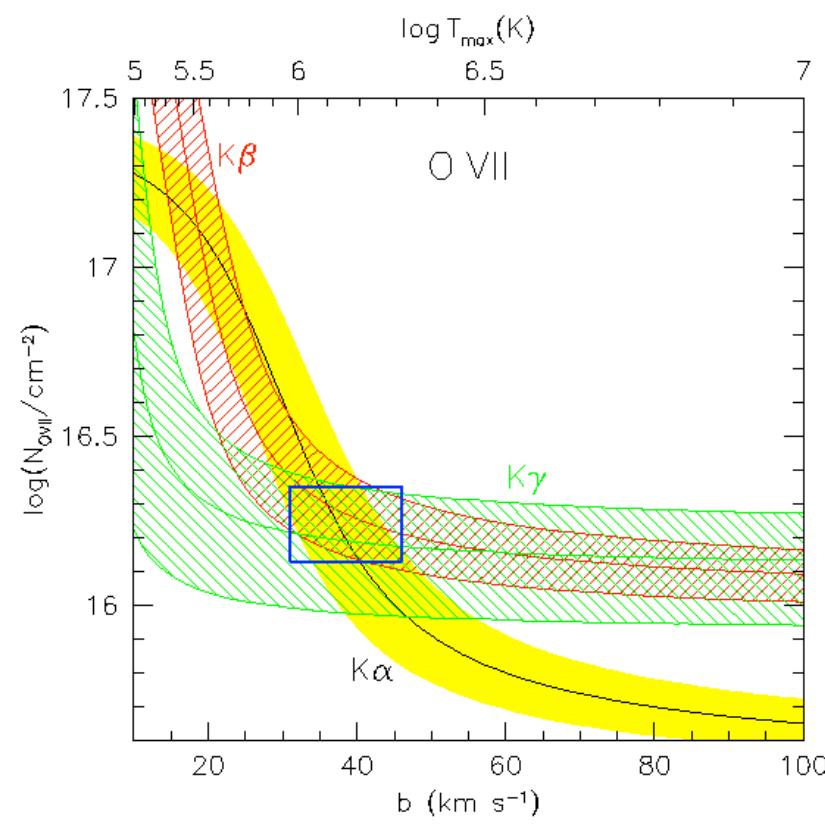


FUSE

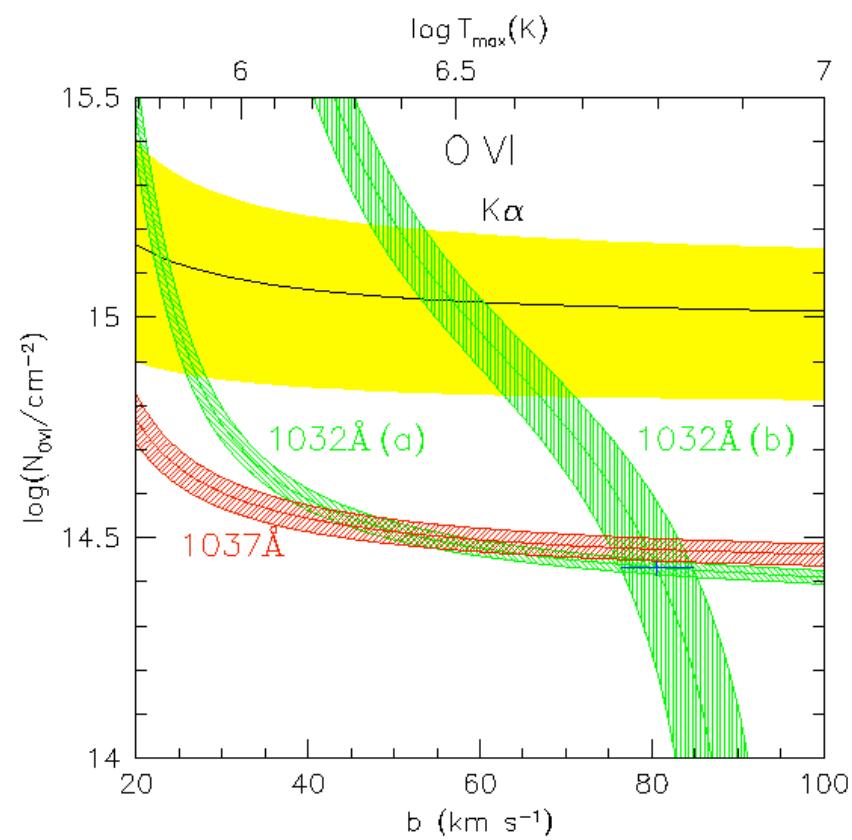


Doppler Parameter

OVII Gas \neq LV-OVI Gas



(Williams+05)



