Accretion through cosmic time

Joss Bland-Hawthorn
(University of Sydney)

Searching for clues relating to structure formation across the hierarchy…

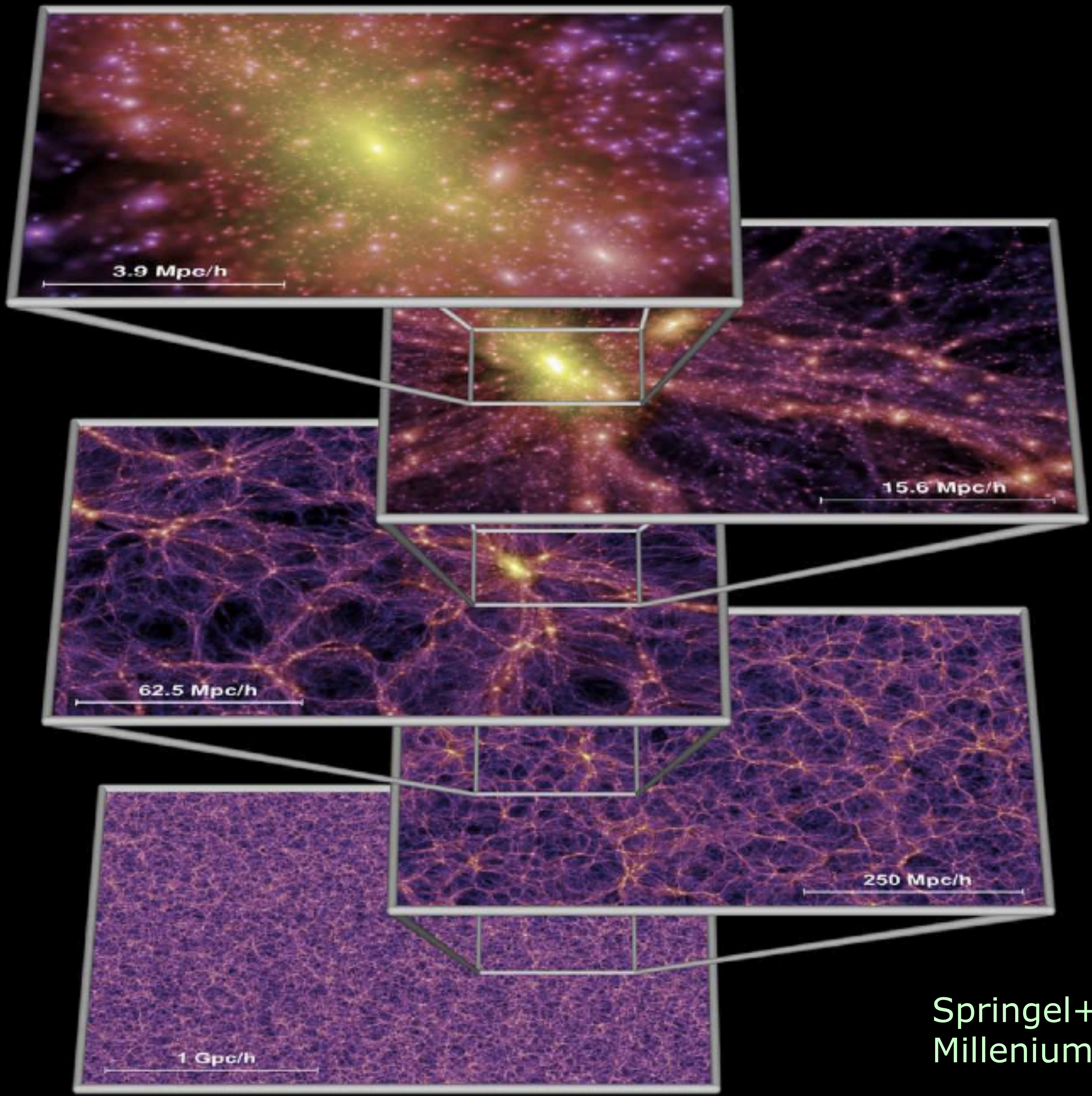
Overview

Earliest fossil of accretion
Press-Schechter approximation
Dark age accretion
Spherical top-hat model
Virialization
Gunn & Gott accretion model
Void evolution
Zel'dovich approximation
Burger's equation
Pichon's asymmetric collapse

"Nature operates by rules we can discover in successive approximations…"

So is there evidence for accretion when atoms formed immediately after decoupling?

We now introduce the Press-Schechter approximation as this has been widely used to calculate HI 21cm signals during the Dark Ages



Springel+ 2005
Millenium

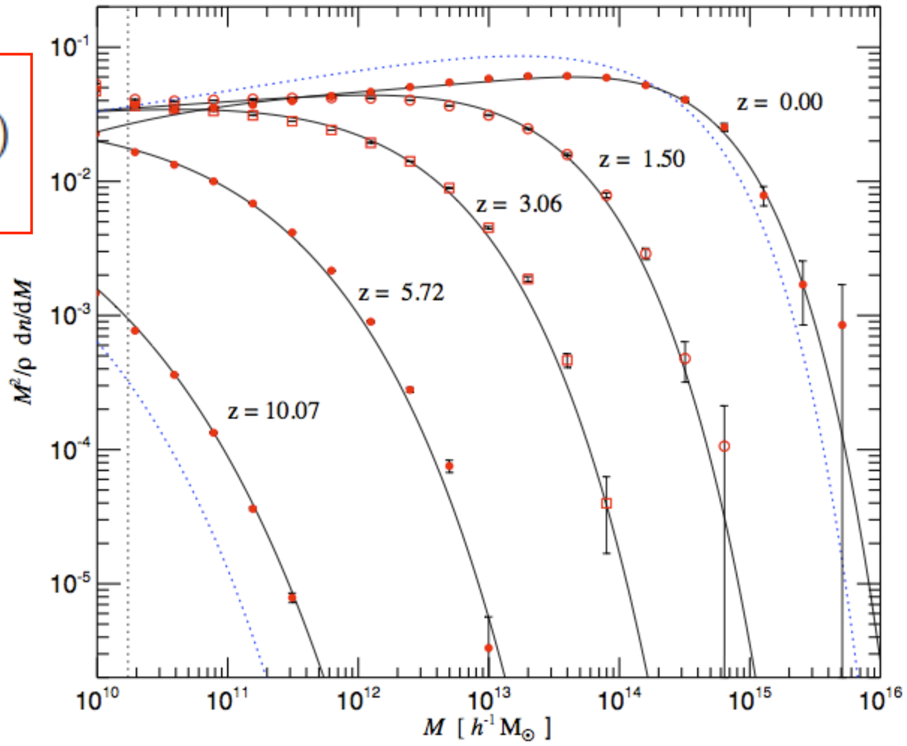
Press-Schechter approximation (1974)

For a **cumulative mass function** *N(>M)* = comoving no. density of bound structures with mass > *M*,

PS arrived at a useful approximation for the **differential mass function** *n(M) = dN(>M) / dM* that is hard to justify rigorously. But it works!

$$n(M) = \frac{\delta_{\rm crit} d\sigma(M)/dM}{\sqrt{2\pi} M \sigma^2(M)} \exp(-\delta_{\rm crit}^2 / 2\sigma^2(M))$$

This formula has a long and chequered history; see convoluted discussions in *Peacock 99*, Longair 07 and Binney & Tremaine 08. An excellent review is *Cooray & Sheth 02.*



Springel+ 2005

Press-Schechter approximation (1974)

To arrive at their formula, they needed two quantities.

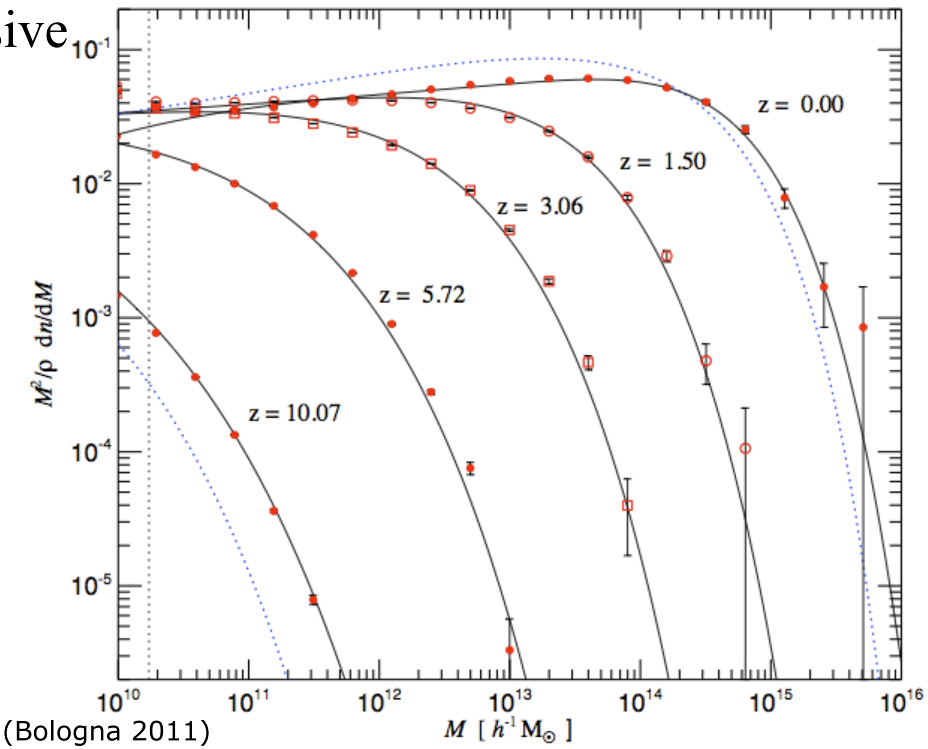
1. Fraction of space *f* where the initial density contrast $\delta > \delta_{crit}$ after it has been filtered with a kernel of mass M (these are non-linear condensations)

$$f(\delta > \delta_{\rm crit}; M) = \int_{\delta_{\rm crit}/\sigma(M)}^{\infty} \frac{d\nu}{\sqrt{2\pi}} \exp(-\nu^2/2).$$

2. Fraction of mass in objects more massive than M,

$$f(> M) = \int_M^{\infty} dM\, M\, n(M)$$

(1) and (2) are differentiated wrt M and made equal to arrive at the PS approx.



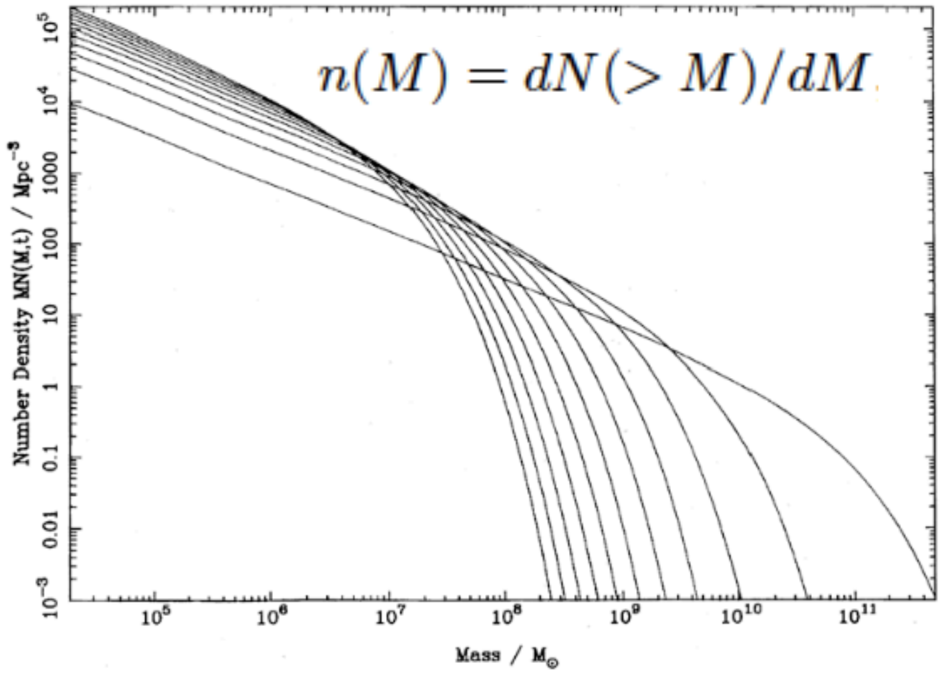
Bland-Hawthorn (Bologna 2011)

Press-Schechter approximation (1974)

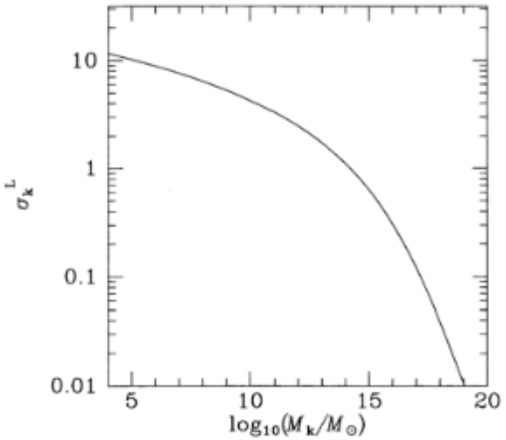
The roll over at large M and the time evolution both come from the factor

$$\delta_{\rm crit}/\sigma(M)$$

CDM variance is smaller at large M, larger at small M, at all times ➔ non-linear evolution



Blain 2007



$$n(m)\mathrm{d}m = \sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}}{m^2}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln m}\right|\frac{\delta_c(z)}{\sigma(m)}\,\exp\left[-\frac{\delta_c^2(z)}{2\sigma^2(m)}\right]\mathrm{d}m.$$

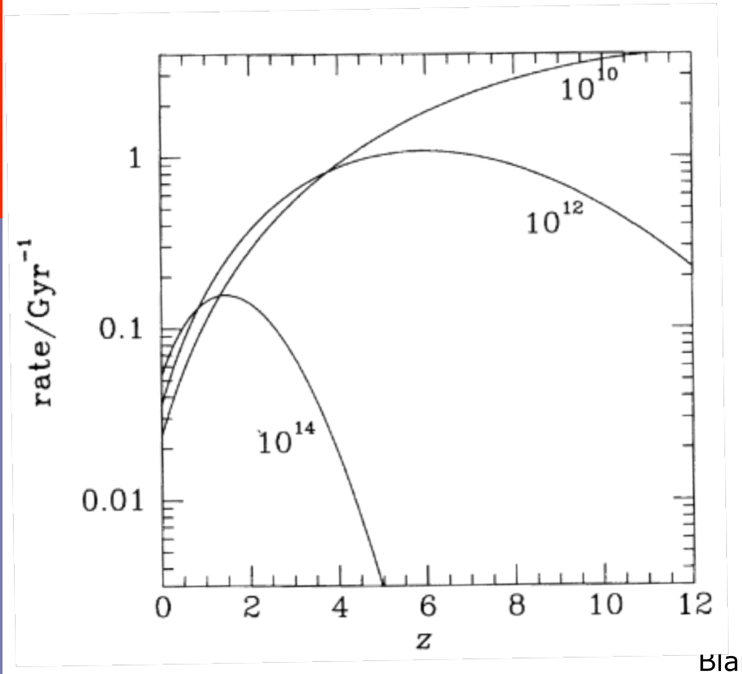
Press-Schechter approximation (1974)

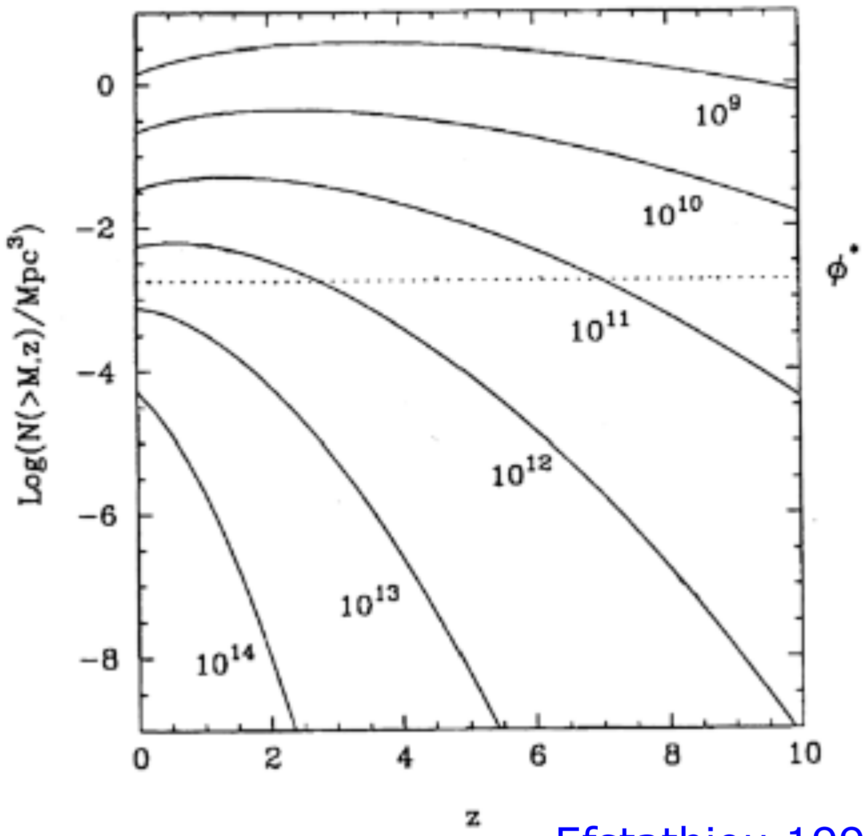
Other projections:

1. evolution of comoving no. density of objects more massive than M

$$n(M) = dN(> M)/dM$$

2. evolution of equal mass mergers





Efstathiou 1995

Lacey & Cole (1993): open source code "extended" to identify progenitors in merger trees

Sheth and van de Weygaert 2004

Good discussion, probably
clearer than BT08, Longair07
on EPS, voids, clouds, etc. Nice
illustrations also.