Ionization Balance

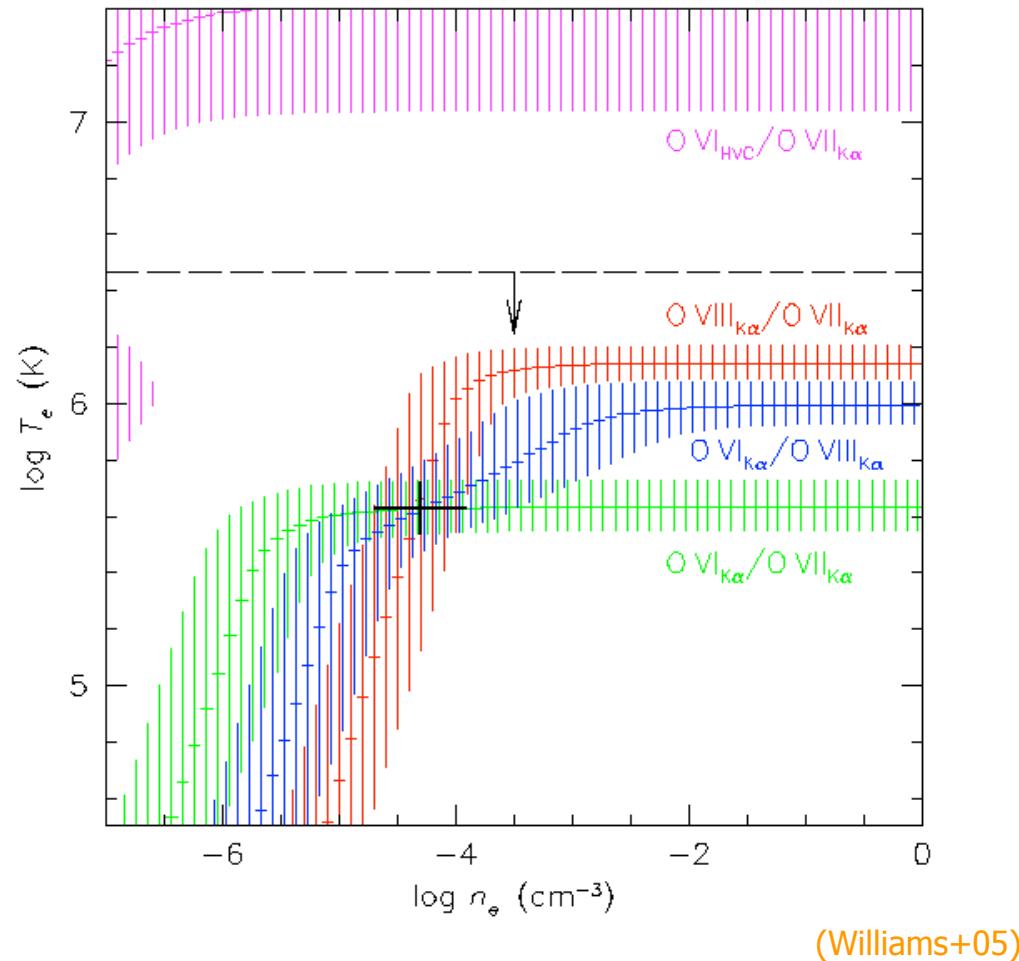
For 2 ions from the same element

$$\frac{\xi_{X^i}}{\xi_{X^{[i+n]}}} = \frac{N_{X^i}}{N_{X^{[i+n]}}} \propto \frac{EW_{X^i}^b}{EW_{X^{[i+n]}}^b}$$

For 2 ions from different elements

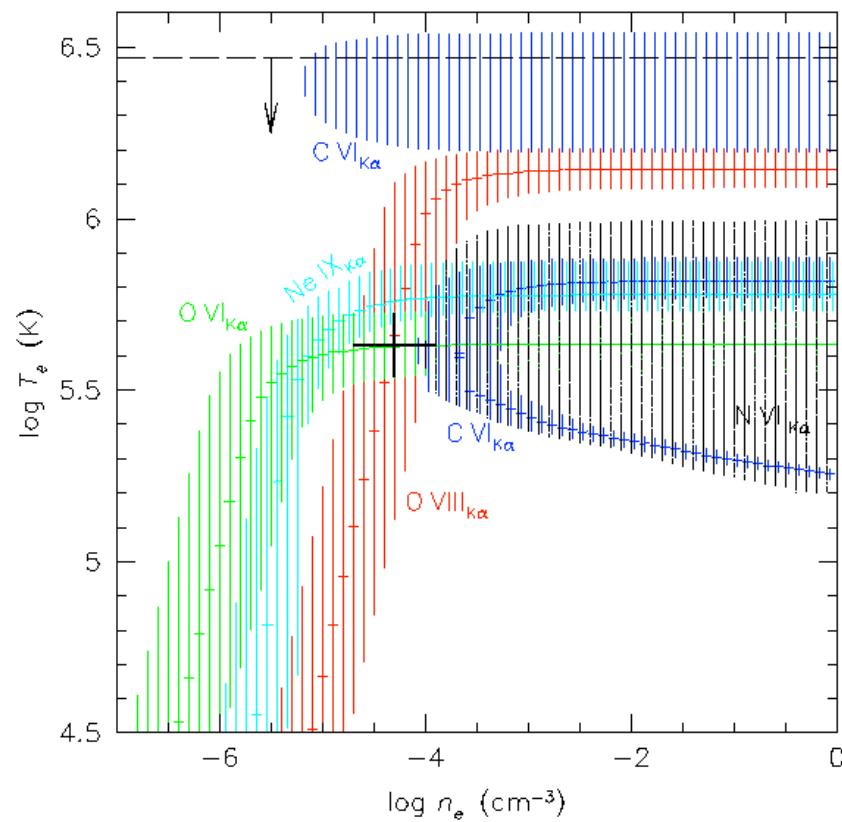
$$\frac{\xi_{X^i}}{\xi_{Y^j}} = \frac{N_{X^i}}{N_{Y^j}} \times \frac{A_Y}{A_X} \propto \frac{EW_{X^i}^b}{EW_{Y^j}^b} \times \frac{A_Y}{A_X}$$

Low-Density Solution The OVI Problem



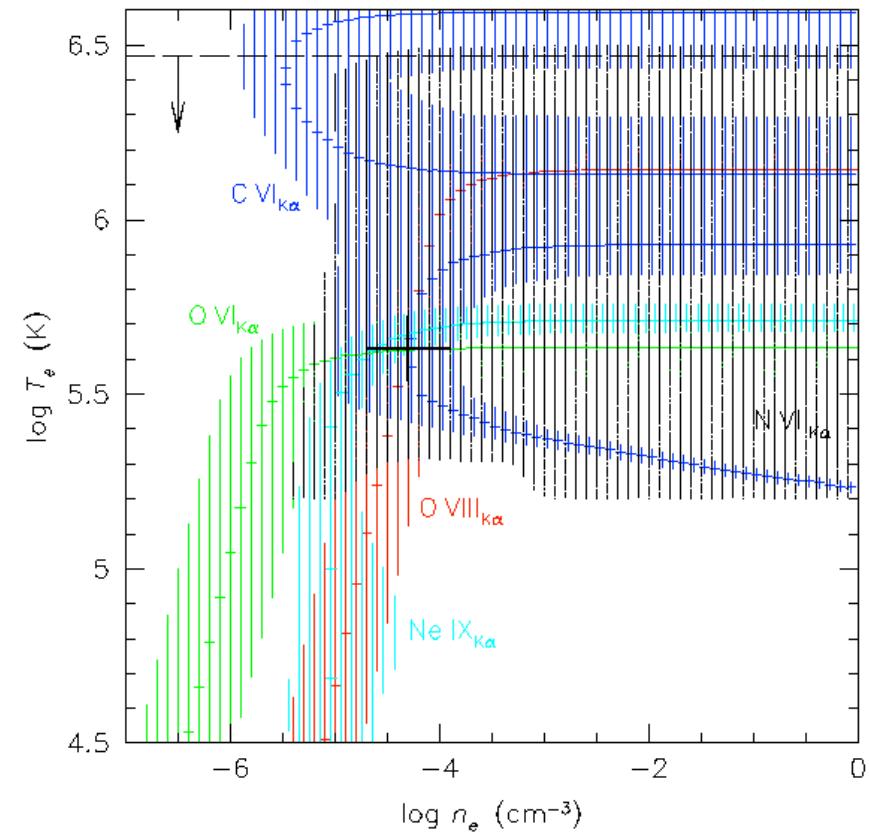
The [Ne/O] Problem

[Ne/O] = 0



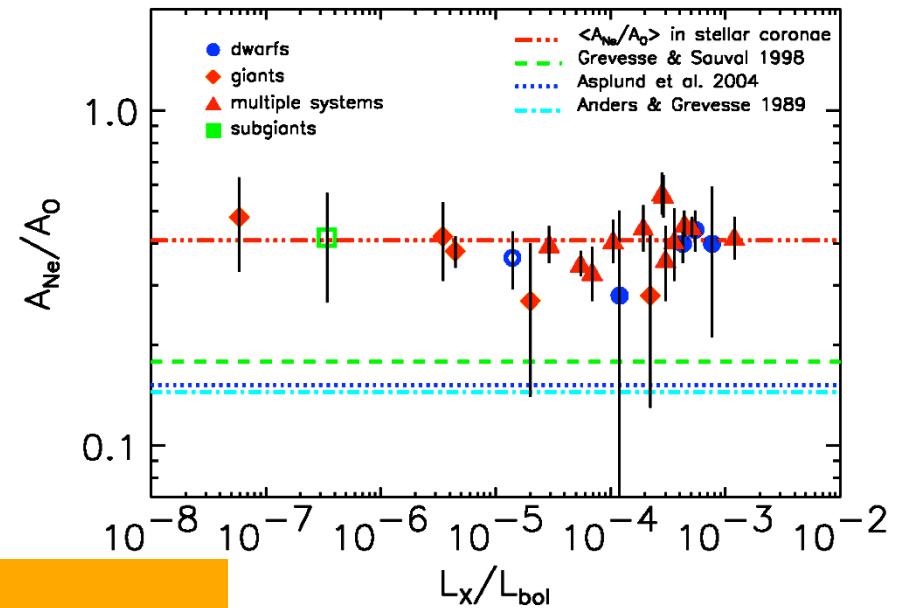
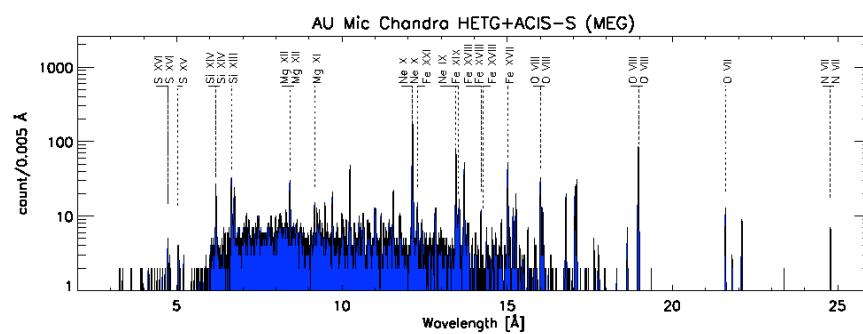
(Williams+05)