Work up in future lectures.

524 *R. K. Sheth and R. van de Weygaert*

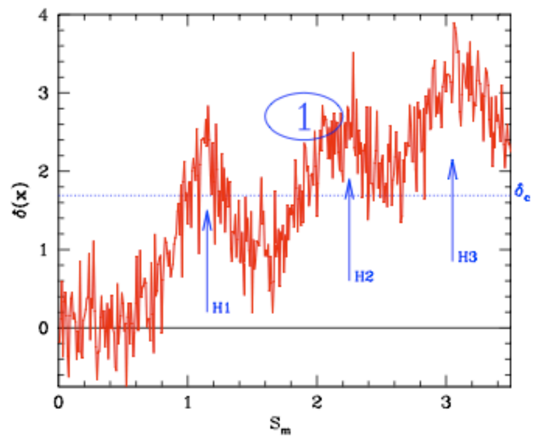


Figure 5. Excursion set formalism, illustrated for the formation of a halo. Random walk exhibited by the average overdensity δ centred on a randomly chosen position in a Gaussian random field, as a function of smoothing scale, parametrized by S_m (large volumes are on the left, small volumes on the right). Dashed horizontal line indicates the collapse barrier δ_c. The largest scale (smallest value of S) on which $\delta(S)$ exceeds δ_c is an estimate of the mass of the halo that will form around that region.

A hierarchy of voids 525

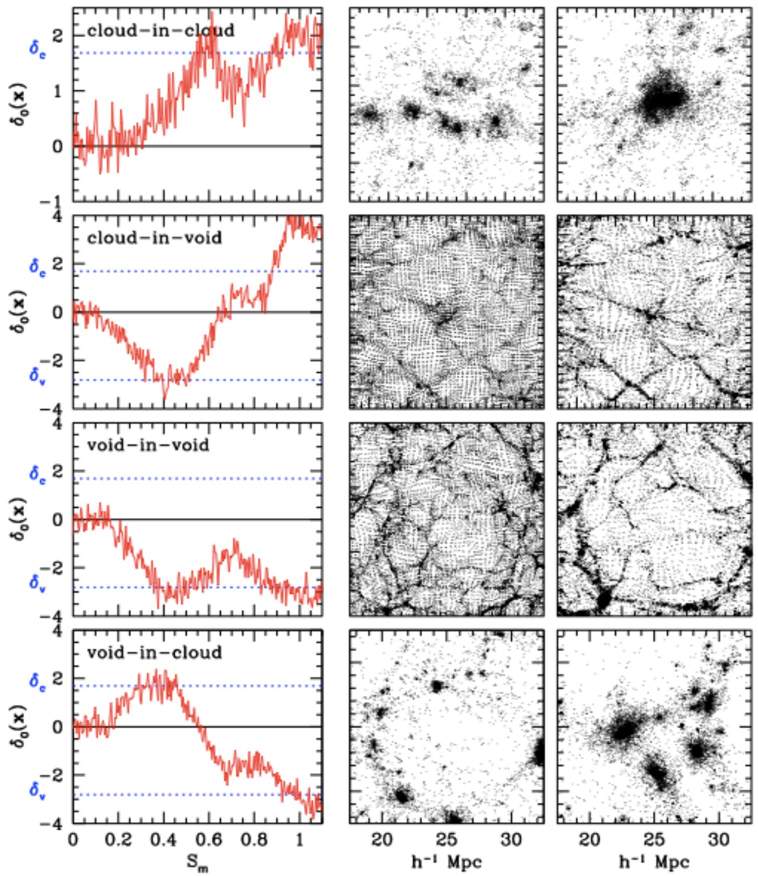


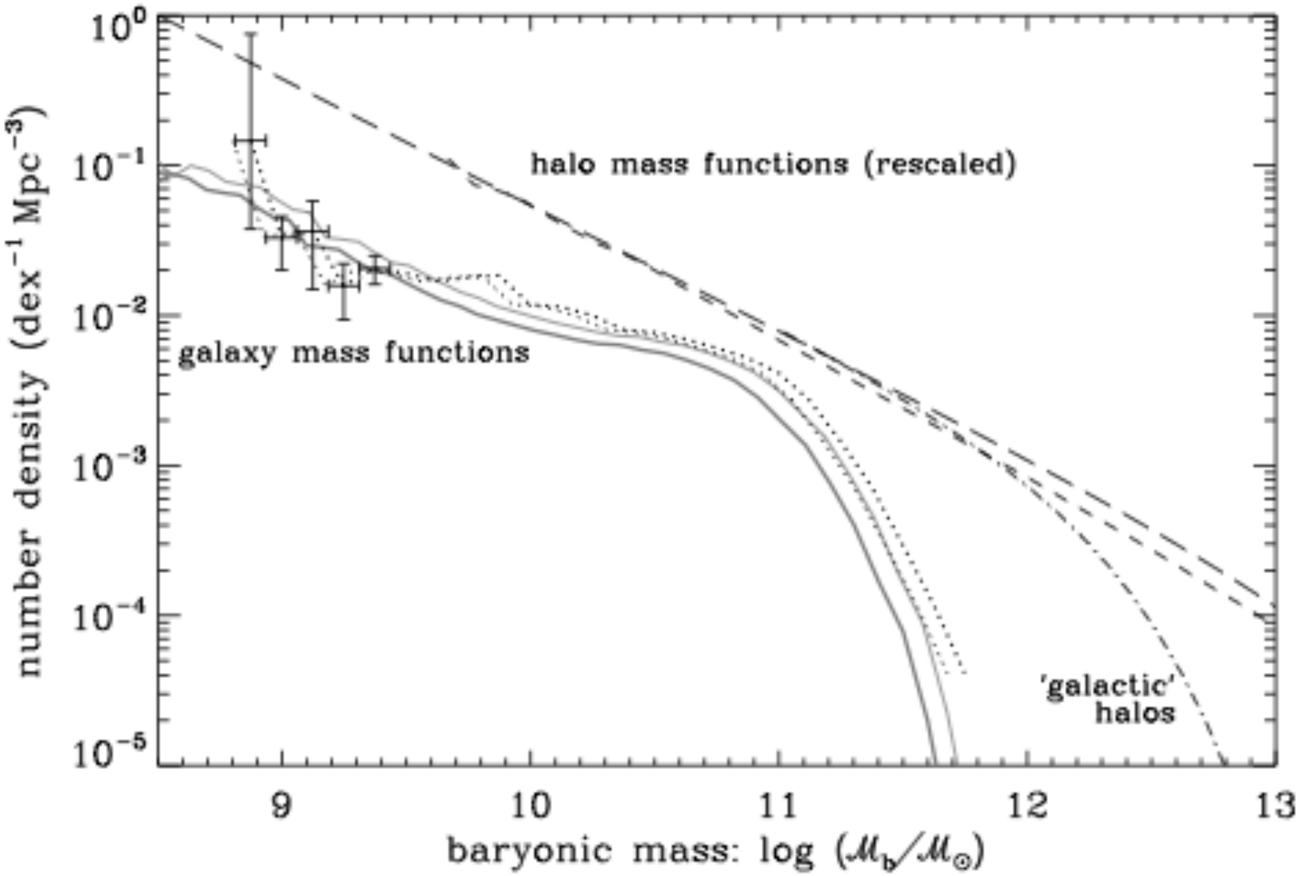
Figure 6. Four-mode (extended) excursion set formalism. Each row illustrates one of the four basic modes of hierarchical clustering: the *cloud-in-cloud* process, *cloud-in-void* process, *void-in-void* process and *void-in-cloud* process (from top to bottom). Each mode is illustrated using three frames. Leftmost panels show 'random walks': the local density perturbation $\delta_0(\boldsymbol{x})$ as a function of (mass) resolution scale S_m (cf. Fig. 5) at an early time in an N-body simulation of cosmic structure formation. In each graph, the dashed horizontal lines indicate the *collapse barrier* δ_c and the shell-crossing *void barrier* δ_v. The two columns on the right show how the associated particle distribution evolves. Whereas haloes within voids may be observable (second row depicts a halo within a larger void), voids within collapsed haloes are not (last row depicts a small void which will be squeezed to small size as the surrounding halo collapses). It is this fact which makes the calculation of void sizes qualitatively different from that usually used to estimate the mass function of collapsed haloes.

FOOTNOTE: Biggest fluctuations collapse earliest with PS plot to show too...

- Best derivation is probably BT08 ch. 9, L07 also good

FOOTNOTE:

PS does **not** match galaxy or cluster mass functions today… a portent of gas processes to come.



Bland-Hawthorn (Bologna 2011)

Mon. Not. R. Astron. Soc. **325,** 468–482 (2001)

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A model

truncated

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[2003MNRAS.341...81I]

Mon. N

Cosmology at Low Frequencies: The 21 cm Transition and the High-Redshift Universe

2006

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Non-linear clustering during the cosmic Dark Ages and its effect on the 21-cm background from minihaloes

Ilian T. Iliev,[1]★ Evan Scannapieco,[1] Hugo Martel[2] and Paul R. Shapiro[2]

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[2]*Department of Astronomy, University of Texas, Austin, TX 78712-1083, USA*

sphere and the *singular* isothermal sphere approximations for the post-collapse objects. The

recently proposed to resolve this conflict.

Dark Ages: z~1089 to z~200, i.e. 370,000 to 6 million years?

What options do we have to see anything post recombination?

THE
© 200

Phys. Rev. D 72, 083002 (2005) [9 pages]

Ionizing radiation from hydrogen recombination strongly suppresses the lithium scattering signature in the CMB

Abstract | References | Citing Articles (10)

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Eric R. Switzer* and Christopher M. Hirata
Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

Received 6 July 2005; published 7 October 2005

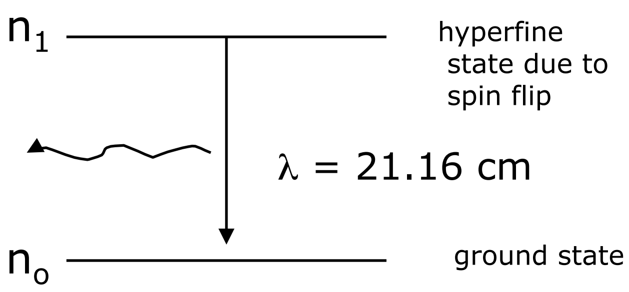
It has been suggested that secondary CMB anisotropies generated by neutral lithium could open a new observational window into the universe around the redshift $z\sim 400$, and permit a determination of the primordial lithium abundance. The effect is due to resonant scattering in the allowed Li I doublet ($2s^2S_{1/2}$-$2p^2P_{1/2,3/2}$), so its observability depends on the formation history of neutral lithium. Here we show that the ultraviolet photons produced during hydrogen recombination are sufficient to keep lithium in the Li II ionization stage in the relevant redshift range and suppress the neutral fraction by ~ 3 orders of magnitude from previous calculations, making the lithium signature unobservable.

We have only the HI 21cm hyperfine spin-flip transition

Dark Ages: a serious black out from z~1089 to z~200?

In a neutral hydrogen universe, we really have only T_s to work with and it needs to be different from T_{CMB}

$$\frac{n_1}{n_0} = 3\exp\left(-\frac{T_*}{T_S}\right)$$



CMB photons keep T_S close to T_{CMB} until z~200 through ICS so the universe is effectively dark to us.

To detect HI, we need to decouple, but how?

$$T_S = \frac{T_{\rm CMB} + y_\alpha T_\alpha + y_c T_K}{1 + y_\alpha + y_c}$$

Two ways:

(A) Spin exchange between H atoms (Purcell & Field 1956)

efficiency depends on local T_K and density

(B) Lyα pumping which then decays to hyperfine level (Field-Wouthuysen Effect)

efficiency depends on local UV field (T_α = Lyα temperature)

So what about accretion after decoupling?

The expanding universe was filled with cooling CMB and cooling HI…

CDM density

Baryon density

Baryon temperature (T_K)

Photon temperature (T_{CMB})

Why the Dark Ages?

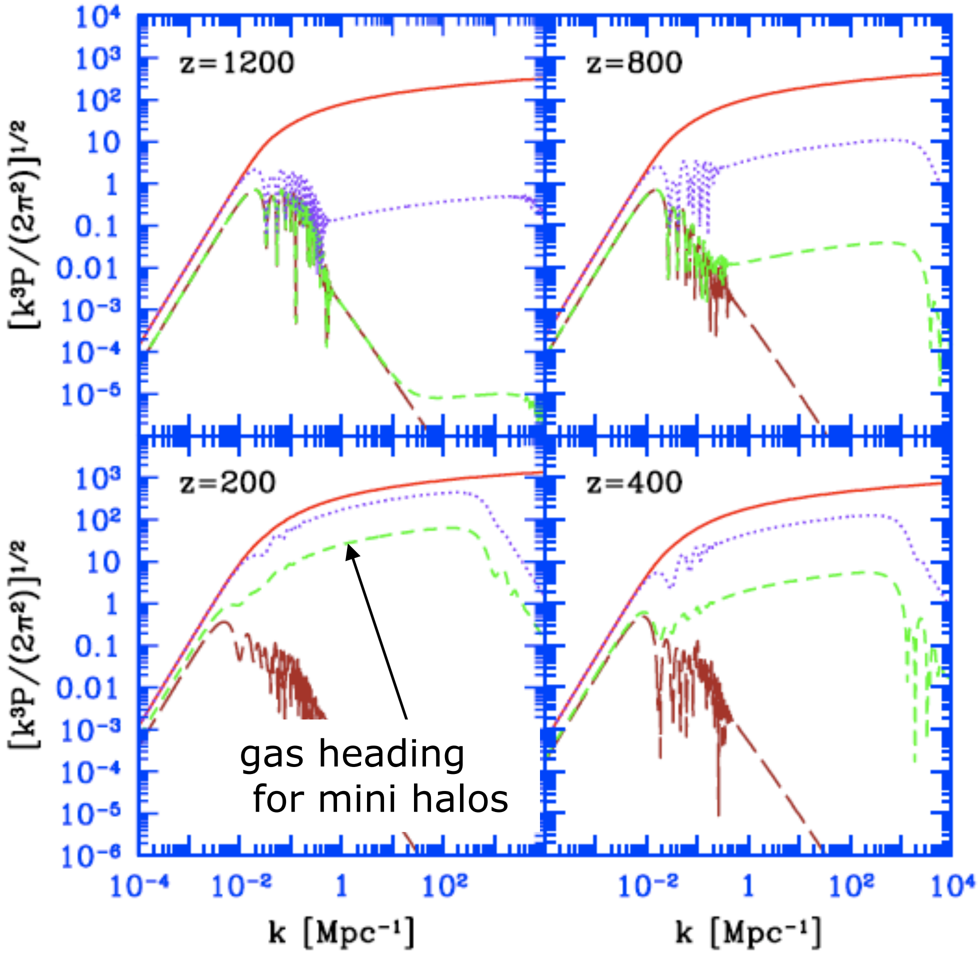
T_K is bound on largest scales to T_{CMB} by ICS until z~200 but decouples after that.