[Ne/O] = 1



Sun Structure and Metallicity

22 nearby stars observed with Chandra-HETG



$$[\text{Ne}/\text{O}] = 0.43 \text{ (2.7 x Solar)}$$

together with the recently revised C, N, O abundances (Asplund+04) brings the solar model back into agreement with Helioseismology

(Drake & Testa, 2005, Nature)

4

The Future: Need for a Large- Throughput High-Resolution X-Ray Spectrometer

Warm-Hot Intergalactic Medium Explorer (WHIMex)

To explore the unseen Universe.

To find the missing matter in deep space.

To learn how galaxies and stars came to be.

WHIMex

Science Objectives

Warm-Hot Intergalactic Medium (WHIM)

- Find the missing baryons in the low-mid-redshift universe.
- Understand the cycles of matter on a cosmological scale.
- Constrain models for the evolution of large-scale structure

Active Galactic Nuclei (AGN)

- Measure the outflow of matter from the nuclei of galaxies
- Examine the role of feedback for the chemical enrichment and heating of interstellar and intergalactic media

Galactic Sources

- Probe shocks, winds, coronae, and accretion disks of stars
- Study exotic (black-hole & neutron) stars and environs
- Characterize the hot component of the interstellar medium

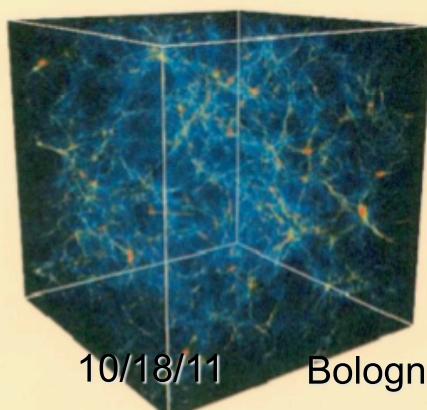


An Explorer 2011 proposal

Submitted by:
University of Colorado at Boulder

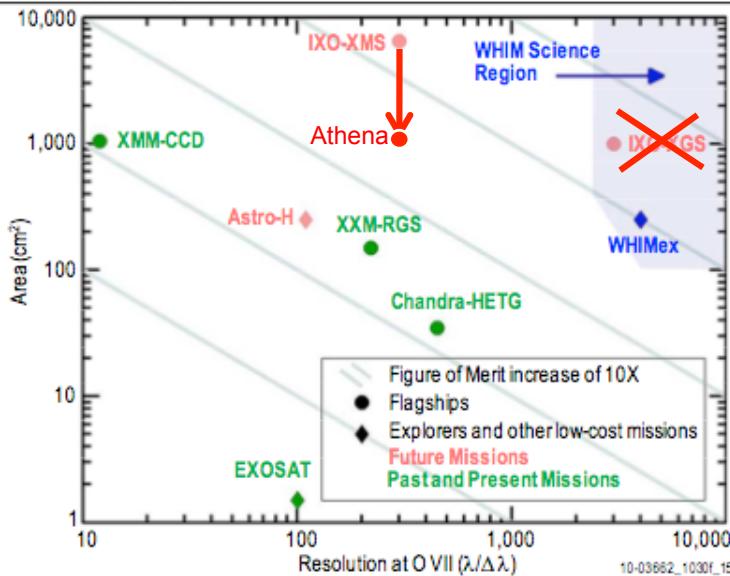
In response to:
AO NNH11ZDA0020

16 February 2011



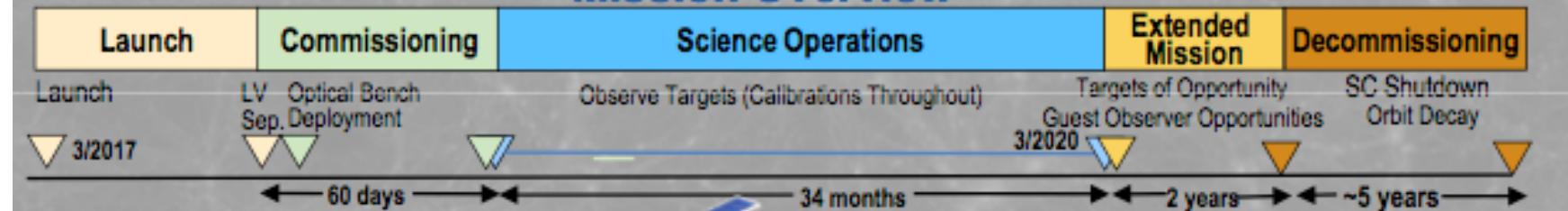
Bologna: PhD C

Proposing Organization:
University of Colorado at Boulder
Principal Investigator:
Professor Webster Cash



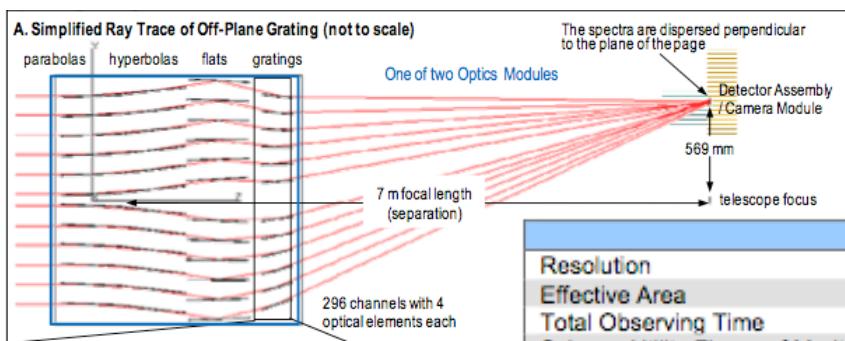
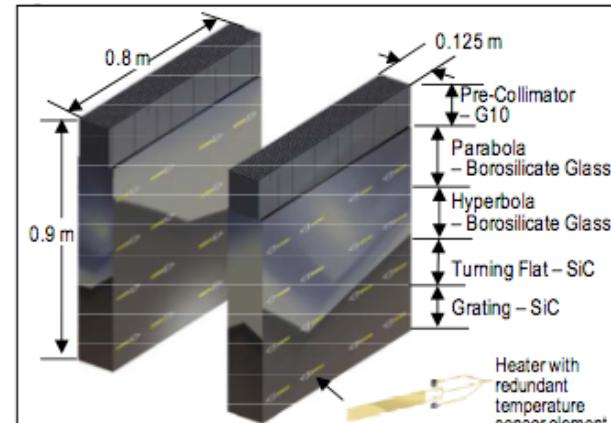
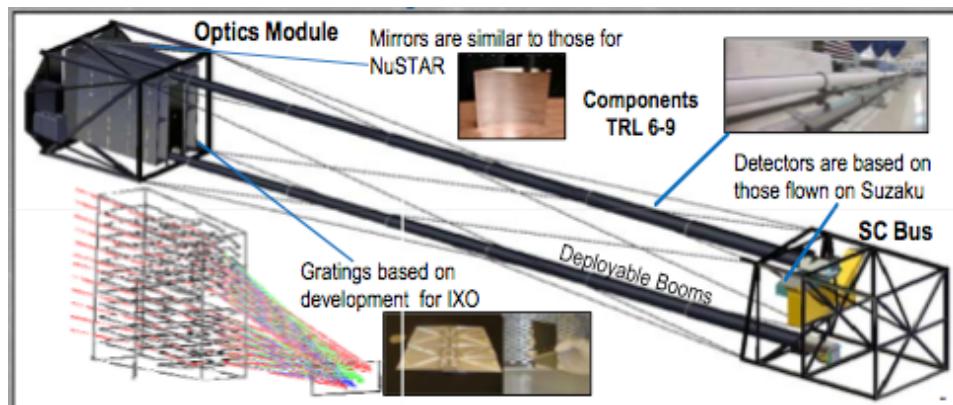
U Colorado	Principal Investigator (PI) Responsibility for the scientific success of WHIMex
MSFC	Project Office; Calibration of X-ray Spectrometer
U Iowa	Deputy PI; Instrument PI for X-ray spectograph
GSFC	Production of Optics Modules, mirror fabrication & alignment
NG	Development & Integration of the entire Flight System; extensible bench
MIT	Grating replication & testing; support for detector acquisition
Sierra Nevada Corp.	Development & integration of Spacecraft Bus.
CASA	Science Operations Center (SOC)
LASP	Mission Operations Center (MOC)
SAO	Public Data Archive (Science Enhancement Option)
Open U	International Partner (UK), detector electronics & assembly
Osaka U	International Partner (Japan), CCD procurement & screening

Mission Overview



Key Instrument Payload Characteristics:

- Spectrograph with resolving power $R \approx 4000$ and effective area $A_{\text{eff}} \approx 360 \text{ cm}^2$ around 0.5 keV
- Telescope focal length $F = 7 \text{ m}$, using an extensible optical bench
- Two Optics Modules, each with ≈ 300 sets of primary, secondary, and flat mirrors plus radial groove gratings
- Spectrograph read-out detector (CCD array) giving high efficiency and separation of spectral orders