Then $T_S \sim T_K < T_{CMB}$ so HI seen absorption against CMB until z~30.

After that, collisions are rare, HI invisible again, except that H_2 cooling inside mini-haloes starts?

First stars, Lyα pumping, HI emission.

Power spectrum of n, T fluctuations vs. co-moving wavenumber



nature

article

Nature **216**, 976 - 978 (09 December 1967); doi:10.1038/216976a0

Molecular Hydrogen in Pre-galactic Gas Clouds

WILLIAM C. SASLAW & DAVID ZIPOY

Department of Applied Mathematics and Theoretical Physics, University of Cambridge

During the collapse of pre-galactic gas clouds through a density of about 10^4 particles/cm^3, hydrogen molecules are produced and dominate the subsequent cooling.

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15. Partridge, B. , and Peebles, P. J. E. , *Astrophys. J.*, **148**, 377 (1967). | Article | ISI |

Also Doug Lin, Saas Fee lectures; picked up by Tom Abel in his work (but not acknowledged?).

$z \sim 150$: T_K falls below T_{CMB} for the first time

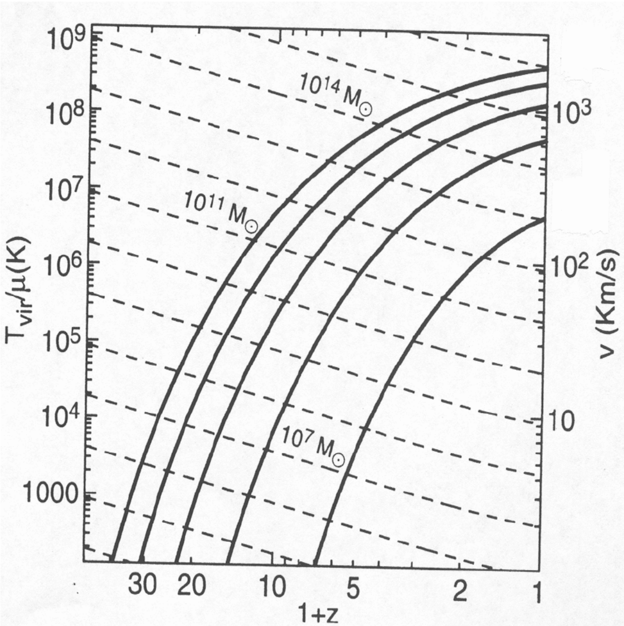
i.e. mean density is sufficient to collisionally couple T_S to T_K such that HI universe is mostly in absorption…

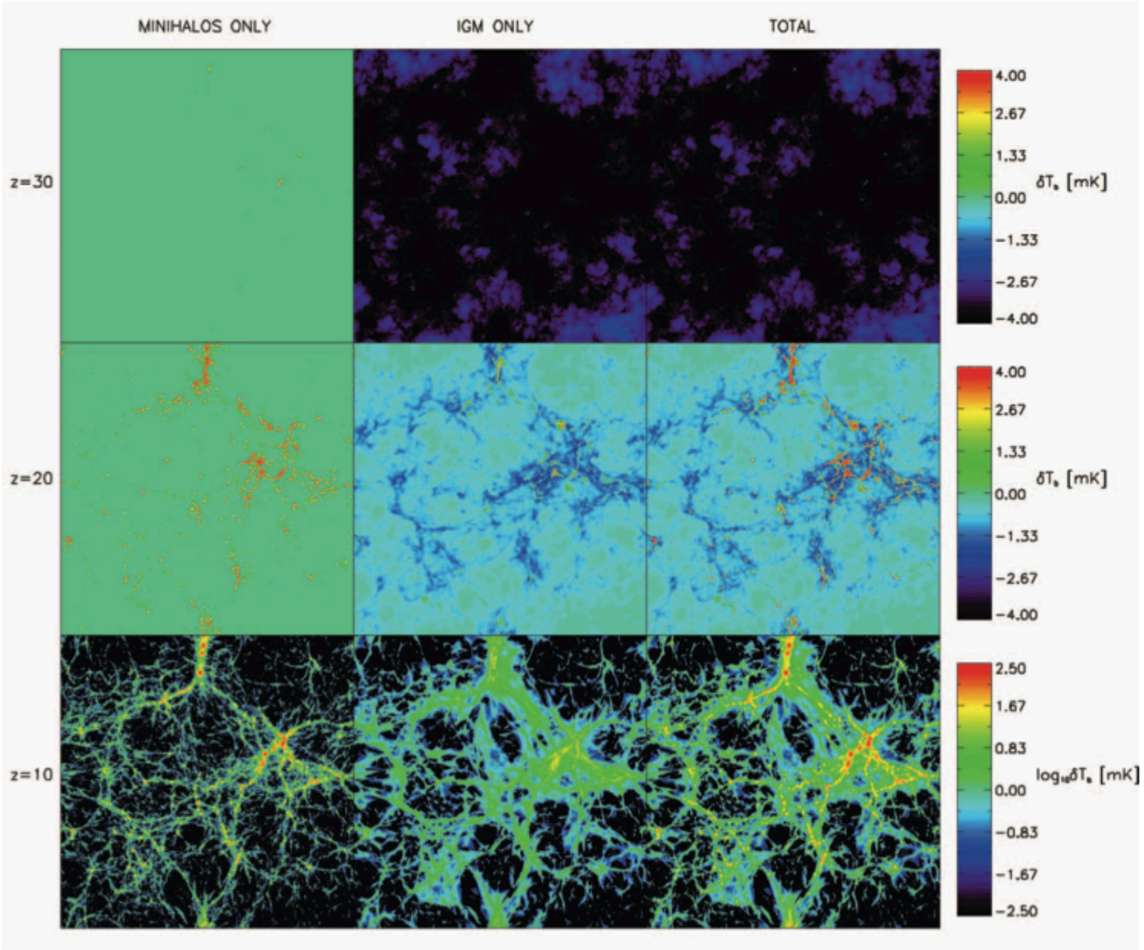
Furlanetto & Loeb (2004): strongest HI **emission** from gas compressing into large-scale IGM shocks

Shapiro et al (2006): strongest HI **emission** from gas compressing into mini halos ($10^4 < M < 10^8$); they find large-scale shocks of lesser importance

$\delta T = T_S - T_{CMB}$

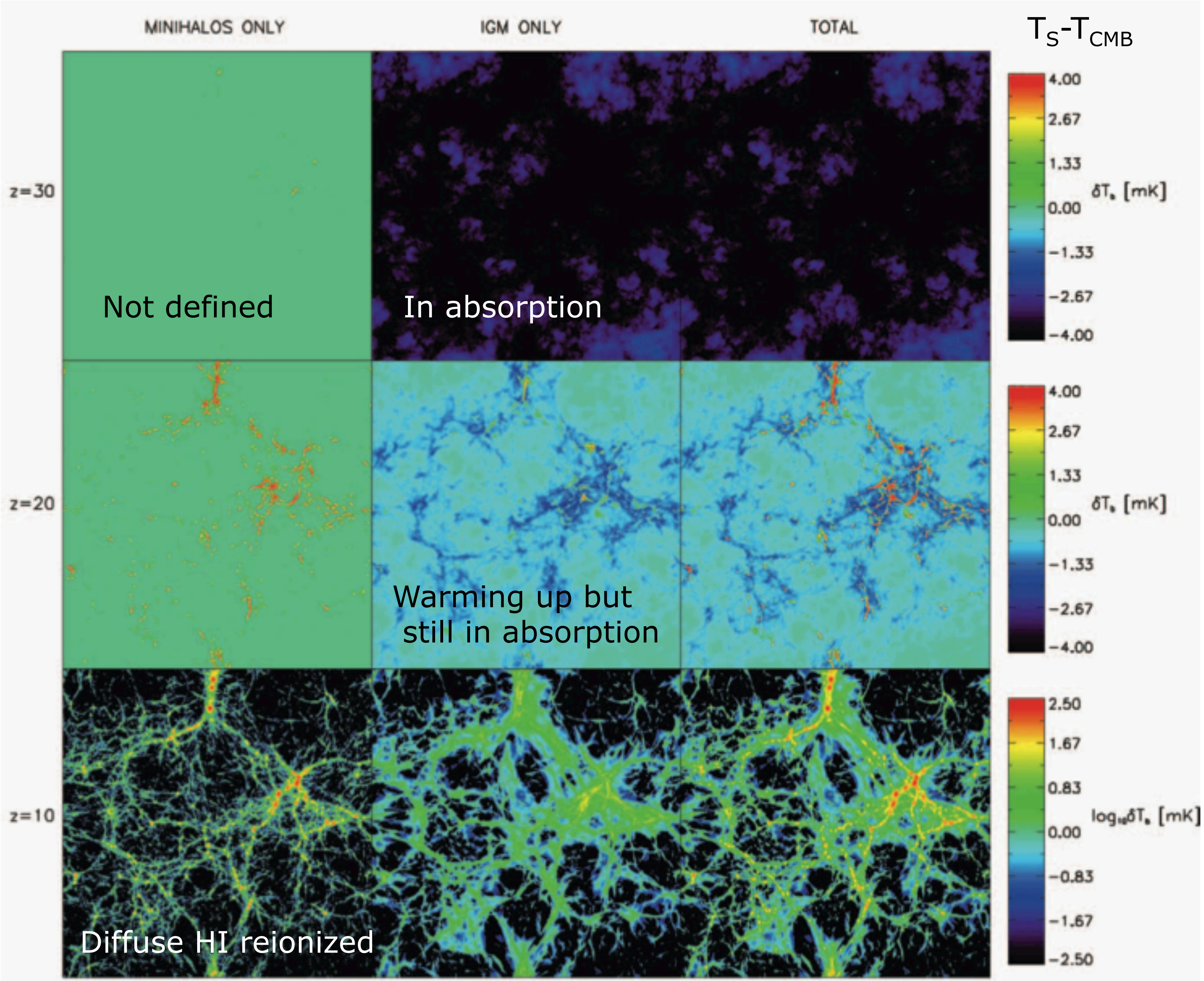
0.5 Mpc co-moving box





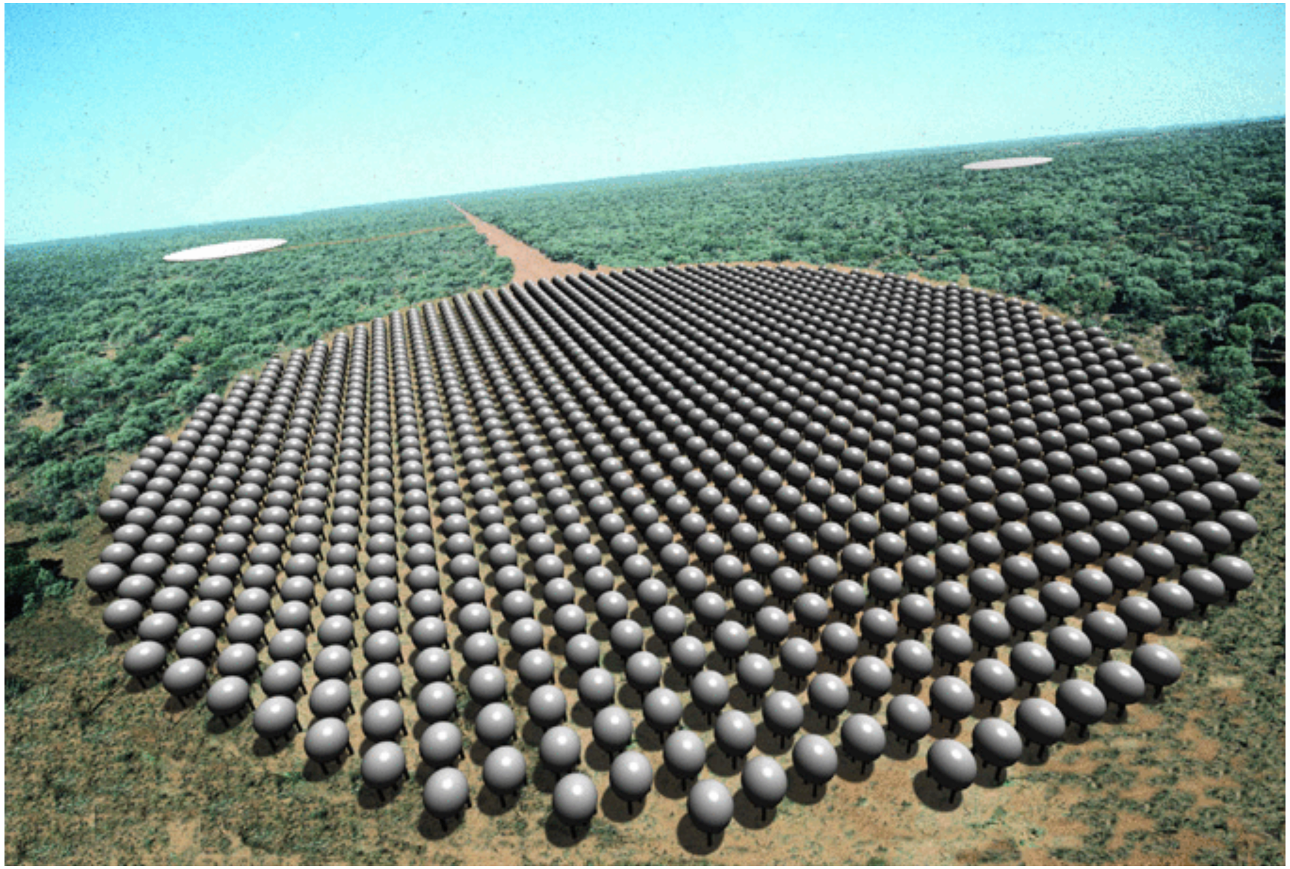
Bla

z<20: Lyα pumping from first sources takes over…



MINIHALOS ONLY
IGM ONLY
TOTAL
T_S-T_{CMB}
z=30
z=20
z=10
Not defined
In absorption
Warming up but
still in absorption
Diffuse HI reionized
4.00
2.67
1.33
0.00
−1.33
−2.67
−4.00
δT_b [mK]
2.50
1.67
0.83
0.00
−0.83
−1.67
−2.50
log₁₀δT_b [mK]

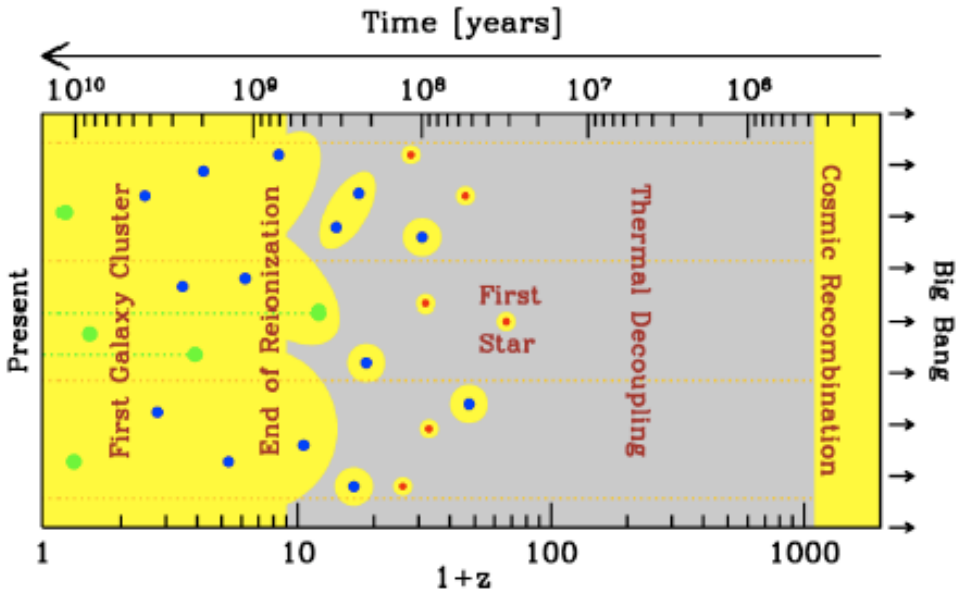
Main science driver of Square Kilometer Array (SKA) ~ 2020+