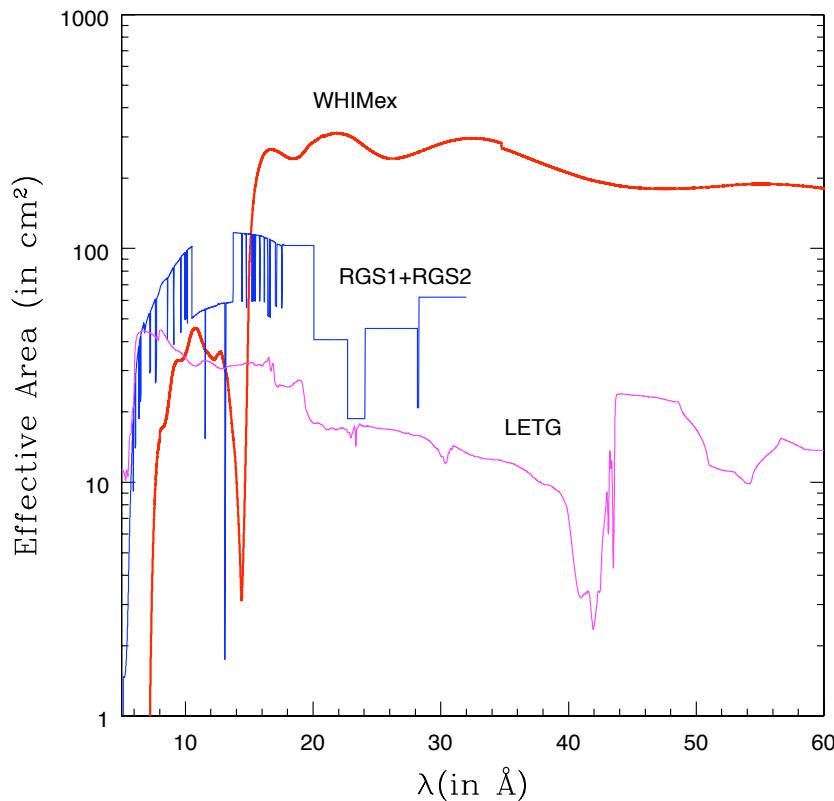
Component	Number
Precollimator	2
Parabola/Hyperbola pairs (X-ray mirrors, 0.4 mm thick)	592 pairs, 1184 elements
Turning flats (1 mm thick)	592
Gratings (1 mm thick)	592
Thermal Control Subsystem	64 heaters, 128 sensors
Structure	2

10-03882-10316-154

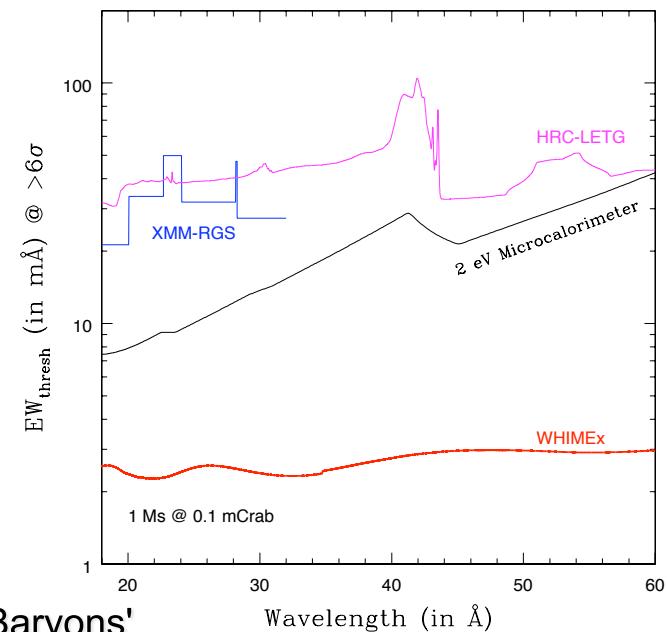
	Threshold	Baseline	Enhanced	Comment
Resolution	$\lambda/\Delta\lambda$	2,500	4,000	Average over band
Effective Area	cm^2	100	250	500
Total Observing Time	Ms	15	45	100
Science Utility Figure of Merit	$\times 10^8$	6	45	200
Number of Targets	WHIM	12	26	50 At 66% observing efficiency Resolution*Area*Time

WHIMEx Eff. Area & FOM

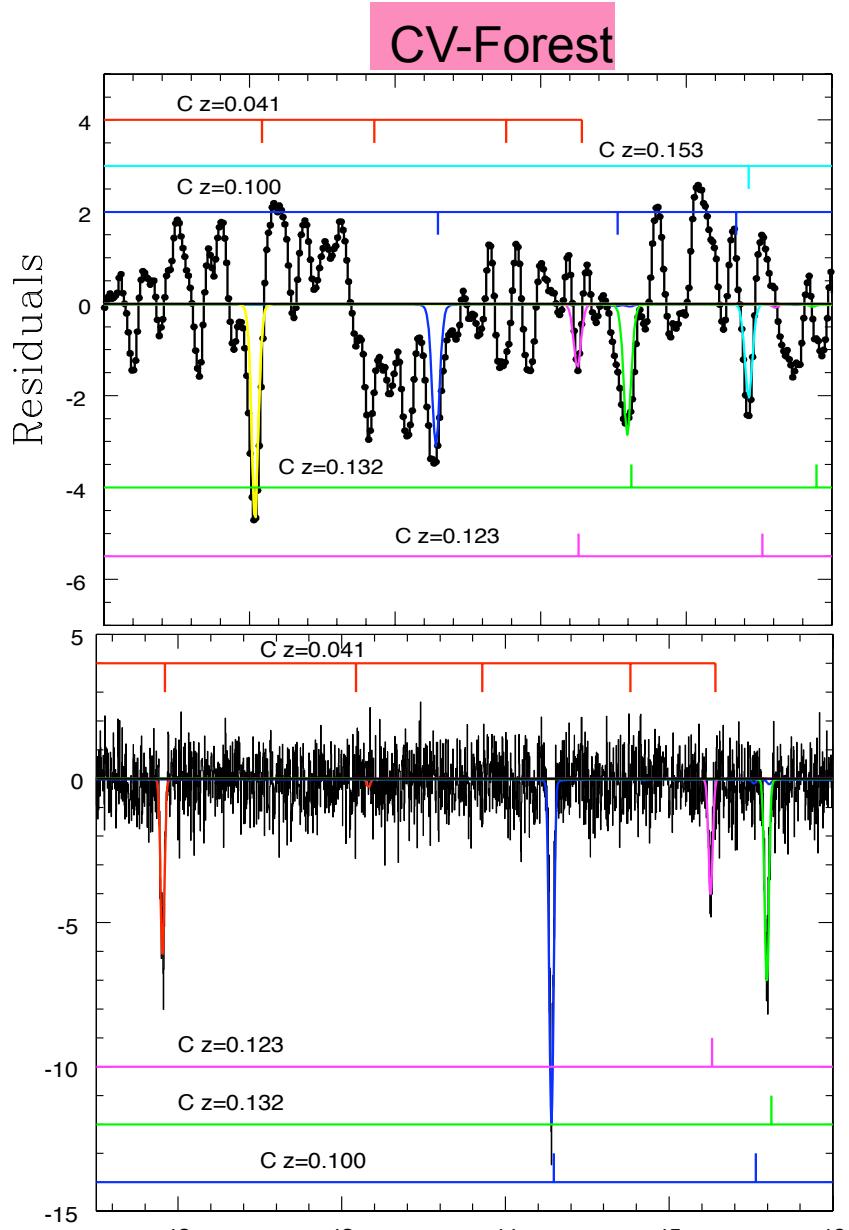
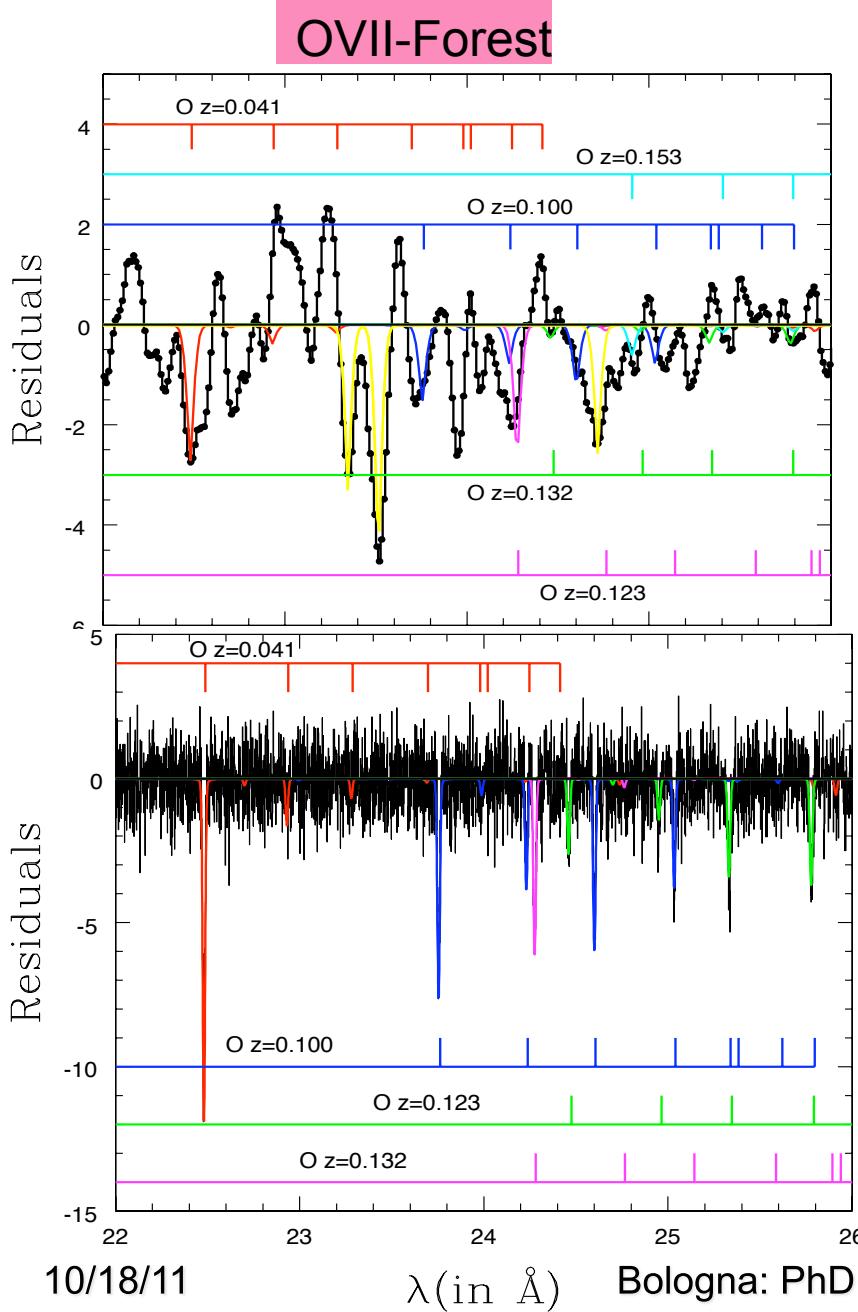
X-Ray Spectral Diagnostics