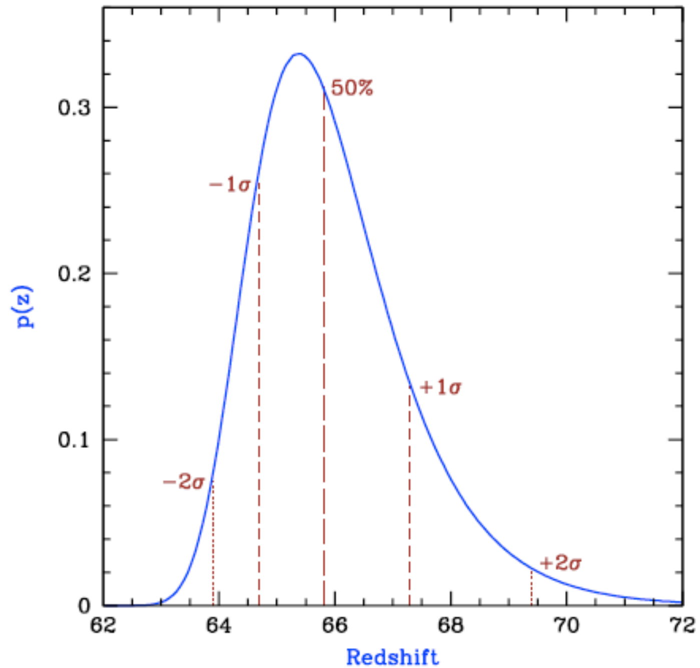


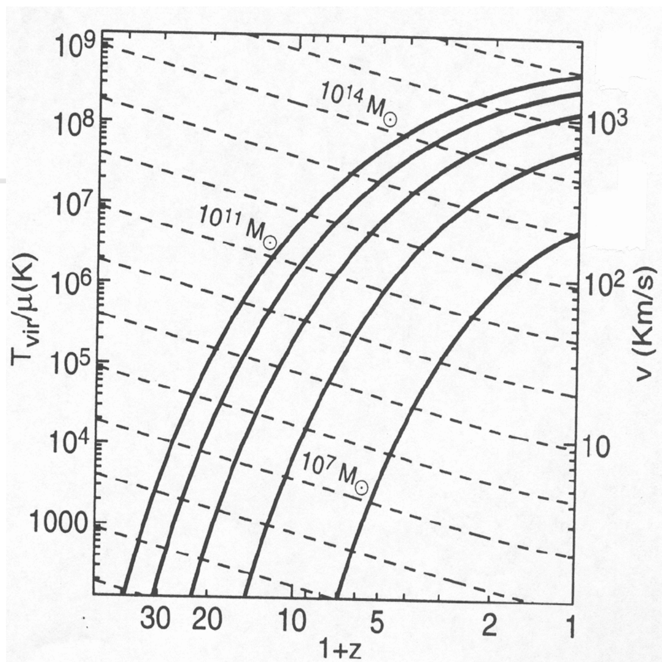
Accretion studies will be greatly advanced after the SKA comes on line.

We will need the intervening decade+ to properly treat gas physics in cosmological simulations. **This is a topic of the future!**

When did the first star form?







Maybe as early as z~65?

But certainly by z~20-30

Bland-Hawthorn (Bologna 2011)

NATURE | LETTER

◂ previous abstract next abstract ▸

FAR FIELD

T_o=12.90 Gyr

A luminous quasar at a redshift of $z = 7.085$

Daniel J. Mortlock, Stephen J. Warren, Bram P. Venemans, Mitesh Patel, Paul C. Hewett, Richard G. McMahon, Chris Simpson, Tom Theuns, Eduardo A. Gonzáles-Solares, Andy Adamson, Simon Dye, Nigel C. Hambly, Paul Hirst, Mike J. Irwin, Ernst Kuiper, Andy Lawrence & Huub J. A. Röttgering

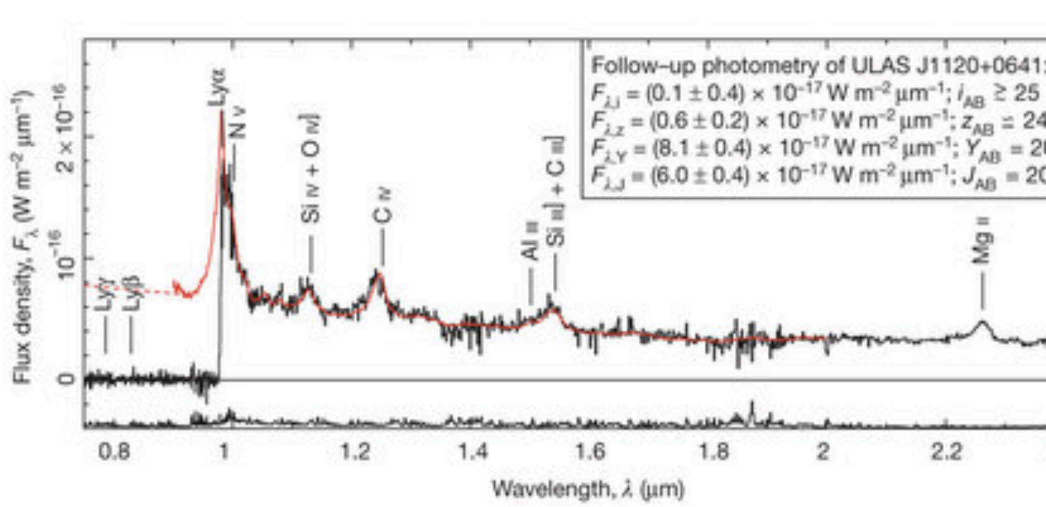
Affiliations | Contributions | Corresponding author

Nature **474**, 616–619 (30 June 2011) | doi:10.1038/nature10159
Received 11 March 2011 | Accepted 28 April 2011 | Published online 29 June 2011

The intergalactic medium was not completely reionized until approximately a billion years after the Big Bang, as revealed[1] by observations of quasars with redshifts of less than 6.5. It has been difficult to probe to higher redshifts, however, because quasars have historically been identified[2, 3, 4] in optical surveys, which are insensitive to sources at redshifts exceeding 6.5. Here we report observations of a quasar (ULASJ112001.48+064124.3) at a redshift of 7.085, which is 0.77 billion years after the Big Bang. ULASJ1120+0641 has a luminosity of $6.3\times10^{13}L_{\odot}$ and hosts a black hole with a mass of $2\times10^{9}M_{\odot}$ (where $L_{\odot}$ and $M_{\odot}$ are the luminosity and mass of the Sun). The measured radius of the ionized near zone around ULASJ1120+0641 is 1.9megaparsecs, a factor of three smaller than is typical for quasars at redshifts between 6.0 and 6.4. The near-zone transmission profile is consistent with a Lyα damping wi[…] neutral fraction of the intergalactic medium in front of ULASJ1120+0641 exceed[…]

Subject terms: Astronomy • Physics

Figure 1: Spectrum of ULAS J1120+0641 and a composite spectrum [...]



Blueward of 1.005μm the spectrum was obtained using the FORS2 on t[…] 1.0" wide longslit and the 600z holographic grism, which has a resolutio[…] 1.6×10^{-4}μm per...

Letter

Nature **443**, 186-188 (14 September 2006) | doi:10.1038/nature0510
18 July 2006

FAR FIELD

T_o=12.88 Gyr

A galaxy at a redshift *z* = 6.96

Masanori Iye[1,2,3], Kazuaki Ota[2], Nobunari Kashikawa[1], Hisanori Furusawa[4], Tetsuya Hashimoto[2], Takashi Hattori[4], Yuichi Matsuda[5], Tomoki Morokuma[6], Masami Ouchi[7] & Kazuhiro Shimasaku[2]

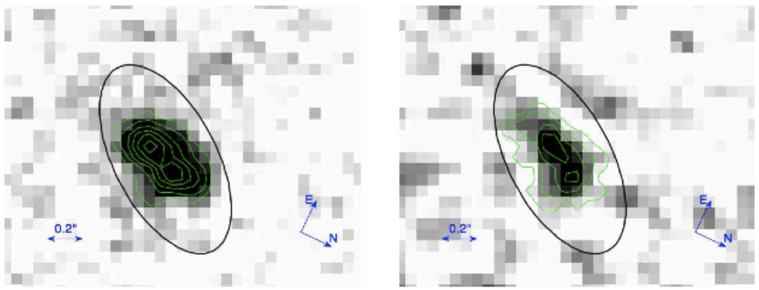
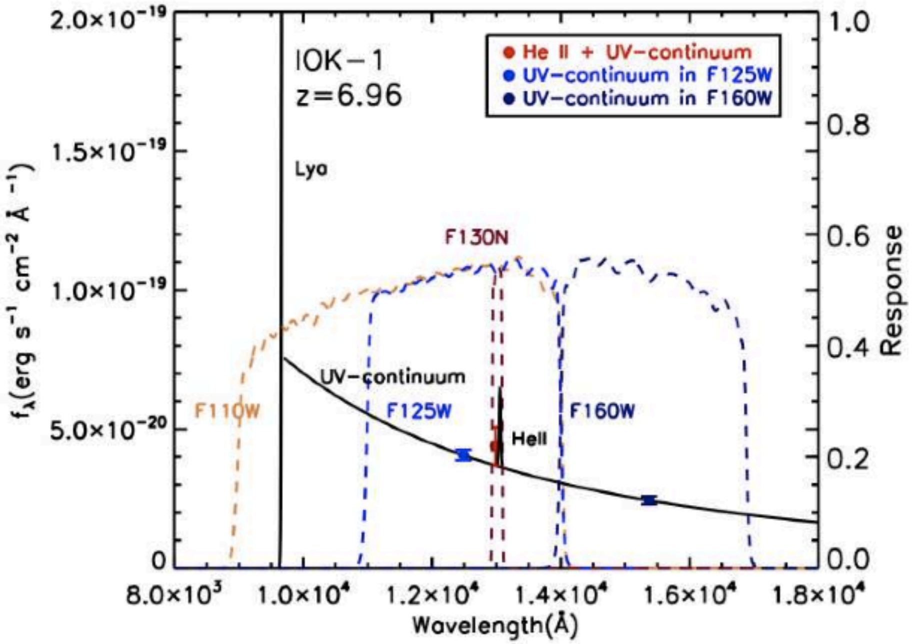


Fig. 1.— High resolution images of galaxy IOK-1 in the F125W (left) and the F130N (right) bands, with contours spaced by 1.4 sky rms for the F125W image and 1 sky rms for the F130N image. The black elliptical aperture is determined by the F130N image. IOK-1 is clearly resolved into two components separated by $\sim 0.''2$. Photometric analysis of the F130N image reveals a flux density excess of 1.2σ.



Draft version August 15, 2011
Preprint typeset using LaTeX style emulateapj v. 11/10/09

SPECTROSCOPIC CONFIRMATION OF TWO LYMAN BREAK GALAXIES AT REDSHIFT BEYOND 7

E. Vanzella,[1] L. Pentericci[2], A. Fontana[2], A. Grazian[2], M. Castellano[2], K. Boutsia[2], S. Cristiani[1], M. Dickinson[3], S. Gallozzi[2], E. Giallongo[2], M. Giavalisco[4], R. Maiolino[2], A. Moorwood[5], D. Paris[2], and P. Santini[2]
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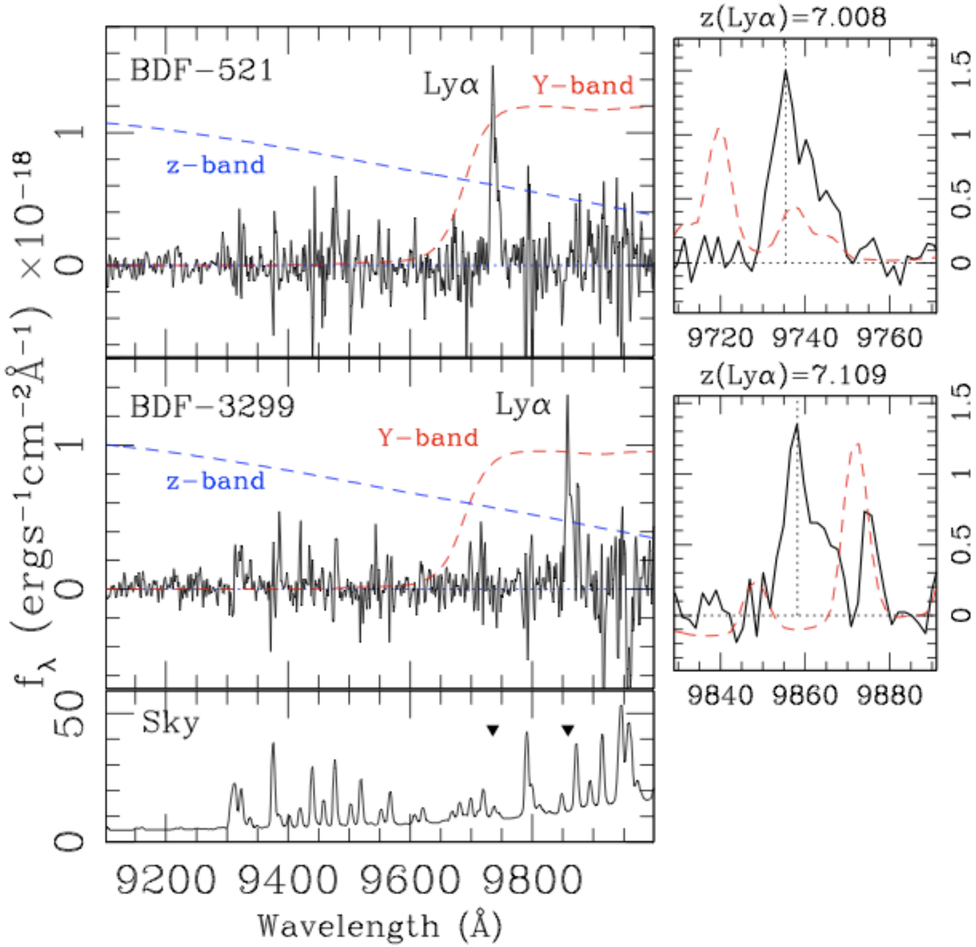
[2]INAF Osservatorio Astronomico di Roma, Via Frascati 33,00040 Monteporzio (RM), Italy
[3]National Optical Astronomy Observatory, PO Box 26732, Tucson, AZ 85726, USA
[4]Department of Astronomy, University of Massachusetts, 710 North Pleasant Street, Amherst, MA 01003 and
[5]European Southern Observatory, Karl-Schwarzschild Strasse, 85748 Garching bei Munchen, German
Draft version August 15, 2011

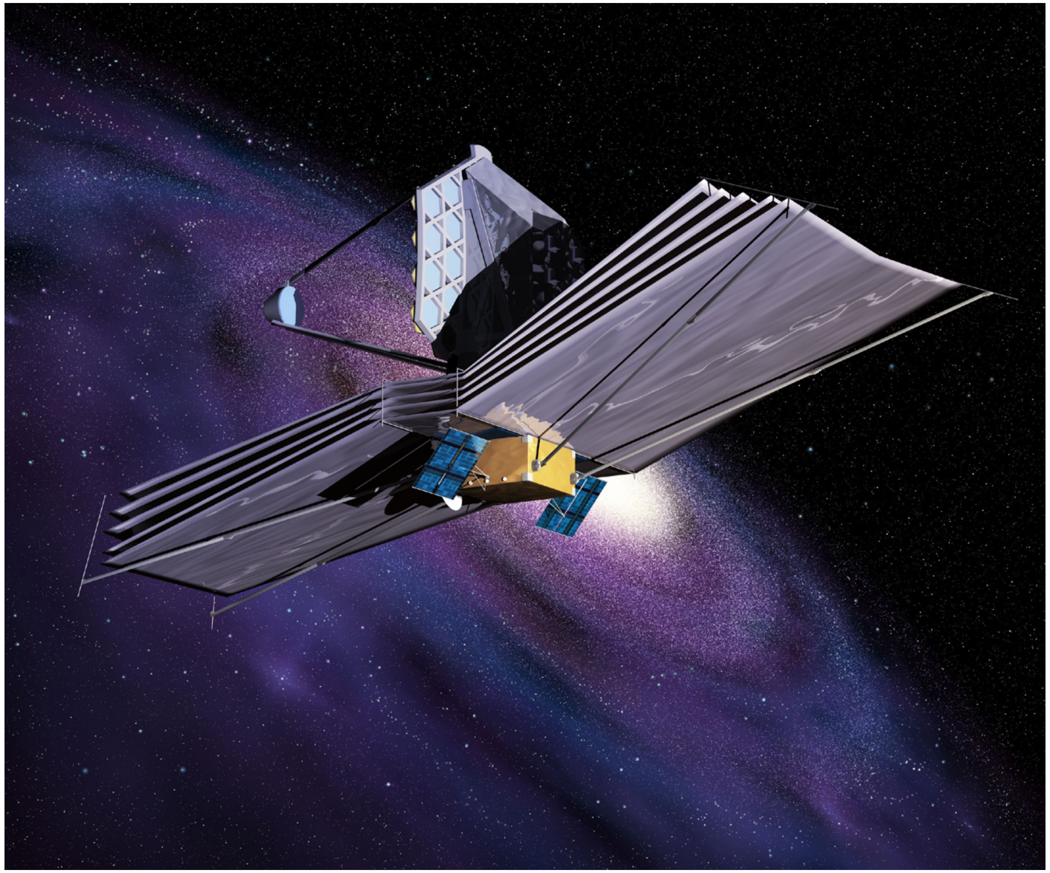


ABSTRACT

f two Lyman break galaxies at redshift > 7. The galaxies
oscopic campaign with FORS2 at the ESO/VLT for the
didates selected from our VLT/Hawk-I imaging survey.
at 9735Å and 9858Å respectively: the lines have fluxes
ibit a sharp decline on the blue side and a tail on the
comparable to the observed asymmetry in $z \sim 6$ Lyα
in the IGM truncates the blue side of the emission line
hat the galaxies are instead at lower redshift and we are
: however from the spectroscopic and the photometric
lausible identifications, except for Lyα at redshift > 7,
st redshift determination for galaxies in the reionization
and photometry, we derive limits on the star formation
he two galaxies. We argue that these two galaxies alone
surroundings.
shifts - galaxies: high-redshift - galaxies: formation

is this believable?

Main science driver of the James Webb Space Telescope (JWST) ~ 2018+



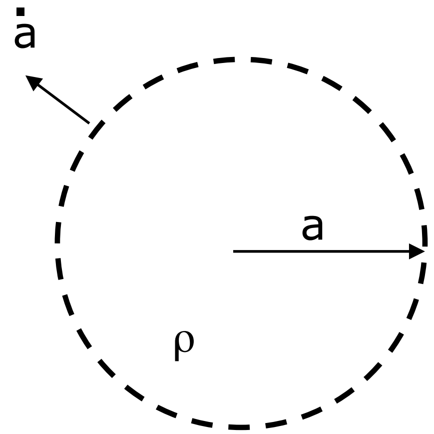
How to think really big!

NASA appears to be launching this satellite out to z ~ 30 kpc

I will now introduce some theoretical concepts that have been useful in understanding how non-linear structure forms out of linear perturbations.

We will meet important ideas that are used in comparing simulations with data.

Some basic theory:



We consider how a density perturbation evolves in the expanding universe. Consider a uniformly expanding sphere with scale factor (i.e. radius) a and mean density ρ. The sphere obeys the CONTINUITY EQUATION $\rho a^3 =$ constant.

BIRCHOFF's THEOREM:

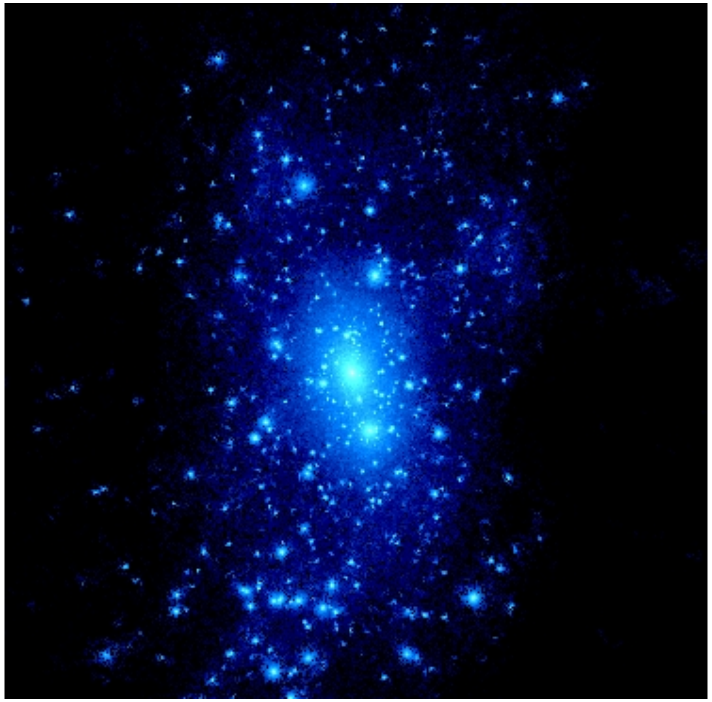
Newtonian gravity provides the correct description of a finite sphere in the limit $a \to 0$. This is because of Birchoff's theorem (the relativistic equivalent of Gauss' law): for a spherically symmetric mass distribution, the gravity within some shell is independent of the matter distribution outside.

ACCELERATION EQUATION:

From Gauss' Law, the acceleration of particles at the surface of the expanding sphere is

$$\ddot{a} = -\frac{4}{3}\pi G \rho a \tag{1}$$

Bland-Hawthorn (Bologna 2011)



ENERGY EQUATION:

We now integrate (1) to get the energy equation

$$\dot{a}^2 = \frac{8}{3}\pi G\rho a^2 + 2E_o \tag{2}$$

where E_o is a constant. A solution to (2) is

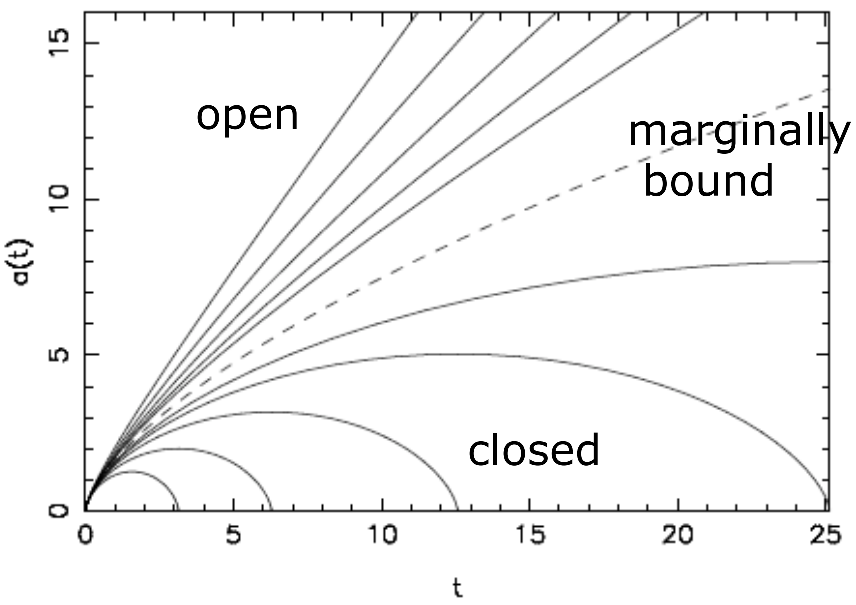
$$a(\eta) = \frac{GM}{2|E_o|}(1-\cos\eta)$$

$$t(\eta) = \frac{GM}{(2|E_o|)^{3/2}}(\eta - \sin\eta)$$

such that the expansion velocity is

$$\dot{a} = \frac{da/d\eta}{dt/d\eta} = \sqrt{2|E_o|}\frac{\sin\eta}{1-\cos\eta}$$

where $M = \frac{4}{3}\pi\rho a^3$.



HUBBLE PARAMETER AND CRITICAL DENSITY:

The expansion rate is

$$H = \dot{a}/a \tag{5}$$

The critical density for a marginally bound universe (Einstein-de Sitter, $\Omega \approx 1$, $\Lambda \approx 0$) is

$$\rho_{\rm crit} = \frac{3H^2}{8\pi G} \tag{6}$$

In EdS universe, $a(t) \propto t^{2/3}$

We will adopt EdS universe as it simplifies algebra…

Bland-Hawthorn (Bologna 2011)

THE SPHERICAL TOP-HAT PERTURBATION:

Excise a sphere of matter M and replace it with a smaller sphere with identical M. The perturbation obeys (2).

Since M is fixed, $\rho_1 R_1(t)^3 = \rho_2 R_2(t)^3$, and we can write

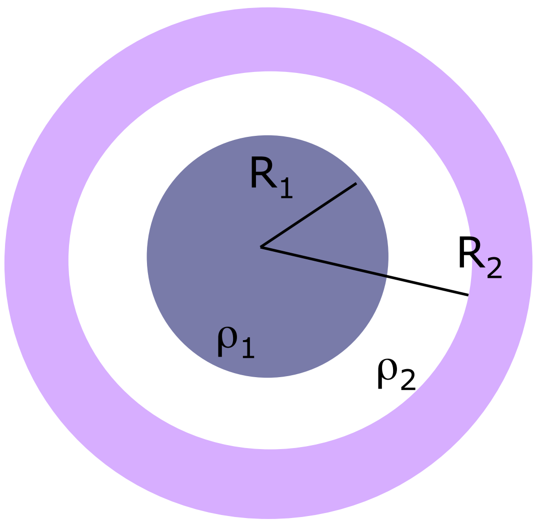
$$\rho_1 = \rho_2\,(1 + \delta\rho/\rho_2) \tag{7}$$

For a small perturbation at a given time ($\delta\rho/\rho \ll 1$),

$$\frac{\delta\rho}{\rho} = -3\,\frac{\delta R}{R} \tag{8}$$

In EdS ($a \propto t^{2/3}$), the energy constant E_o is always negligible and can only dominate at late times in a low density universe.

Spherical Collapse



The inner radius peaks at the "turnaround time"

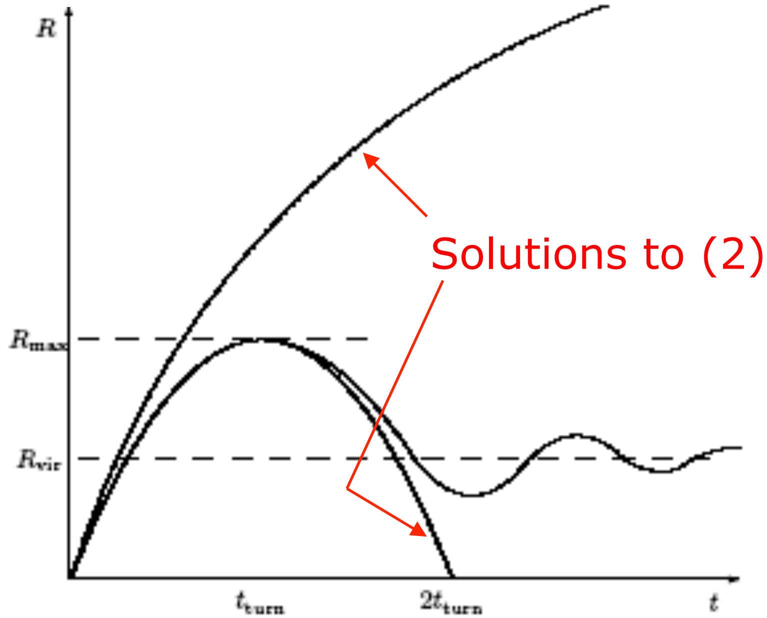
$$t_{\rm turn} = \frac{\pi G M}{(2|E_o|)^{3/2}}$$

when the density is

$$\rho = \frac{3M}{4\pi R_{\rm max}^3} = \frac{3\pi}{32 G t^2}$$

Compare with EdS,

$$\rho_o = \frac{1}{6\pi G t^2}$$



Bland-Hawthorn (Bologna 2011)

In EdS, the overdense perturbation will turn around with density contrast

$$(\rho/\rho_o)_{\rm turn} = \frac{9\pi^2}{16} = 5.55 \qquad (13)$$

A perfectly spherical overdensity would collapse to a black hole at $2t_{\rm turn}$. More realistically, a collapsing "blob" becomes increasingly aspherical and virialized after collapsing by a factor of 2 in radius. The system will virialize with a density ~ 8 times larger than at turnaround.

VIRIALIZATION: reaching a state where the radius is no longer contracting.

In the time frame $t_{\rm turn}$ to $2t_{\rm turn}$, the EdS background has expanded by $2^{2/3}$ and will therefore have decreased in density by a factor 4, giving a density contrast of

$$(\rho/\rho_o)_{\rm virial} = 18\pi^2 = 5.55 \times 8 \times 4 = 180 \qquad (14)$$

So the virial radius of a halo is defined to be the radius at which the density is about 200 times the critical density. Matter has mostly stopped collapsing within here.

While we derived this for a matter-dominated EdS universe, the scale radius is about the same in most other cosmologies of interest.

Bland-Hawthorn (Bologna 2011)

The only thing the interior "knows" about the exterior is the time since the Big Bang. For a virialized object,

$$v_{\rm circ}^2 = \frac{GM}{R_{180}} = 180 \times \frac{4}{3}\pi G \rho_o R_{180}^2$$
$$= 90 H^2 R_{180}^2$$

$$\sigma_v = v_{\rm circ}/\sqrt{2} = 7\ H\ R_{180} \qquad (15)$$

We see this in the definition of the NFW halo

A rich cluster has $\sigma_v \sim 1000$ **km s**$^{-1}$, such that $R_{180} \approx 1.9$ **Mpc.**

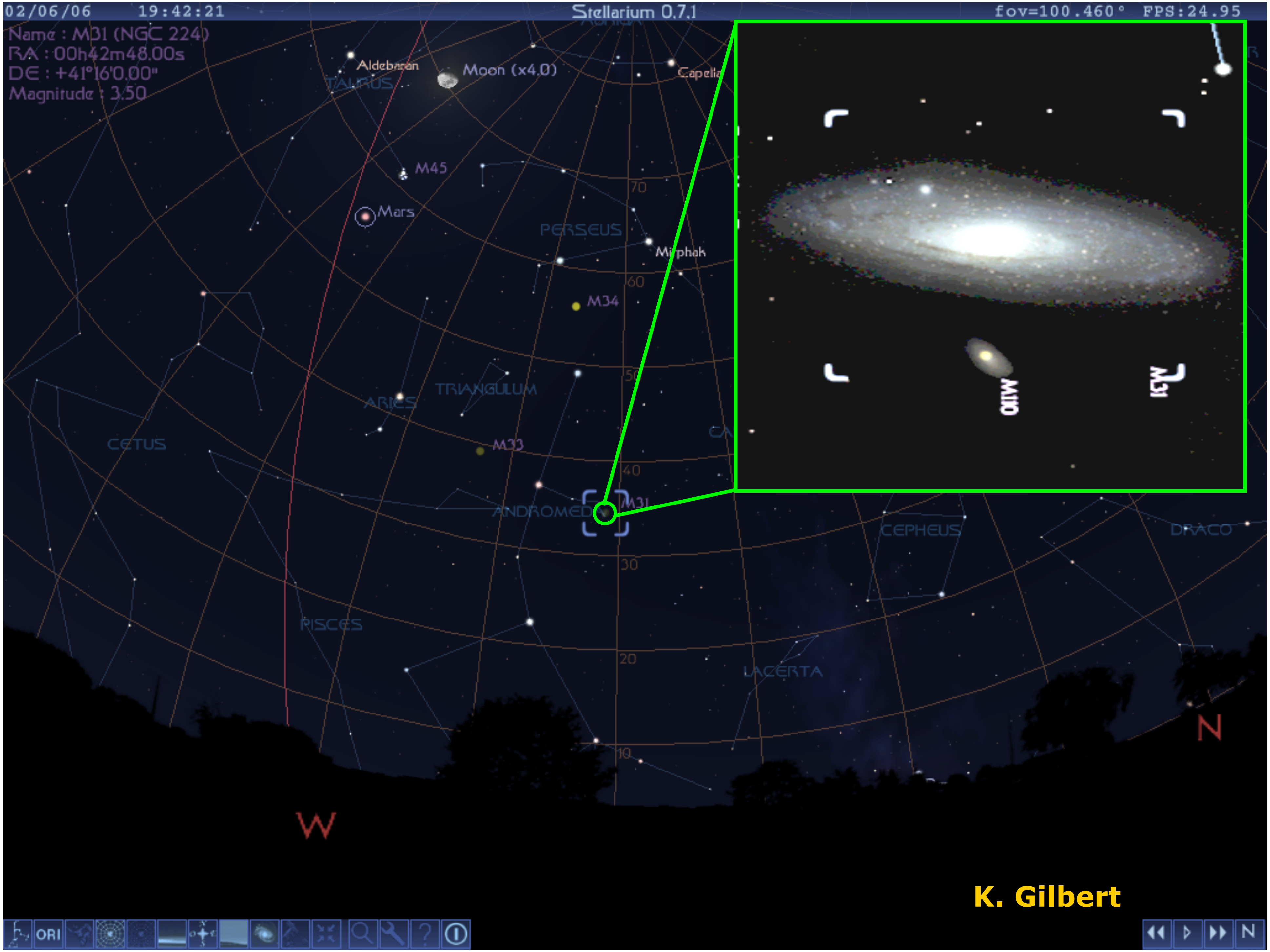
The Milky Way has $v_{\rm circ} = 200$ **km s**$^{-1}$, such that $R_{180} \approx 270$ **kpc.**

Nowadays, R_{200} is more conventional within ΛCDM

So we know something of the size and internal dynamics of a dark matter halo, but what about the internal structure?