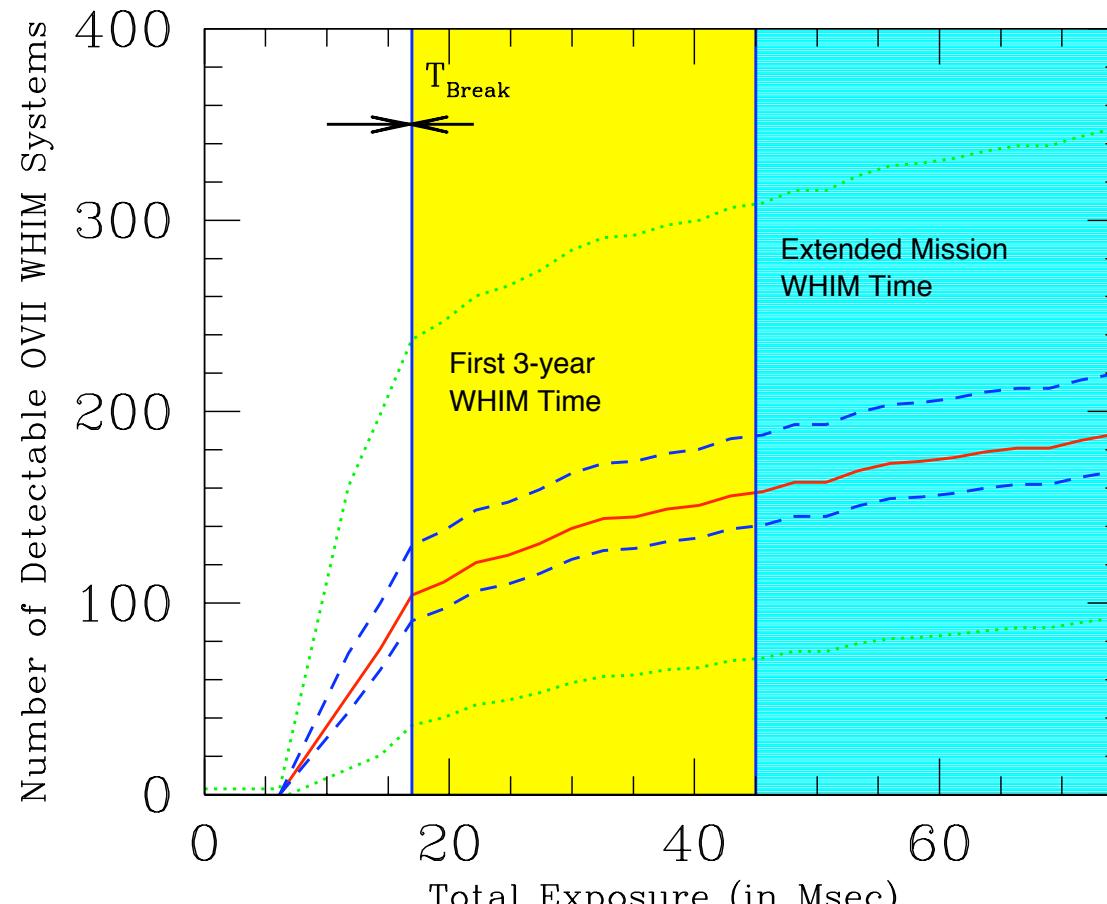
Line	E in keV (z=0-0.5)	Science Value Derived from Line
O VIIα	0.57-0.38	The strongest WHIM line and the key line for WHIM science. Number of absorbers per unit redshift (dn/dz)
O VIIIα	0.65-0.44	Ionization parameter (U) and temperature (T) and hence density
C Vα	0.31-0.2	Strong WHIM line: dn/dz and T
O VIIβ	0.67-0.44	Bulk motion (b) and column density (N_{ion}); Needed for saturation
C Vβ	0.35-0.23	b and N_{ion} ; b from 2 elements gives T
C VIα	0.37-0.24	Absorption ratio, U , and T . Strong at high T
C Vγ	0.37-0.24	b and N_{ion}
O VIIγ	0.7-0.45	b and N_{ion}
Si X/Si X*	0.245	N_{ion} , Electron number density (n_e), distance to Black Hole
Mg VIII/Mg VIII*	0.21	N_{ion} , n_e , distance to Black Hole



The WHIM with WHIM-Ex



The WHIM with WHIMex



$$104^{+26}_{-13}$$