Amazing fact:
The DM halo of M31 is tens of degrees in size as "seen" from Earth. See what follows.

Bland-Hawthorn (Bologna 2011)



02/06/06 19:42:21
Stellarium 0.7.1
fov=100.460° FPS:24.95
Name : M31 (NGC 224)
RA : 00h42m48.00s
DE : +41°16'0.00"
Magnitude : 3.50
Aldebaran
Moon (x4.0)
TAURUS
Capella
M45
Mars
PERSEUS
Mirphak
M34
TRIANGULUM
ARIES
CETUS
M33
ANDROMEDA
M31
CEPHEUS
DRACO
PISCES
LACERTA
M110
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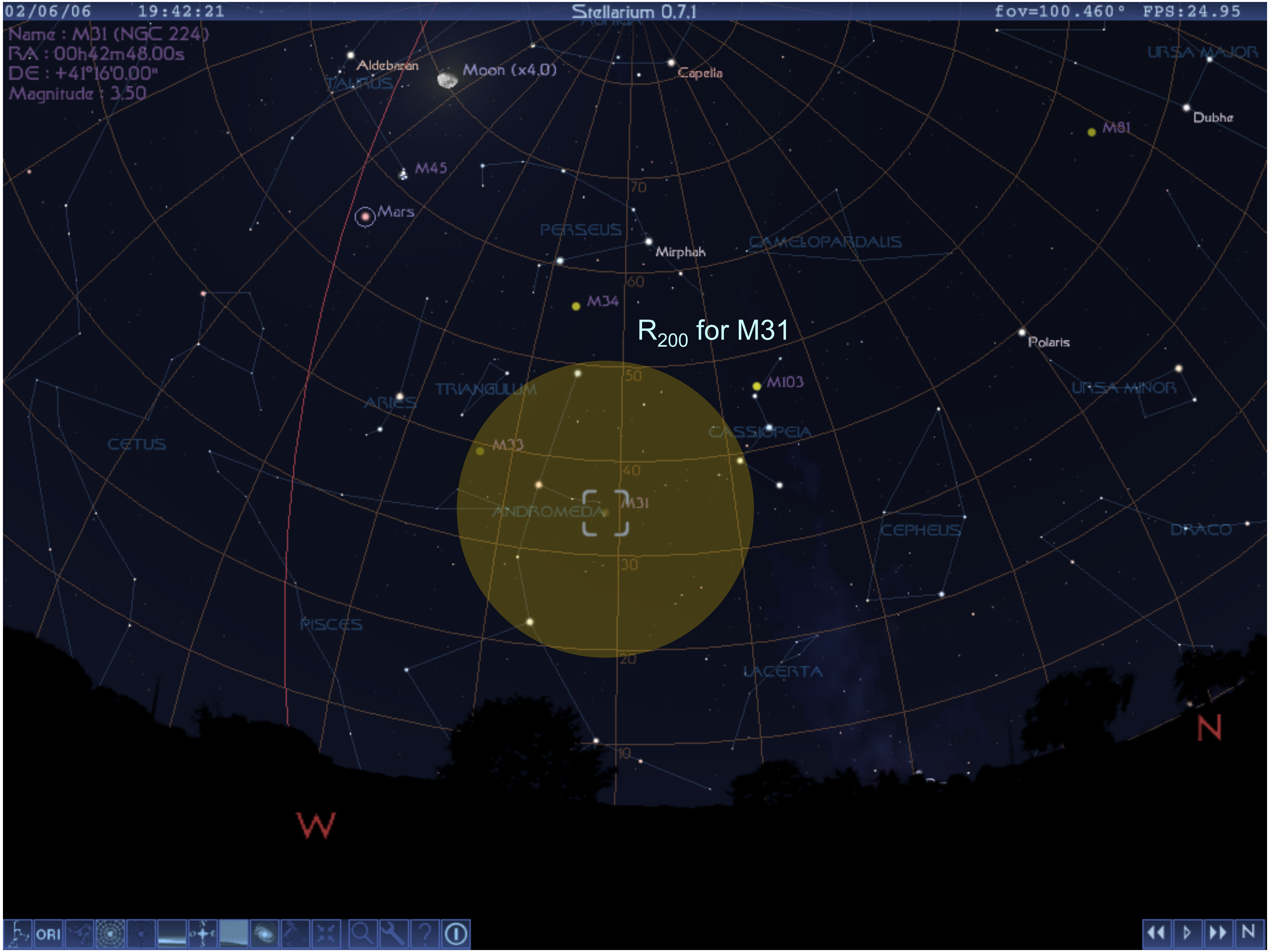

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R200 for M31
Polaris
M103
URSA MINOR
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ARIES
CASSIOPEIA
CETUS
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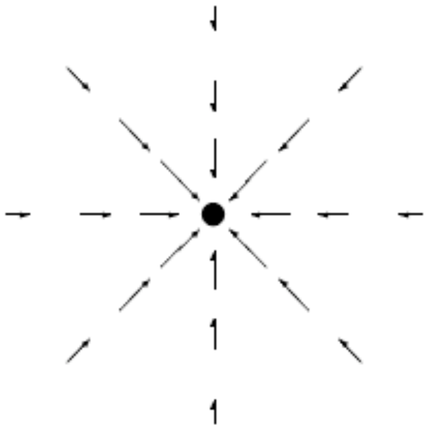
ON THE INFALL OF MATTER INTO CLUSTERS OF GALAXIES AND SOME EFFECTS ON THEIR EVOLUTION*

JAMES E. GUNN
Hale Observatories, California Institute of Technology, Carnegie Institution of Washington

AND

J. RICHARD GOTT III
Princeton University Observatory
Received 1972 January 24



THE GUNN-GOTT SPHERICAL ACCRETION MODEL (1972)

Consider a point-like seed of mass M_o in an otherwise uniform EdS background. At large radii, the mass will induce a peculiar acceleration

$$\ddot{R} = \frac{GM_o}{R^2} \qquad (16)$$

After a Hubble time, this will generate a peculiar infall velocity

$$\delta v \sim \ddot{R}t \sim \frac{GM_o}{HR^2} \qquad (17)$$

This kind of $\delta v \propto 1/R^2$ flow is "divergence free" such that there is no change in density at large distances. The flux of matter across a surface $=$ velocity $\times$ area which is independent of R.

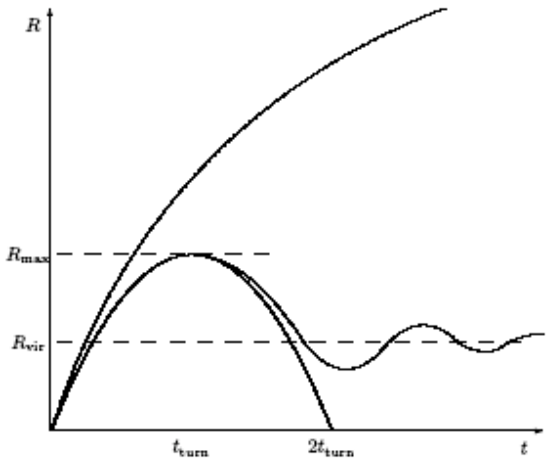
From (2) and (17), the amount of mass convected across a shell in a Hubble time is $\delta M \sim 4\pi R^2 \rho_o t\, \delta v \sim M_o$.

The only build-up is at the centre and this grows as $\delta M / M_o \propto a(t)$ as we have seen. This mass represents a density contrast of order unity at a physical radius R such that $\delta M \sim \frac{4}{3}\pi R^3 \rho_o$.

This is close to the turnaround radius,

$$R_{\rm turn} \sim (\delta M / \rho_o)^{1/3} \propto a^{4/3} \qquad (18)$$

As time marches on, progressively larger shells will turn around, collapse and virialize in some complicated way, shell crossing, etc.



Gunn & Gott trace the binding energy of shells and argue that the initial and final values must be comparable. Thus they show that

$$\delta\phi_{\rm initial} = GM_o/R \tag{19}$$

can be equated with

$$\delta\phi_{\rm final} = GM(R_{\rm final})/R_{\rm final} \tag{20}$$

such that

$$M(R_{\rm final}) \propto R_{\rm final}^{3/4} \tag{21}$$

The density within radius $R_{\rm final}$ is then

$$\rho_{\rm final} \sim M(R_{\rm final})/R_{\rm final}^3 \propto R_{\rm final}^{-9/4} \tag{22}$$

This is very close to an isothermal sphere, i.e. $\rho \propto R^{-2}$, to explain flat rotation curves and the distribution of galaxies in clusters.

M_o is a constant

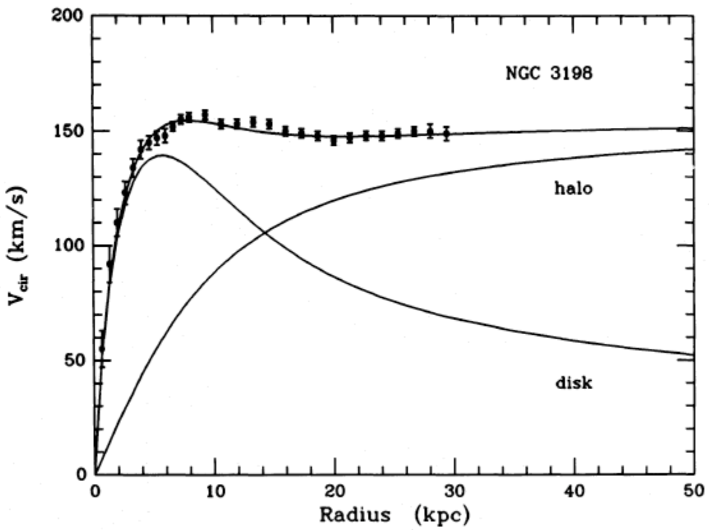
$R \propto M^{1/3}$ since ρ within infalling shells is a constant

$$\frac{GM}{R} = v^2(R) = const$$

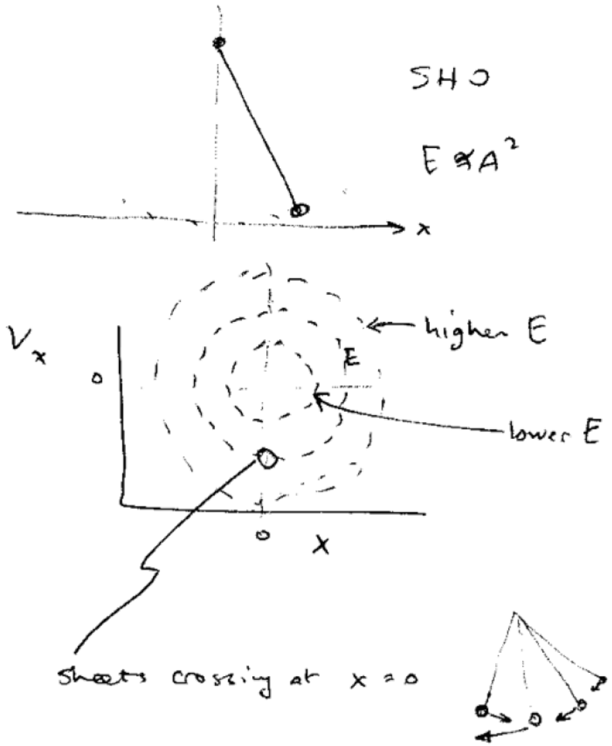
$$\rho R^2 = const$$

This is one of a much wider class of similarity solutions for collapse (e.g. Bertschinger 1984; Fillmore & Goldreich 1984)

DISTRIBUTION OF DARK MATTER IN NGC 3198



~N+1 Sheets problem : draw on board



Infinite parallel sheets experiment (1D kinematics)

N sheets start together at x=0, expand with Hubble flow, then collapse under mutual self gravity.

Simple equation of motion depends only on sheets either side of sheet ***i***

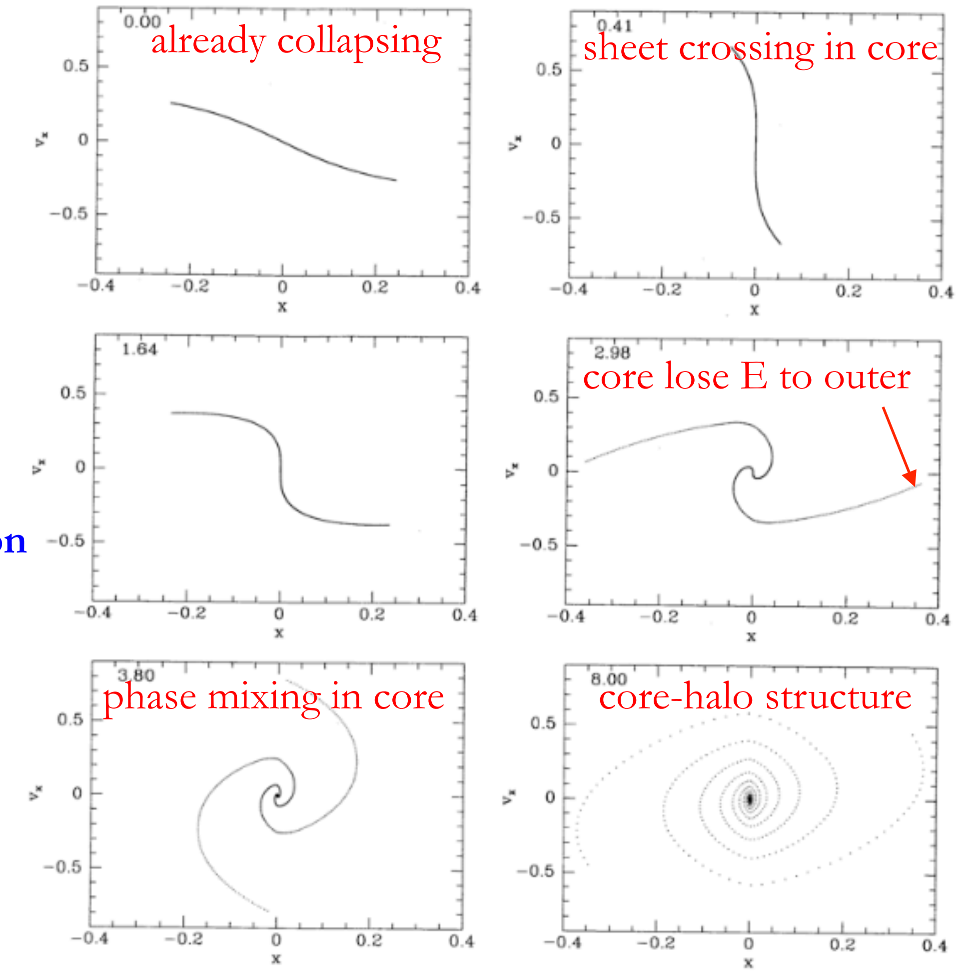
$$\ddot{x} = 2\pi G\mu(N_{+i} - N_{-i})$$

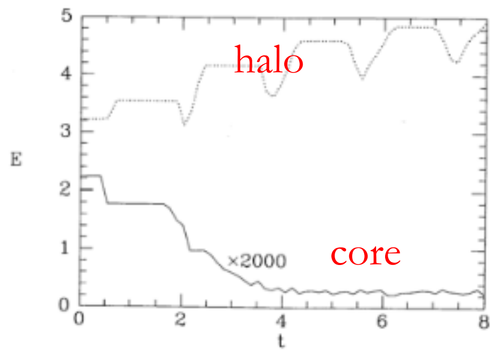
There is only a linear distance dependence and we can track sheets that have crossed.

The inner sheets have to climb out of a deeper well than when they fell in; the delayed outer sheets infall and climb out of a shallower well. This transfers energy to the outer sheets.

Virialization, phase mixing, violent relaxation
➔ core-halo structure

PHASE SPACE



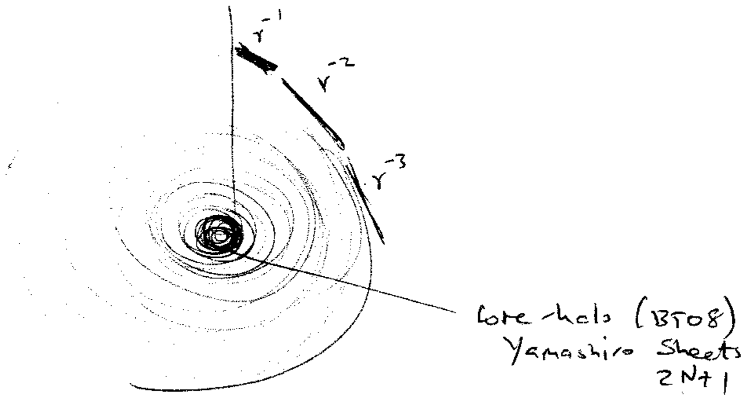


Yamashiro; BT08

DM: core, halo structure

3 different kinds:

- Burkert
- NFW
- Einasto



Need to stress that r^{-2} diverges mass

However, there's not sufficient time? to grow a virialized system beyond a certain radius

$\delta = 200$ for a galaxy.

Hence $r^{-2} \rightarrow r^{-3}$ reflects truncation imposed by infall time

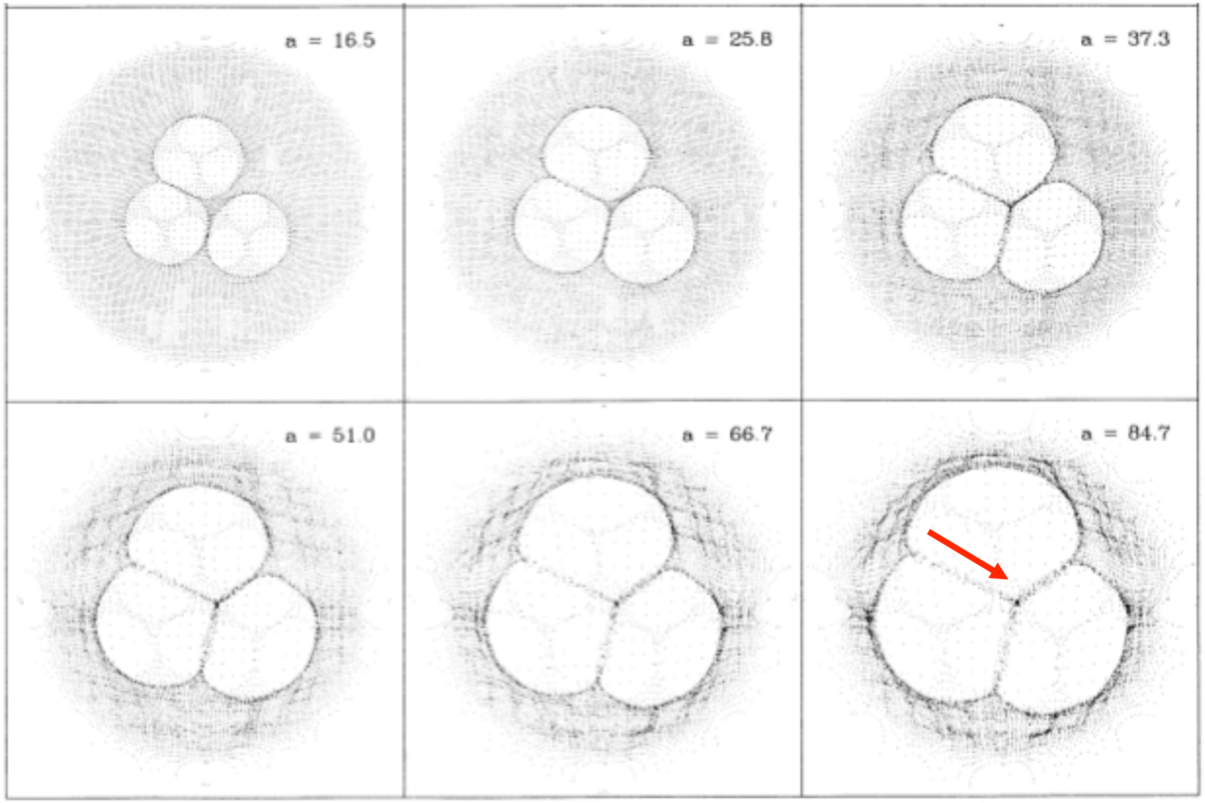
Evolution of voids

Underdense regions are not held back by internal gravity, and therefore expand faster than the universe as a whole. Smaller voids are swept aside.

They promote the rapid development of sheets and filaments. Flow takes place along these into the collapsed interstices.

In the 3rd lecture, we will see how this process may assist cold mode accretion.

In any event, our simple pictures of spherical or 1D sheet accretion must be modified.



Dubinski et al 1993

Note how the small voids are swept aside by the larger rapidly expanding voids.

520 *R. K. Sheth and R. van de Weygaert* 2004

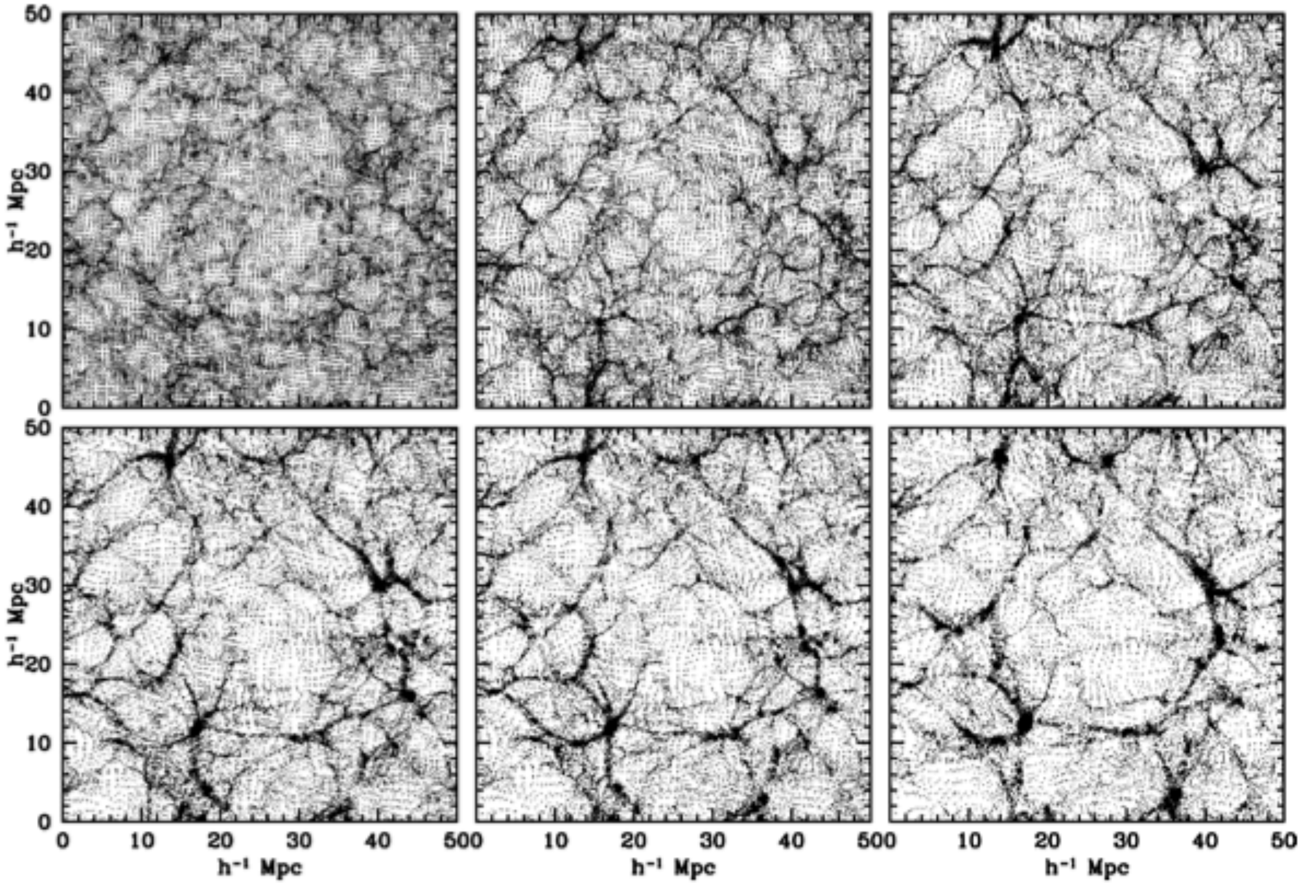


Figure 2. Void evolution. Six time-steps in the evolution of a void region in a 128^3 particle *N*-body simulation of structure formation in an SCDM model: top left to bottom right shows expansion factors $a_{exp} = 0.1$, 0.2, 0.3, 0.35, 0.4 and 0.5 (the present time has $a_{exp} = 1.0$). Initial conditions were defined such that they would focus in on a 3σ ($4\ h^{-1}$ Mpc) void, using a constrained random field code (van de Weygaert & Bertschinger 1996). The sequence shows the gradual development of a large void of diameter $\approx 25\ h^{-1}$ Mpc as the complex pattern of smaller voids and structures that had emerged within it at an earlier time merge with one another. This illustrates one aspect of the evolving void hierarchy: the *void-in-void* process.

Until now, we have been limited to spherical accretion. But as void evolution demonstrates, this cannot be true in general.

Elliptic perturbations have been discussed (Peacock & Heavens 95; Sheth & Tormen 1999; Jenkins et al 2001; cf. Jang-Condell & Hernquist 2001).

Zeldovich (1970) showed how collapse can be followed into the non-linear regime. Collapse happens fastest along the shortest axis (Lin et al 1965).

Gravitational Instability: An Approximate Theory for Large Density Perturbations

YA. B. ZELDOVICH
Institute of Applied Mathematics, Moscow

Received September 19, 1969

An approximate solution is given for the problem of the growth of perturbations during the expansion of matter without pressure. The solution is qualitatively correct even when the perturbations are not small. Infinite density is first obtained on disc-like surfaces by unilateral compression.

The following layers are compressed first adiabatically and then by a shock wave. Physical conditions in the compressed matter are analysed.

The power of a concise abstract.

Hello Malcolm,

I've reached the end of a graduate lecture series here in Bologna where I taught structure formation from inflation down to the scales of galaxies.

I happened to pick up a copy of your very nice revised ed on galaxy formation, and useful it was too. For example, I consulted you (and others like Cooray & Sheth) on Press-Schechter approx.

As a fan of your writing style, I thought I would point out what I think was a possible missed opportunity in your section on the Zeldovich approximation.

To my mind, and I am willing to be wrong, the Lagrangian comoving frame of Zeldovich is inertial, and one can define a new timescale ~ a(t) such that dr/dt ~ constant. So now motion is purely ballistic (kinematic) rather than dynamic. That's why Zeldovich sims always look like hot dark matter sims, for example.

So in practical terms, at any given epoch, I can generate large-scale structure using the known power spectrum, correlation fn, sigma_8, beta parameter, purely as data points in a 3D volume. And it looks much like an N-body sim. But now, I want to generate redshift diagrams, let's say, with fingers of God, Kaiser flattening.

So I can do that by moving to the comoving frame, applying some local smoothing, and then determining local density gradients in 3-space from which I get the 3-space ballistic motion. And not a single N-body calculation in sight. I like your comment about the cleverness of b(t) ensuring all perturbations growing at the same rate.... Yes.

Of course, you're covering a huge topic that it's not possible to do justice to everything, and you leave scope for students to explore further.

My best wishes to your wife whom I remember as a very interesting lunch partner years ago!

Best wishes, Joss.

p.s. I like your email address; it sounds like a broomstick from Hogwarts School.

Zeldovich approximation (1970)

Perturbations are following in Lagrangian coordinates **r** (**x** are proper coordinates)

$$\mathbf{x} = a(t)\mathbf{r} + b(t)\mathbf{p}(\mathbf{r})$$

p(r) describes the deviations as viewed from a comoving observer and a(t) describes the background expansion

In the reference frame of an ellipsoidal perturbation, the deformation tensor is

$$D = \begin{bmatrix} a(t) - \alpha b(t) & 0 & 0 \\ 0 & a(t) - \beta b(t) & 0 \\ 0 & 0 & a(t) - \gamma b(t) \end{bmatrix}$$

The density in the vicinity of any particle is

mean universal density

$$\varrho[a(t) - \alpha b(t)][a(t) - \beta b(t)][a(t) - \gamma b(t)] = \bar{\varrho} a^3(t)$$

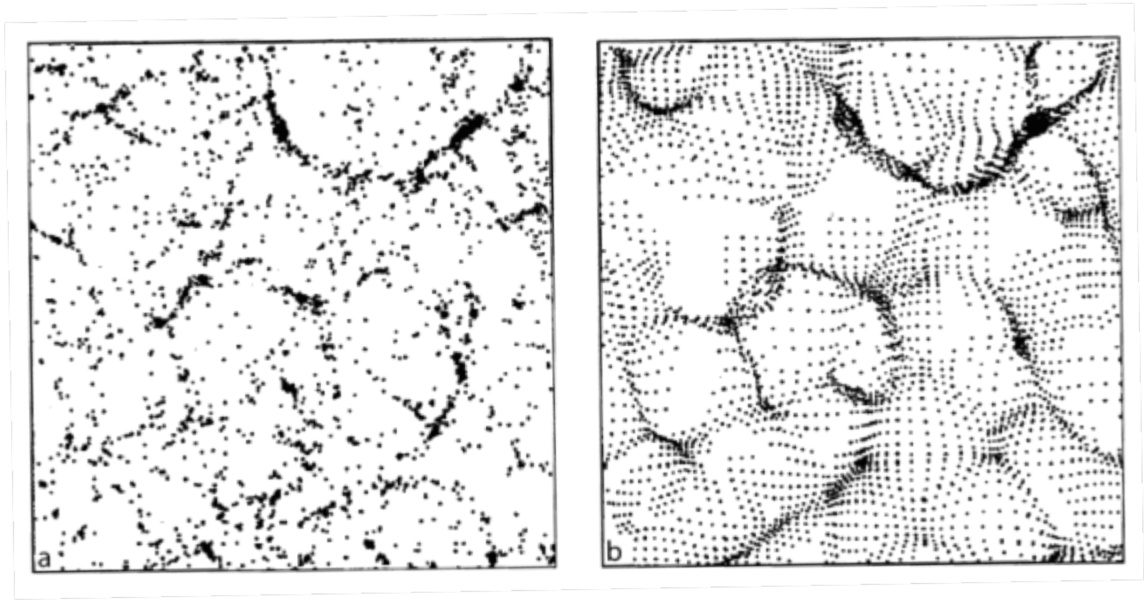
mass conserved

α, β, γ vary from point to point depending on local density variations, but a(t), b(t) are the same for all particles, **i.e. perturbations grow at the same rate everywhere!**
They can have different shapes but direction of motion depends on (smooth) density depending on ellipsoidal kernel you've chosen to determine α, β, γ .

Once one of the diagonal terms has gone to zero, a pancake is formed and we have infinite local density (but finite surface density).

The solution then breaks down, but it does a good job.

The coeffs α, β, γ have to be evaluated at every point. The ZA particles move purely under gravity, i.e. no pressure.



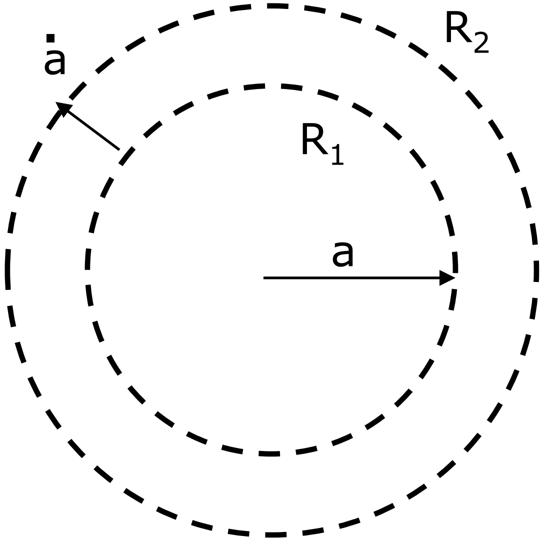
Left: CDM, full 3D gravity. **Right:** Zeldovich approx. based entirely on ballistic motion dep. on local density

But note something very interesting about motion within comoving coordinates which is what Zeldovich spotted.

Gravitational Instability: An Approximate Theory for Large Density Perturbations

YA. B. ZELDOVICH
Institute of Applied Mathematics, Moscow

1970



For growing perturbations in EdS, the velocity difference between particles at R_1 and R_2 is

$$- \delta v = H\delta R - R\delta H = \frac{2}{3} H R \frac{\delta\rho}{\rho}$$

response to Hubble flow

response to perturbation

$$\propto 1/v \propto \sqrt{R} \propto t^{1/3} \qquad (23)$$

$$\frac{\delta H}{H} = -\frac{1}{3}\frac{\delta\rho}{\rho} \propto t^{2/3}$$

In fact, we get the same time dependence in linear theory (i.e. kinematic, no gravity), such that

$$\vec{v}(\vec{r}, t) = (t/t_o)^{1/3}\, \vec{v}_o(\vec{r}) \qquad (24)$$

where $\vec{r}$ is a comoving spatial coordinate and $\vec{v}_o$ is the peculiar velocity at some initial time. The peculiar velocity field just grows with time at the same rate $\vec{v} \propto t^{1/3}$ at all points in space.

The physical displacement of a particle in time dt is $d\vec{x} = \vec{v}\,dt$ so the comoving displacement is

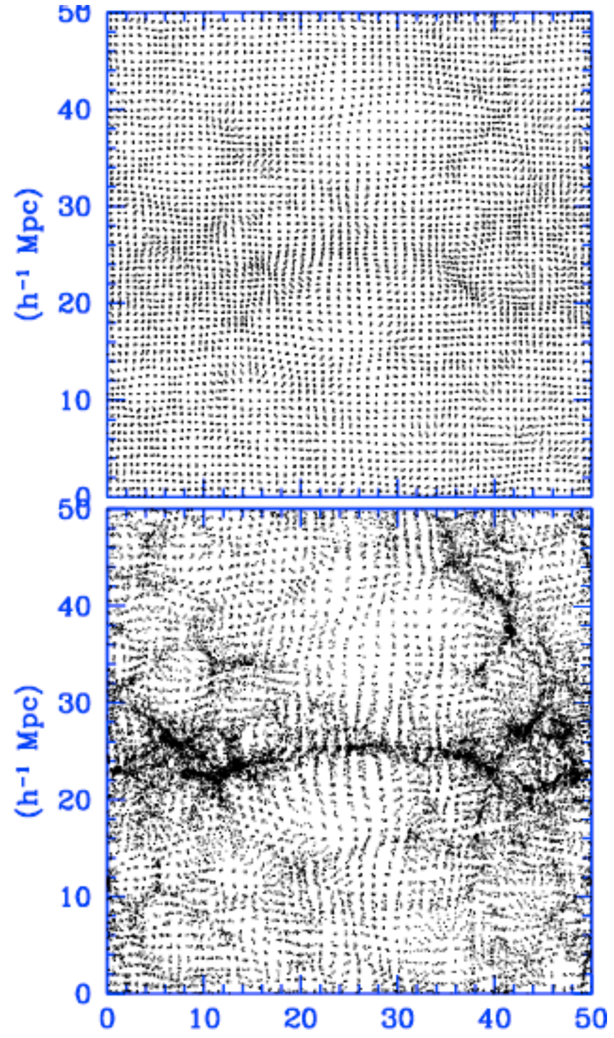
$$d\vec{r} = \frac{d\vec{x}}{a} = \frac{\vec{v}}{a}dt = \vec{u}\,dt \tag{25}$$

where the comoving peculiar velocity is $\vec{u} = \vec{v}/a$.

The rate of change of comoving position with a is then

$$\begin{aligned} \frac{d\vec{r}}{da} &= \frac{\vec{v}}{a}\frac{dt}{da} \\ &\propto t \end{aligned} \tag{26}$$

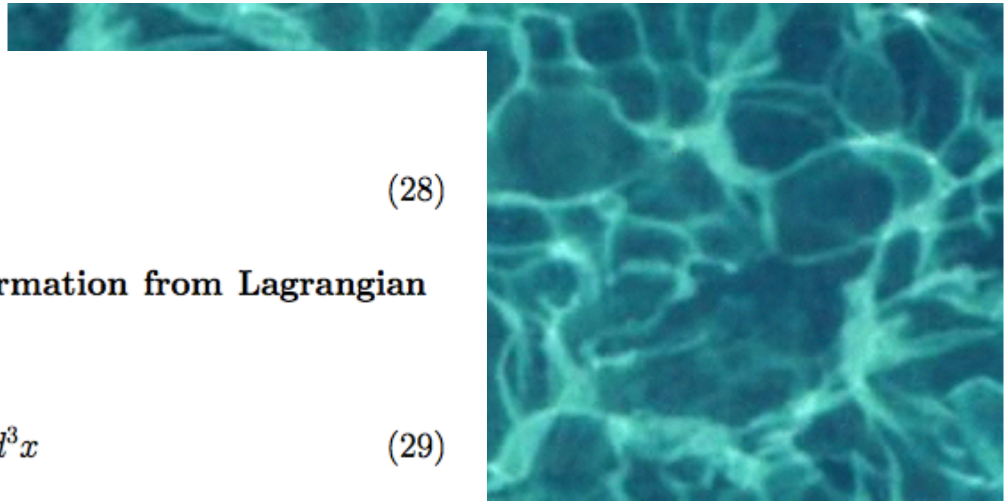
If we define a new time $\tau \propto a$ such that $d\vec{r}/d\tau$ = constant, then particles move ballistically (i.e. unaccelerated) in comoving coordinate space.



Zel'dovich's approximation is to assume that this motion continues into the non-linear regime.

The comoving frame has no forces acting! It's just kinematic, not dynamic, motion depending on local density field!

So ZA simulations have particles shooting through structures unaccelerated in comoving coordinates, hence often resemble **hot dark matter** simulations (e.g. 1 keV particles).



Until caustics form, the density is

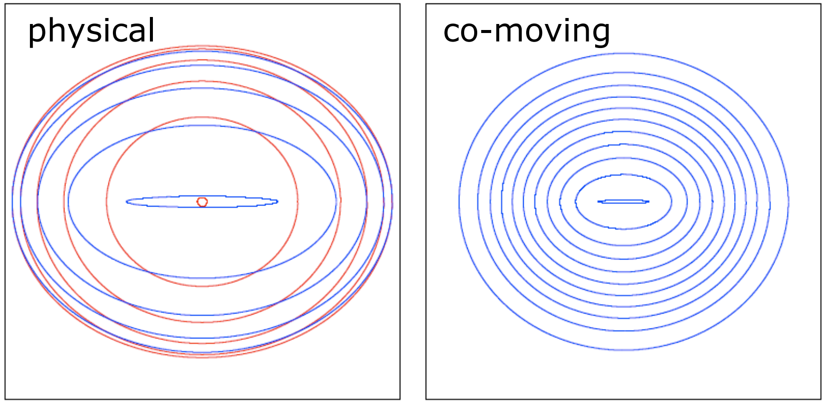
$$\rho \propto \left|\frac{\partial x_i}{\partial r_j}\right|^{-1} \tag{28}$$

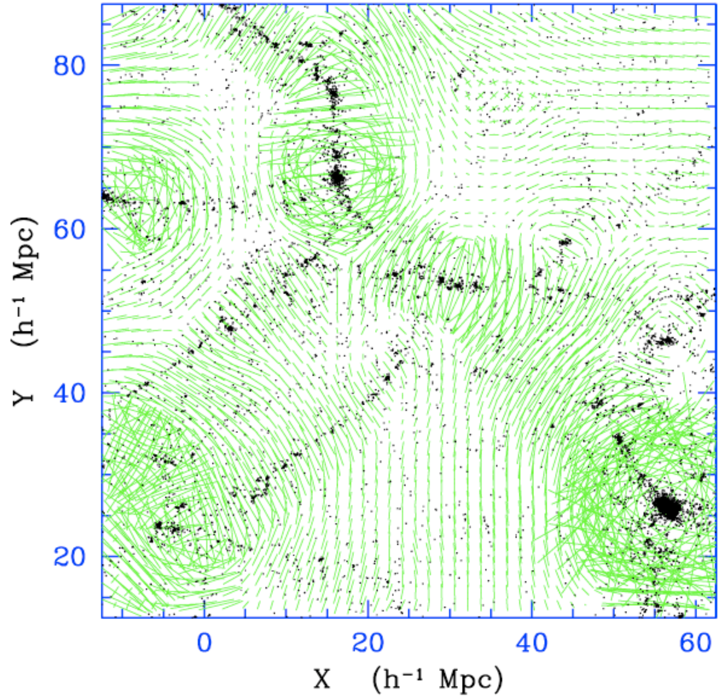
where $|\partial\vec{x}/\partial\vec{r}|$ is the Jacobian of transformation from Lagrangian to Eulerian, i.e. mass conservation,

$$dM = \rho_L d^3 r = \rho_E d^3 x \tag{29}$$

The transformation leads to a Lagrangian mapping resulting in the formation of caustics (i.e. infinitely dense regions). We can write the actual comoving (Eulerian) coordinate $\vec{x}$ as a function of the initial (Lagrangian) coordinate $\vec{r}$ as

$$\vec{x}(\vec{r}) = \vec{r} + \tau\vec{u}(\vec{r}) \tag{27}$$





Now from (27)

$$\frac{\partial x_i}{\partial r_j} = \delta_{ij} + \tau \, \frac{\partial u_i}{\partial r_j}$$

so we can write

$$\rho \propto \frac{1}{(1+\tau\lambda_1)(1+\tau\lambda_2)(1+\tau\lambda_3)}$$

where λ_i are eigenvalues of the deformation tensor $D_{ij} = \partial u_i/\partial r_j$.

ZEL'DOVICH PANCAKES:

If, say, λ_1 goes negative, the density of a small comoving volume of matter will collapse to infinite density along this axis in a time $\tau = -1/\lambda_1$. These pancakes grow rapidly and intersect to form a cellular network of walls and filaments. The matter in these regions drain into nodes (interstices) where all three eigenvalues go negative.

BURGER's EQUATION:

Unaccelerated motion after shell crossing is clearly unrealistic. A useful modification is for particles to stick together when they cross. This is also unrealistic but receives some justification in a universe where the sheet is being stretched by adiabatic expansion.

If the sheet thickness is T and surface density Σ, the acceleration of a particle at the surface is $\ddot{r} \sim G\Sigma$ and

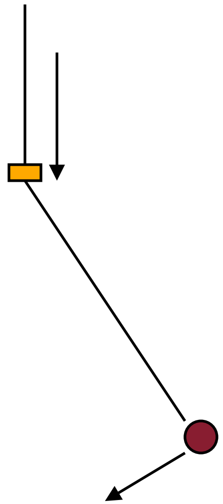
$$\omega \sim \sqrt{\frac{\ddot{r}}{T}} \sim \sqrt{\frac{G\Sigma}{T}} \tag{32}$$

But for a sheet expanding in the Hubble flow, $\Sigma \propto 1/a^2$. We can apply the law of adiabatic invariance $r = A\cos(\omega t)$ with amplitude $A \propto 1/\sqrt{\omega}$ and $A \approx T$, the sheet thickness must evolve as

$$T \propto a^{2/3} \tag{33}$$

This increases with time but is slower than $a(t)$, so the sheet does indeed become thin.

Brilliant insight! Solvay 1920?



Interestingly, it is possible to compute the non-linear P(k) from the initial power spectrum within Zel'dovich's approximation, and derive the build up of angular momentum.

Interesting twist on void evolution and Zeldovich pancakes…

Rigging dark halos: why is hierarchical galaxy formation consistent with the inside-out build-up of thin discs?

C. Pichon[1,2,3], D. Pogosyan[4], T. Kimm[2], A. Slyz[2], J. Devriendt[2,3] and Y. Dubois[2]

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August 17, 2011

ABSTRACT

State-of-the-art hydrodynamical simulations show that gas inflow through the virial sphere of dark matter halos is focused (i.e. has a preferred inflow direction), consistent (i.e. its orientation is steady in time) and amplified (i.e. the amplitude of its advected specific angular momentum increases with time). We explain this to be a consequence of the dynamics of the cosmic web within the neighbourhood of the halo, which produces steady, angular momentum rich, filamentary inflow of cold gas. On large scales, the dynamics within neighbouring patches drives matter out of the surrounding voids, into walls and filaments before it finally gets accreted onto virialised dark matter halos. As these walls/filaments constitute the boundaries of asymmetric voids, they acquire a net transverse motion, which explains the angular momentum rich nature of the later infall which comes from further away. We conjecture that this large-scale driven consistency explains why cold flows are so efficient at building up high redshift thin discs from the inside out.

10 *C. Pichon, D. Pogosyan, T. Kimm, A. Slyz, J. Devriendt a*

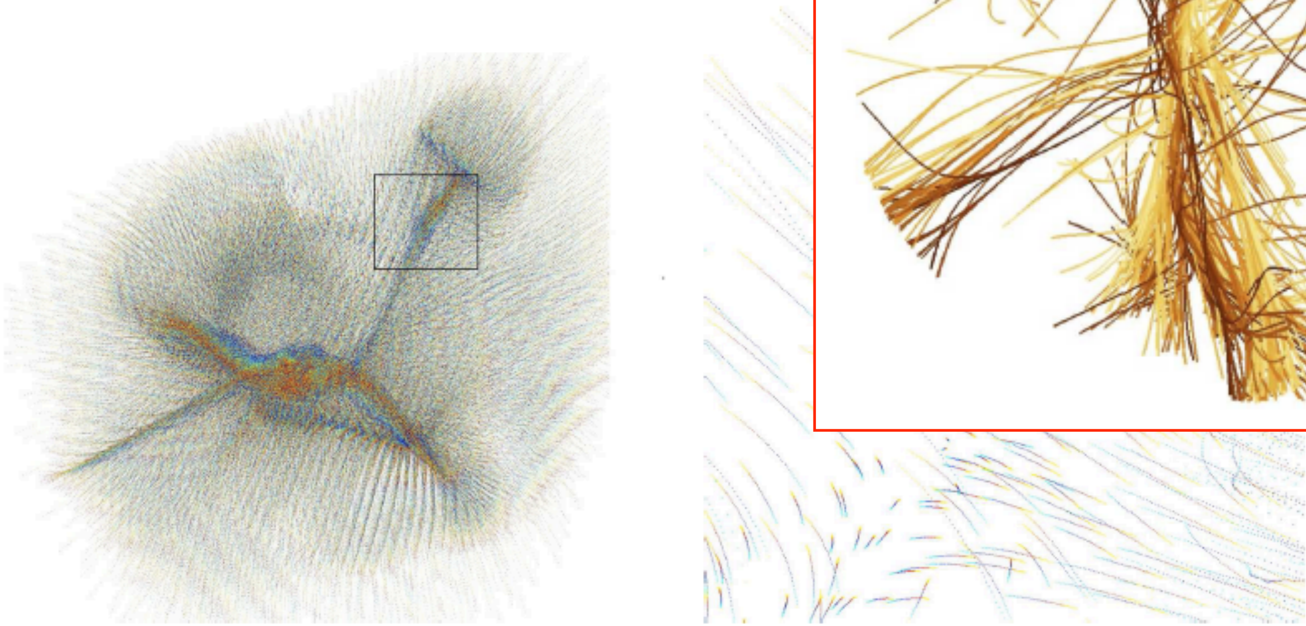

Figure 12. *Left:* example of trajectories of dark matter particles, colour coded by redshift from red (high redshift) to blue (low redshift). These trails first converge towards the filaments and then along the filaments towards the central halo: their motion account for both the orbital angular momentum of the filamentary flow and the spin of satellites formed within those filaments. *Right:* a zoom of the left panel corresponding to the shell crossing between two walls leading to the formation of the north east filament. Some of the angular momentum lost in the transverse motion is given to the spin of structures forming within that filament; the rest is converted into the angular momentum of the filament. The corresponding animation is available for download at `http://www.iap.fr/users/pichon/rig/`.